How to set good examination questions?

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Outline

1 Introduction

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1. Introduction
2. Examination as a summative assessment

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3. Assessment and assessment objectives

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5. Some criteria for good mathematics problems
6. Attributes of a good test item
7. Core skills

Highlights

In today's talk, I briefly touched on:

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2. Problem posing as framework for item design
3. Specific attributes associated to good test items
4. Core skills invoked in setting good examination questions

Beauty pageant

Activity 1 (5 min)

Which of the following test items would be more likely to appear as a (or part of a) question in an 'A' level mathematics examination paper? Justify your choice.

Beauty pageant

Item 1

Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{\ln\left(\frac{1}{3+\sqrt{x}}\right)}\right) = \frac{-1}{2\sqrt{x}\left(3+\sqrt{x}\right)^2}.$$

Beauty pageant

Item 2

Explain why the equation

$$(z+2i)^6 = z^6$$

has five roots.

Beauty pageant

Item 3

Find the numerical value of the derivative of $\lg(x^3)$ when $x = 10$.

Beauty pageant

Item 4

The number of Bye-Bye Panda due to birth and death in a protected bamboo environment is modelled by the differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}t} = kS(k - S),$$

where S is the number of pandas at time t years and k is a constant. Given that $k > S$, show that the differential equation has general solution

$$S = \frac{kA}{A + e^{-k^2 t}},$$

where A is an arbitrary constant.

Beauty pageant

Item 5

A company buys $p\%$ of its electronic components from supplier A and the remaining $(100-p)\%$ from supplier B. The probability that a randomly chosen component supplied by A is faulty is 0.05. The probability that a randomly chosen component supplied by B is faulty is 0.03.

For a general value of p, the probability that a randomly chosen component that is faulty was supplied by A is denoted by $\mathrm{f}(p)$.

Show that $\mathrm{f}(p) = \dfrac{0.05p}{0.02p+3}$. Prove by differentiation that f is an increasing function for $0 \leq p \leq 100$, and explain what this statement means in the context of the question.

Beauty pageant

Item 6

A Geography student is studying the distribution of bullfrogs in a large rural field where there is an average of 500 bullfrogs per 400 km^2. One part of the field is identified and divided into 50 equal squares, each with sides measuring 2 km. The student wishes to model the distribution of bullfrogs in each square by using the Poisson distribution with mean λ.

(i) State the value of λ and one assumption made in using the Poisson distribution.

(v) The student suggests using the same model on another rural field in another country. Comment on the suitability of the model in this situation.

Discussion

Sharing

Please share about your discussion concerning these six items, in the context of the question posed earlier.

Sense of beauty

Maxim

Deep-seeded in us, teachers, is an intrinsic sense of what a good examination question should be like.

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(Black, P.J. William, D. (1998), *Inside the Black Box: Raising standards through classroom assessment.* Kings College, London.)

Differentiated assessment based on purpose

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- Assessment for learning

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- Assessment as learning

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- Assessment for learning
- Assessment as learning
- Assessment of learning

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- compares one student's achievement with standards results can be communicated to the student and parents
- occurs at the end of the learning unit

Evaluation

- judgment made on the basis of a students performance

Formative assessment

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- made to determine a student's knowledge and skills, including learning gaps as they progress through a unit of study

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- used to inform instruction and guide learning
- occurs during the course of a unit of study
- makes up the subsequent phase of assessment for learning

Summative assessment

Assessment

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Summative assessment

Assessment

- that is made at the end of a unit of study to determine the level of understanding the student has achieved
- includes a mark or grade against an expected standard

Summative assessment

Examination is a summative assessment!

Formative assessment

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In this sense,

Examination can have its formative effects!

From theory to practice

Question

How do we actually implement assessment?

From theory to practice

Answer.

Much is dependent on what you want to assess.

From theory to practice

Answer.

Much is dependent on what you want to assess.

This then depends on what you want the students/pupils to learn or acquire.

Assessment objective

Remember

You need to abide by the Ministry's assessment objectives.

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So you need to align your test design to meet these objectives as well.

Assessment objectives

There are three levels of assessment objectives for the examination.

