

# Mathematical Modelling – From Abstract to Practical

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# Outline

## 1 Introduction

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- 1 Introduction
- 2 Key features of mathematical modelling

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- 3 Abstract

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- 3 Abstract
- 4 Practical

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- 1 Introduction
- 2 Key features of mathematical modelling
- 3 Abstract
- 4 Practical
- 5 Complexity

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- 1 Introduction
- 2 Key features of mathematical modelling
- 3 Abstract
- 4 Practical
- 5 Complexity
- 6 Concluding remarks

## FAQ's

# What is mathematical modelling?

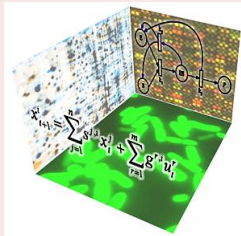
$$\frac{\partial}{\partial \theta} \ln f_{a, \sigma^2}(\xi_1) = \frac{1}{\sigma^2} \left( \xi_1 - a \right) f_{a, \sigma^2}(\xi_1) = \frac{1}{\sigma^2} \left( \xi_1 - a \right) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\xi_1 - a)^2}{2\sigma^2}}$$

Do we *all* have the *same* understanding of this term?

# FAQ's

## Question 2

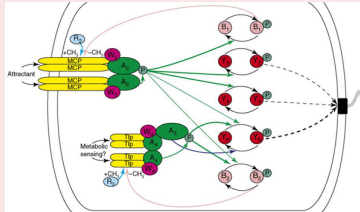
Why mathematical modelling?



# FAQ's

## Question 3

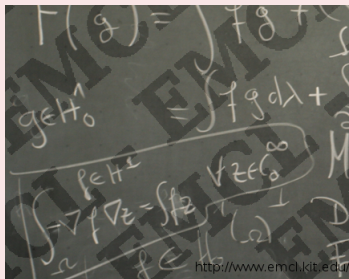
What is the role of mathematical modelling in mathematics?



## FAQ's

### Question 4

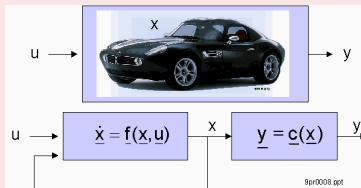
What is the role of mathematical modelling in mathematics education?



## FAQ's

### Question 5

What are some of the ways in which mathematical modelling can be implemented?



# Aims

- 1 Explore mathematical modelling along two different lines: abstract vs practical.

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- 1 Explore mathematical modelling along two different lines: abstract vs practical.
- 2 Make explicit those characteristics of modelling processes associated to each type.
- 3 Supply with a spectrum of modelling endeavors and seek educational relevance in these.

# What is a mathematical model?

## Definition

A **mathematical model** is a description of a system using mathematical concepts and language.

# What is a mathematical model?

## Definition

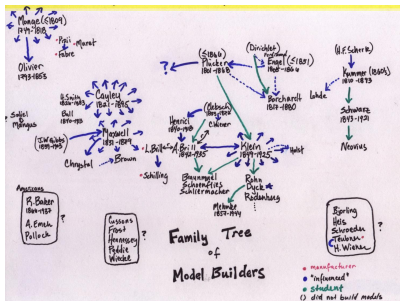
A **mathematical model** is a mathematical structure used to derive logic and reasoning in another mathematical system.

# What is mathematical modelling?

The process of manufacturing a mathematical model is termed **mathematical modelling**.

# What is a mathematical modelling endeavor?

A **mathematical experience** refers to an opportunity, a context or a task which requires mathematical modelling.



# Examples

## Vending machine

A typical can-drinks vending machine accepts coins of different denominations, and upon receiving the user-supplied choice of soft drinks, together with the correct payment, dispenses the correct product.



There are several different products, not necessarily equally priced.

# Examples

## Vending machine



- A soft-drink company organizes a charity activity in which each participant is given a set of  $N$  coins with randomly distributed denominations.