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Assessment objectives

The assessment will test candidates' abilities to:

AO1 understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphic calculators;

AO2 reason and communicate mathematically through writing mathematical explanation, arguments and proofs, and inferences;

AO3 solve unfamiliar problems, translate common realistic contexts into mathematics; interpret and evaluate mathematical results, and use the results to make predictions, or comment on the context.

What is problem solving?

Problem solving inevitably involves:

- Understand the problem

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- Devise a plan
- Implement the plan
- Evaluate the solution

What is problem posing?

Problem posing is centrally about asking sensible and natural (mathematical) questions.

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- Good problems engage students actively in learning mathematics
- Asking and solving good problems pushes research frontiers
- Ensures the very survival of mathematics as a subject
- Enables the implementation of interesting, accurate and useful test-items

Bloom's taxonomy

In connection with setting Higher-Order-Thinking questions, do not forget:

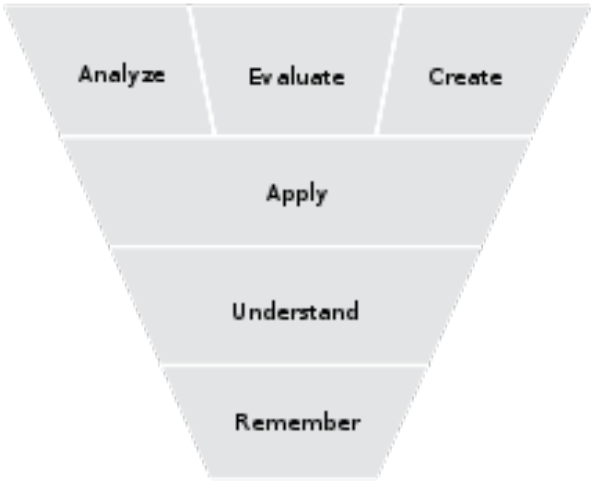

Figure: Bloom's taxonomy

What does good problem posing involve?

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- is NOT an easy task.
- requires one to understand thoroughly the related content materials.
- requires one to analyze and reprocess the existing information or problems.

Good problem posing

We set out to systematically examine various methods of posing mathematical problems.

Criteria for good math problems

A good mathematics problem possesses one or more of the following characteristics:

Criteria for good math problems

1. It is clearly stated and easy to understand.

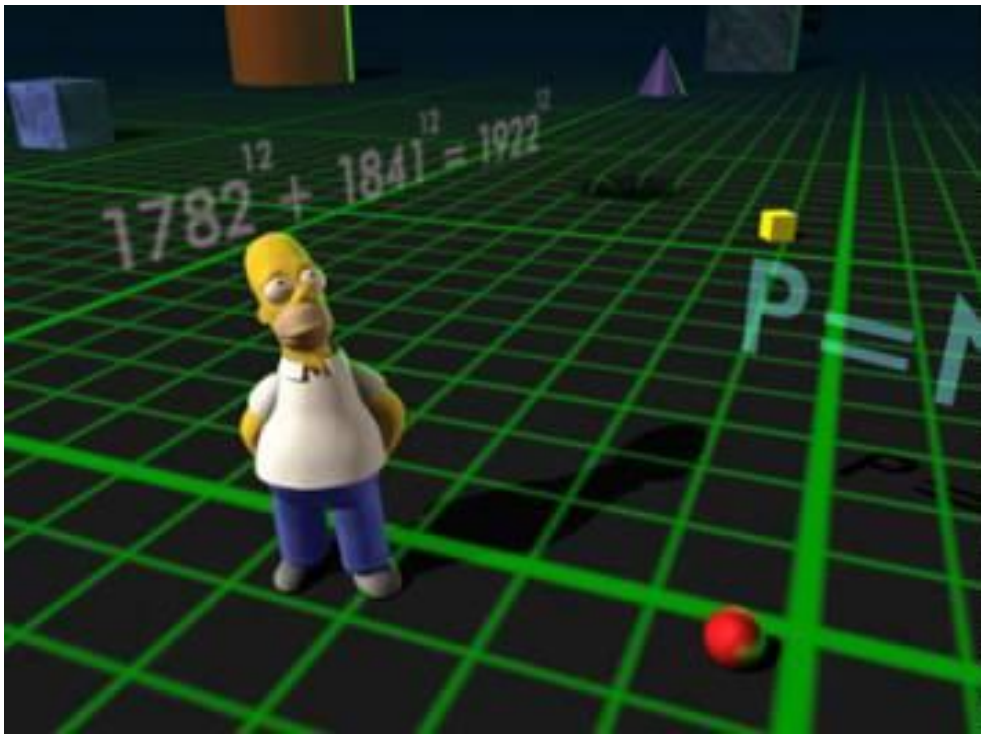
Criteria for good math problems

A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street. – David Hilbert (1862 – 1943)