# Examples

## Vending machine



- A soft-drink company organizes a charity activity in which each participant is given a set of  $N$  coins with randomly distributed denominations.
- The coins are then inserted into a prototypical vending machine.

# Examples

## Vending machine

- If the amount inserted is sufficient, the participant gets to choose the product he or she desires. Otherwise, the participants tops up with his or her own money for the desired product. This top-up is then donated to charity.

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*Design* a feasible distribution of the denominations for each participant, giving a suitable fixed number  $N$ .

# Examples

## Snowplow deployment in Maryland

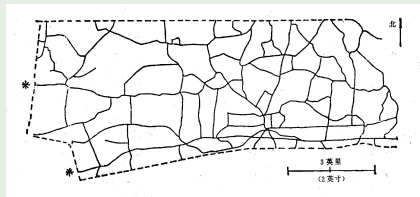
In the county of Wicomico in the state of Maryland, snowplows are deployed to clear snow after heavy snowstorms.



## Examples

### Snowplow deployment in Maryland

In a particular scenario, two snowplows are dispatched respectively from a garage about 4 miles west of each of the two points demarcated by \*'s on the dual-carriageway layout map below:



*Propose* an efficient deployment scheme for the snowplows in clearing the snow?

# Examples

## Water tank flow



Some state water-right agencies require from communities data on the rate of water use, in gallons per hour, and the total amount of water used each day. Many communities do not have equipment to measure the flow of water in or out of the municipal tank. Instead, they can measure only the level of water in the tank, within 0.5% accuracy, every hour.

# Examples

## Water tank flow

More importantly, whenever the level in the tank drops below some minimum level  $L$ , a pump fills the tank up to the maximum level,  $H$ ; however, there is no measurement of the pump flow either. Thus, one cannot readily relate the level in the tank to the amount of water used while the pump is working, which occurs once or twice per day, for a couple of hours each time.

# Examples

## Water tank flow

*Estimate* the flow out of the tank  $f(t)$  at all times, even when the pump is working, and estimate the total amount of water used during the day.

# Examples

## Water tank flow

Table 1 gives real data, from an actual small town, for one day.

Time (s)	Height (0.01 feet)	Time (s)	Height (0.01 feet)	Time (s)	Height (0.01 feet)
0	3175	35932	Pump operating	68535	2842
3316	3110	39332	Pump operating	71854	2767
6635	3054	39435	3550	75021	2697
10619	2994	43318	3445	79154	Pump operating
13937	2947	46636	3350	82649	Pump operating
17921	2892	49953	3260	85968	3475
21240	2850	53936	3167	89953	3397
25223	2797	57254	3087	93270	3340
28543	2752	60574	3012	32284	2697
64554	2927				

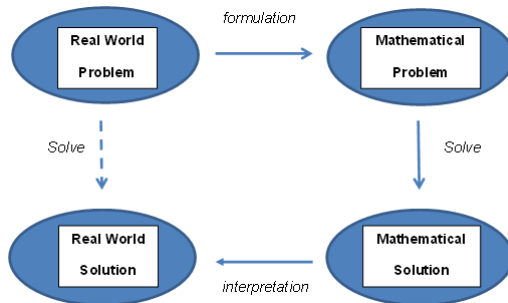
# Examples

## Water tank flow

The table gives the time, in, since the first measurement, and the level of water in the tank, in hundredths of a foot. For example, after 3316 seconds, the depth of water in the tank reached 31.10 feet. The tank is a vertical circular cylinder, with a height of 40 feet and a diameter of 57 feet. Usually, the pump starts filling the tank when the level drops to about 27.00 feet, and the pump stops when the level rises back to about 35.50 feet.

# Nature of mathematical modelling

Mathematical modelling is characterized by the following cycle:



# Variables

In general, the original system in the problem involves different kind of variables:

- decision variables (independent variables)

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# Objective functions

Objectives and constraints of the system and its users can be represented as functions of the output variables or state variables. The objective functions will depend on the perspective of the model's user.