Criteria for good math problems

2. It is difficult but not completely inaccessible.

Criteria for good math problems



Criteria for good math problems

Problem

For integers $n > 2$ the equation

$$a^n + b^n = c^n$$

cannot be solved with positive integers a, b and c.

Criteria for good math problems

3. Its solution not only enhances our understanding of existing paradigm, but also leads us to solutions of other problems and to the discovery of new tools or theories.

Criteria for good math problems

Of course, the above list is far from exhaustive.

Activity 2 (2 min)

In your small groups of 3,

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Activity 2 (2 min)

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1. can you add at least two more items to this?
2. share with us these additional criteria?

Brown and Walter's method

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What-If-Not scheme

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4. Select some of these attributes and ask the important question:

   What might the outcome be if these selected attributes were replaced by different ones?

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The WIN scheme constitutes of the following sequence of actions:

1. Observing an existing result.
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3. Breaking these attributes down into more subtle ones (if necessary).
4. Select some of these attributes and ask the important question:

   What might the outcome be if these selected attributes were replaced by different ones?
5. Repeat this above procedure as many times as possible.

What-If-Not

Question

Let's make pairs and let's try to find ways to win the following game:

Game 1: Twenty units of yellow coloured tubes are connected with one unit of black tube. Two students, based on the rock-scissors-paper method, determine who starts first. Then, the students take turns to take from one to three cubes. The student who takes the last cube is the winner.

What-If-Not

Activity 3 (3 min)

Let's change the game in whatever way you'd like (e.g., to modify the game after seeing the original game) and try to investigate a corresponding strategy.

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What-If-Not

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2. Two students will take turn to take at least one and up to three cube of he same colours. They take cubes from either right or left side.
3. The student who takes the black cube will be the loser.

What-If-Not

9740/N09/II/Q10(ii,iii)

A company supplies sugar in small packets. The mass of sugar in one packet is denoted by X grams. The masses of a random sample of 9 packets are summarised by

$$\sum x = 86.4, \quad \sum x^2 = 835.92.$$

The mean mass of sugar in a packet is claimed to be 10 grams. The company directors want to know whether the sample indicates that this claim is incorrect.

What-If-Not

9740/N09/II/Q10(ii,iii)

(ii) Stating a necessary assumption, carry out a t-test at 5% significance level. Explain why the Central Limit Theorem does not apply in this context.

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9740/N09/II/Q10(ii,iii)

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(iii) Suppose now that the population variance of X is known, and the assumption made in part (ii) is still valid. What change would there be in carrying out the test?

Inverse problems

Another general method for posing mathematics problems is to formulate the inverse problem of a given one.

Inverse problems

Roughly speaking,

if problem A is ...

Given $p_1, p_2, \ldots, p_n$, find $q_1, q_2, \ldots, q_m$,

Inverse problems

Roughly speaking,

if problem A is ...

Given $p_1,\ p_2, \ldots, p_n$, find $q_1,\ q_2, \ldots, q_m$,

then the problem B

Given $q_1,\ q_2, \ldots, q_m$ and $p_1,\ p_2, \ldots, p_k$, find $p_{k+1},\ p_{k+2}, \ldots, p_n$.

may be regarded as an inverse problem of **A**.

Inverse problems

Prelim item

Old MacDonald keeps hens, ducks, cats and pigs on his farm. One day, he counted a total of 53 heads and 186 legs for these animals. He has also recorded the amount of food consumed and the waste produced by each animal on that particular day. The findings are recorded in the following table.

Inverse problems

Prelim item

	Hen	Duck	Cat	Pig	Total
Amt of food consumed by each animal (kg)	0.15	0.33	0.29	?	21.89
Amt of waste produced	0.034	0.076	0.087	0.12	4.543

Assume each of the hens consumes the same amount of food and produces the same amount of waste. This assumption applies to the ducks, cats and the pigs as well.

Inverse problems

Prelim item

He was unable to record the amount of food consumed by each pig as the pigs ate too quickly. Given that there are 10 hens on the farm, determine the amount of food consumed by each pig.

A meaningful quote

"The process of selecting item ideas goes on simultaneously with the process of inventing them. Skill in item writing depends not only upon prolific inventiveness but also upon discriminating judgment in the selection. In selecting item ideas, the writer must consider their appropriateness, importance, and probable discriminating ability." – Robert Ebel

Writing guidelines

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2. Item Structure: Layout, question stems & answer options

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Writing guidelines

1. Item Composition: Alignment & focus, cognitive level
2. Item Structure: Layout, question stems & answer options
3. Item Mechanics: Syntax & usage
4. Bias

Rule Number 1

"Only ask people questions they can answer."

– Van Mondfrans, 1985

Equations and inequalities

Prelim item

A certain economy of three industries that produce three products: X, Y and Z. In a given week, it produces $\$x$ worth of X, $\$y$ of Y and $\$z$ of Z. The production of $\$1$ worth of any particular product requires the use of some of its own product, and some of the two other products, as shown in the table below.

Equations and inequalities

Prelim item

The production of \$1 worth of ...

Requires the use of	X	Y	Z
X	\$0.10	\$0.25	\$0.20
Y	\$0.30	\$0.40	\$0.50
Z	\$0.10	\$0.15	\$0.10

Equations and inequalities

Prelim item

For instance, to produce \$1 worth of X, one needs to use \$0.10 of X, \$0.30 of Y and \$0.10 of Z.
Other than providing internally for the three industries, each industry must also meet its external demand. During the given week, the economy has an external demand of \$50,000 worth of X, \$75,000 worth of Y and \$125,000 of Z.

(i) By considering how the industry for X meets both its internal provision and external demand, show that

$$0.9x - 0.25y - 0.2z = 50000.$$

Leonteif Model

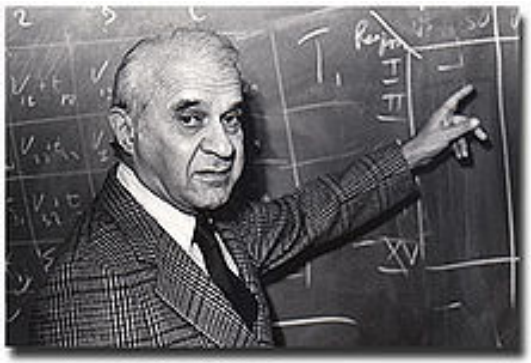

Figure: Wassily Leontief (1905–1999)

Sequence and series

Prelim item

A sequence of numbers, x_n, satisfy the relation

$$x_{n+1} = \frac{1 - x_n}{x_n - 4}, \text{ for } n \in \mathbb{Z}^+.$$

(i) If the sequence converges to a number L, show that L satisfies the equation $L^2 - 3L - 1 = 0$.

(iv) Show that $x_{n+1} - L = \frac{(L+1)(L-x_n)}{x_n-4}$. Deduce that

$$|x_{n+1} - L| < \frac{1}{5}|x_n - L| \text{ if } x_n < L.$$

Relevance

Test items should be structured so that a relevant (non-trivial) result emerges.

Maclaurin's series

9740/N08/II/Q1

Let $f(x) = e^x \sin x$.

(i) Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$.

Maclaurin's series

9740/N08/II/Q1

Let $\mathrm{f}(x) = e^x \sin x$.

(i) Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$.

(ii) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x, up to an including the term in x^3.

Denote the answer to part (ii) by $\mathrm{g}(x)$.

(iii) On the same diagram as in part (i), sketch the graph of $y = g(x)$. Label the two graphs clearly.

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(iii) On the same diagram as in part (i), sketch the graph of $y = g(x)$. Label the two graphs clearly.

(iv) Find, for $-3 \leq x \leq 3$, the set of values of x for which the value of $g(x)$ is within ± 0.5 of the value of $\mathrm{f}(x)$.