# Vending machine

There are a few variables in this modelling experience:

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# Vending machine

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- The number of coins  $N$
- The distribution of coin denominations
- The price of each available product
- The reachability of a purchasing state
- The operational mode of the vending machine, e.g., does it give a change if the payment exceeds the price of the product?

# Vending machine

## Question

What do we aim to achieve when we solve this design problem?

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## Objective function

Assigning to each denomination distribution a state-reachability output together with a probabilistic measure.

# Different classifications of models

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## ① Linear vs nonlinear

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- ① Linear vs nonlinear
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- ③ Static vs dynamic
- ④ Discrete vs continuous
- ⑤ Deductive vs inductive

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This problem can be formulated using a **graph-theoretic** model. Because the distances between junctions are important in the consideration of efficiency in this problem, it is best to employ weighted graphs. Thus, the model chosen is

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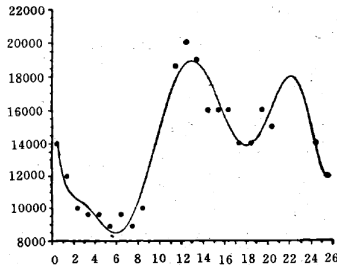
# Snowplow deployment in Maryland

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- deterministic
- static
- discrete, and
- deductive.

## Water tank flow

This problem demands a solution for the rate of water flow  $f(t)$ , from which the volume  $V(t)$  of water in the tank at any time  $t$  can be determined. A fairly direct approach is to figure out the variational plot of the height of the water level against time.



# Water tank flow

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# Water tank flow

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- inductive.

## Black box vs white box

Different models work on varying degree of a priori information, i.e., available information which is independent of experience.



Black box model: no a prior information at all

White box model: full a prior information

# Modelling in abstract contexts

We start with the white box model.

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## Term model for typed $\lambda$ -calculus

The raw terms of the calculus can be given by the *Backus Naur Form*, i.e., BNF below:

$$M ::= x \mid \lambda x. M \mid M(M) \mid \underline{n} \mid \\ \text{succ}(M) \mid \text{pred}(M) \mid \text{ifz}(M, M, M) \mid \text{fix}(M)$$

# $\lambda$ -calculus

The operational semantics of this language is given by

$$\frac{}{x \Downarrow x}(\text{VAR}) \quad \frac{}{V \Downarrow V}(\text{VAL})$$

$$\frac{M \Downarrow \lambda x. T \quad T[N/x] \Downarrow V}{M(N) \Downarrow V}(\text{APPL}) \quad \frac{M \Downarrow \underline{n}}{\text{succ}(M) \Downarrow \underline{n+1}}(\text{SUCC})$$

$$\frac{M \Downarrow \underline{0}}{\text{pred}(M) \Downarrow \underline{0}}(\text{PRED}) \quad \frac{M \Downarrow \underline{n+1}}{\text{pred}(M) \Downarrow \underline{n}}(\text{PRED})$$

# $\lambda$ -calculus

The operational semantics of this language is given by

$$\frac{M \Downarrow \underline{0} \quad M_1 \Downarrow V}{\text{ifz}(M, M_1, M_2) \Downarrow V} (\text{IF-ZERO}) \quad \frac{M \Downarrow \underline{n+1} \quad M_2 \Downarrow V}{\text{ifz}(M, M_1, M_2) \Downarrow V} (\text{IF-ZERO})$$

$$\frac{M(\text{fix}(M)) \Downarrow V}{\text{fix}(M) \Downarrow V} (\text{FIX})$$

# $\lambda$ -calculus

## Addition & Factorial

The term-model can be implemented on the functional language `HASKELL` as follows:

```
add :: Int -> Int -> Int
add x y = add (x + 1) (y - 1)
```

```
fact :: Int -> Int
fact n = if (n == 0) then 1 else fact (n-1)
```

# $\lambda$ -calculus

## Program correctness and equivalence

- How can one verify if the given program works?

# $\lambda$ -calculus

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- How can one verify if the given program works?
- How can one tell if two programs are operationally equivalent?