Sequence and series

Prelim item (H2)

A sequence of positive real numbers $x_1, x_2, x_3, \ldots$ satisfies the recurrence relation

$$x_{n+1} = \frac{1}{e^{x_n}}, \quad n \geq 1.$$

(ii) Given that $x_1 < 1$, show that $\sum_{n=1}^{N} x_n < N$, where N is a constant.

Correctness and rigour

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Correctness and rigour

Test items must be vetted and verified over and over again to be

1. mathematically sound,
2. correct, and
3. accurate.

Sequence and series

Prelim item (H2)

Find A and B be such that

$$\frac{2k+1}{k^2(k+1)^2} = \frac{A}{k^2} + \frac{B}{(k+1)^2}.$$

Show that the sum of the series

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \cdots + \frac{2n+1}{n^2(n+1)^2} = 1 - \frac{1}{(n+1)^2}.$$

Deduce that sum to infinity of this series $\frac{3}{2^4} + \frac{5}{3^4} + \frac{7}{4^4} + \ldots$ is less than 1.

Differential equation

Prelim item (H2)

The variables x and u are related by

$$xe^u \frac{\mathrm{d}u}{\mathrm{d}x} + e^x = 4x. \quad (I)$$

(i) By means of the substitution $y = xe^u$, obtain a differential equation relating y and x.

(ii) Hence show that the general solution of (I) is

$$u = \ln\left(2x + \frac{k}{x}\right), \ k \text{ is an arbitrary constant.}$$

Graph theory

Prelim item (H3)

Let F be a 3-regular graph of order n and size m. If F is edge-3-colourable and has a Hamiltonian cycle, prove that it is possible to find precisely k edges which are incident with every vertex of the graph.

Scope of syllabus

Test items must be set within the scope of the stipulated syllabus.

Complex numbers

Consider the following test item:

9233/N05/II/Q1

Verify that $z = i$ is a root of the equation

$$z^4 - 2z^3 + 6z^2 - 2z + 5 = 0.$$

Hence determine the other roots.

Complex numbers

The above question tests on:

	Topic/Sub-topics	Content
4.1	Complex number	conjugate roots of a polynomial equation with real coefficients

Complex numbers

Prelim item (H2)

If $z = i$ is a root of the equation

$$z^3 + (1 - 3i)z^2 - (2 + 3i)z - 2 = 0,$$

determine the other roots.
Hence find the roots of the equation

$$w^3 + (1 + 3i)w^2 + (3i - 2)w - 2 = 0.$$

Aesthetic beauty

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2. symmetry of the situation,
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4. ingenuity of ideas.

A.P./G.P.

9233/N05/II/Q4

It is given that a, b, c are the first three terms of a geometric progression. It is also given that a, c, b are the first three terms of an arithmetic progression.

(i) Show that $b^2 = ac$ and $c = \frac{a+b}{2}$.

A.P./G.P.

9233/N05/II/Q4

It is given that a, b, c are the first three terms of a geometric progression. It is also given that a, c, b are the first three terms of an arithmetic progression.

(i) Show that $b^2 = ac$ and $c = \frac{a+b}{2}$.

(ii) Hence show that $2\left(\frac{b}{a}\right)^2 - \left(\frac{b}{a}\right) - 1 = 0$.

A.P./G.P.

9233/N05/II/Q4

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(i) Show that $b^2 = ac$ and $c = \frac{a+b}{2}$.

(ii) Hence show that $2\left(\frac{b}{a}\right)^2 - \left(\frac{b}{a}\right) - 1 = 0$.

(iii) Given that the sum to infinity of the geometric progression is S, find S in terms of a.

Roots of an equation

Prelim item

By solving the equation $(w-2)^4 = 8\sqrt{2} - (8\sqrt{2})i$, prove that $4\cos\left(-\frac{9\pi}{32}\right)e^{i\left(-\frac{9\pi}{32}\right)}$ is a root to the equation and find its other roots, leaving your answer in similar form.

Qualities of a good item writer

1. The item writer must have thorough mastery of the subject matter being tested.
2. The item writer must be a master of verbal communication.
3. The item writer must be skilled in handling the special technology used for item writing.