# $\lambda$ -calculus

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# $\lambda$ -calculus

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## Solution

- Manufacture a mathematical model for the term calculus
- Model must be fairly abstract, i.e., does not depend on the operational semantics  $\Downarrow$
- Model must be accurate enough to capture the essential aspects of the calculus

# $\lambda$ -calculus

## Denotational semantics

D.S. Scott (late 1960's) invented the Scott model to interpret

- 1 data types as topological orders (called **domains**)

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D.S. Scott (late 1960's) invented the Scott model to interpret

- ① data types as topological orders (called **domains**)
- ② data elements as the elements of domains
- ③ functional programs as continuous functions between domains

This is an example of a 'grey' and deductive model (based on Scott's theory).

# $\lambda$ -calculus

## Modelling computation

In any computation, every finite part of the output depends on only a finite part of the input. The central theme is:

**Continuity.**

# $\lambda$ -calculus

## Surprising fact

For any terms  $M, N : \sigma$ ,

$$\llbracket M \rrbracket = \llbracket N \rrbracket \implies M = N.$$

The converse does not hold.

# Real exact arithmetic

## Question

What is a real number?

## Real exact arithmetic

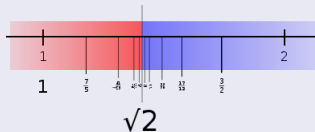
There are nine-thousand answers to this question. Amongst these, we pick out three:

# Real exact arithmetic

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## Dedekind cuts

In mathematics, a Dedekind cut, named after Richard Dedekind, is a partition of the rational numbers into two non-empty parts  $A$  and  $B$ , such that all elements of  $A$  are less than all elements of  $B$ , and  $A$  contains no greatest element.



# Real exact arithmetic

## Decimal

Every real number can be represented by a floating point decimal.

$$\sqrt{2} = 1.41421356237309504880168872420969807856967187...$$

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## Decimal

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## Warning

Problematic representation for computers!

# Real exact arithmetic

## Theorem

*The simple scaling function*

$$f : \mathbb{R} \longrightarrow \mathbb{R}, x \mapsto 9x$$

*can never be realized by any program*

$$\bar{f} : (10)^\omega \longrightarrow (10)^\omega.$$

# Real exact arithmetic

## Signed bit streams

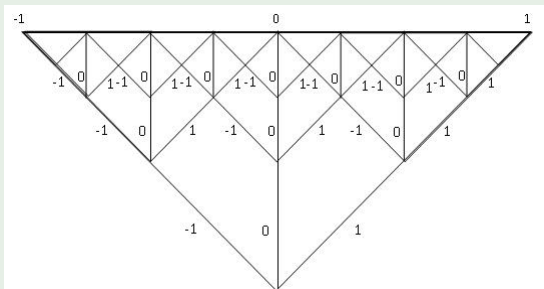
Every real number can be represented by signed bit stream:

$$\{-1, 0, 1\}^\omega.$$

# Real exact arithmetic

## Example

$$\llbracket 0 : -1 : 1^\omega \rrbracket = 0 \cdot \frac{1}{2^1} + (-1) \cdot \frac{1}{2^2} + 1 \cdot \sum_{k=3}^{\infty} \frac{1}{2^k} = 0$$



# Real exact arithmetic

## The negative truth

In actual fact, what we are proposing to be  $\mathbb{R}$  are not  $\mathbb{R}$ !

# Real exact arithmetic

## The negative truth

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## The positive truth

They are all models for  $\mathbb{R}$ !

# Real exact arithmetic

## Surprising fact

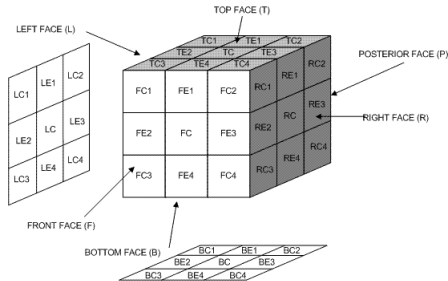
Derivatives are not computable but Riemann integrals are!