Scenario building

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Scenario building

In creating interesting questions, it is sometimes important to know how to recast familiar (existing) problems in a completely fresh setting.
This is called scenario building.

Geometry

Question (Additional Mathematics)

Boat 1 and boat 2, which travel at constant speeds, not necessarily the same, depart at the same time from docks A and C, respectively, on the banks of a circular lake. If they go straight to docks D and B, respectively, they collide. Prove that if boat 1 goes instead straight to dock B and boat 2 goes straight to dock D, they arrive at their destinations simultaneously.

Geometry

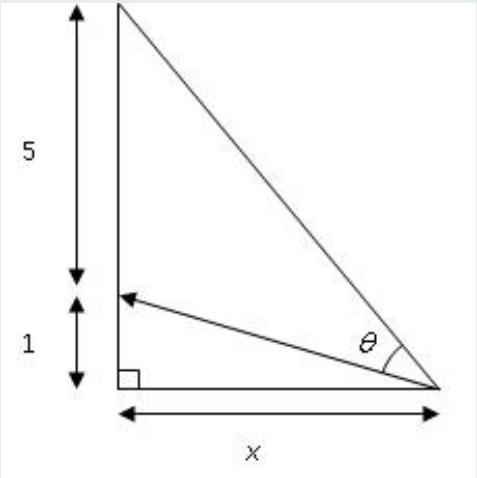
Activity 4 (5 min)

By solving the above question, discover what is the topic that is being tested?

Applications of differentiation

Prelim item

A movie theatre screen which is 5 metres high, has its lower edge 1 metre above an observer's eye. The visual angle of the observe seated x metres away as shown in the diagram below.



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(iii) Suppose that the observer is situated between 2 metres and 15 metres from the screen. Find, to the nearest degree, the smallest visual angle.

Exploiting history of mathematics

We may use our knowledge in the history of mathematics to inject a historical dimension into assessment.

Binomial series

Common test item

(i) Find, up to x^2, the binomial expansion of $(1+x)^{\frac{1}{2}}$.

(ii) State the range of x for which the above expansion is valid.

Binomial series

Common test item

An ancient Indian algorithm for finding the square root of a positive integer S is given in the *Bakhshali manuscript*. This is described as follows:

1. Let the perfect square nearest to S be N^2, $N > 0$.
2. Calculate $d = S - N^2$.
3. Calculate $P = \frac{d}{2N}$.
4. Calculate $A = N + P$.
5. Approximate $\sqrt{S}$ by the formula:

$$\sqrt{S} = A - \frac{P^2}{2A}.$$

Binomial seeries

Common test item

In summary, this amounts to

$$\sqrt{S} \approx N + \frac{d}{2N} - \frac{d^2}{8N^2}\left(N + \frac{d}{2N}\right)^{-1}.$$

The following parts explore how the ancient algorithm works.

(iii) Using (i) and assuming that $\left|\frac{d}{N^2}\right|$, show that

$$(N^2 + d)^{\frac{1}{2}} \approx N + \frac{d}{2N} - \frac{d^2}{8N^3}.$$

1. Hence explain how the ancient Indian algorithm works.

Qualities of a good item writer

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1. The writer must possess a rational and well developed set of thinking processes to determine what is most important to test.
2. The writer must understand the individuals for whom the test or items are intended.
3. The writer must accept that the process of constructing good test items is not simple.

Qualities of a good item writer

It seems that in order to be a good writer, one needs to be a ...

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Figure: Superman!

Practical tips

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- Adapt and modify without compromise for beauty.

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- Be observant to gather possible scenarios and ideas.
- Keep a private question or resource bank.
- Read more mathematics to increase one's depth in content, breadth for ideas and moments of inspirations.

Conclusion

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1. examination as a summative assessment,
2. the steps taken to pose good problems,
3. special qualities found in good test items, and
4. core skills invoked in setting good examination questions.

References

1. Brown, S. and Walter, M., *The art of problem solving*, Lawrence Erlbaum Associates Publishers, 1990.
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3. W.K. Ho, *Using of history of mathematics in the teaching and learning of mathematics in Singapore*, In Proceedings of 1st Raffles International Conference on Education, p. 1–38, 2009.

Acknowledgment

Thank you!