# Rubik's cube

This physical gadget needs no introduction.



# Rubik's cube



## Modelling the moves

Name each  $90^\circ$  clockwise turn of these faces to be

$T, B, F, P, L, R.$

# Rubik's cube

To understand the configurations of the Rubik's cube (i.e., the position of each of the faces of the 'cubies'), it is essential to model the numerous possible moves.

# Rubik's cube

## Modelling the moves of a Rubik's cube

The mathematical model here is the algebraic structure of a group.

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## Modelling the moves of a Rubik's cube

The mathematical model here is the algebraic structure of a group.  
Denote the set

$$S := \{T, B, F, P, L, R\}$$

and the group generated by the set  $S$ ,

$$(\mathbb{G}, *) = \langle S \rangle.$$

where the group operation  $*$  is that of composition of moves.

# Rubik's cube

## Surprising fact

Every scrambled Rubik's cube configuration can be returned to its home state in at most **26** moves.

# Characteristics of modelling in abstract contexts

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- The subject of study is itself a fully-defined mathematical entity in the real world.
- Its behavior is very complex and thus, reasoning directly in/with this entity is difficult/tedious.
- Based on experience with the entity, certain important behavioral aspects are extracted into a theoretical model.
- Interpretation to the real world entity will only be accurate up to those identified salient features.

# Subjective information

Unlike a priori information, subjective information derives from some of the following:

- Intuition

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- Experience
- Expert opinion
- Convenience of mathematical form.

# Engineering problem

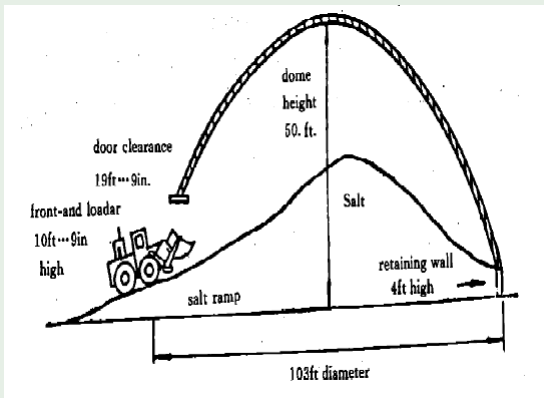


## Salt storage problem

For approximately 15 years, a Midwestern state has stored salt used on roads in the winter in circular domes. Figure 1 shows how salt has been stored in the past. The salt is brought into and removed from the domes by driving front-end loaders up ramps of salt leading into the domes. The salt is piled 25 to 30 ft high (i.e., 7.6 m to 9.2 m), using the buckets on the front-end loaders.

# Engineering problem

## Salt storage problem



# Engineering problem

## Salt storage problem

Recently, a panel determined that this practice is unsafe. If the front-end loader gets too close to the edge of the salt pile, the salt might shift, and the loader could be thrown against the retaining walls that reinforce the dome. The panel recommended that if the salt is to be piled with the use of the loaders, then the piles should be restricted to a maximum height of 15 ft (i.e., 4.6 m).

Construct a mathematical model for this situation and find a recommended maximum height for salt in the domes.

# Logistics problem

## Personnel management system

Hospitals generally categorize surgical operations as major, standard, and trivial cases. Major cases involve complex procedures such as heart bypasses; standard ones cover gastronomical and intestinal procedures; and trivial cases include removal of small external growths. Each category calls for a number of senior (attending) surgeons, anesthesiologists, resident surgeons, operating nurses, and non-operating nurses; as well as time and costs.

# Logistics problem



## Personnel management system

Category	S	A	RS	ON	NN	Time	Aver. costs
M	3	1	1	2	2	1 day	\$30, 000
S	2	1	1	1	2	1/2 day	\$16, 000
T	1	1	0	1	1	1/5 day	\$3, 000

# Logistics problem

## Personnel management system

A certain hospital is staffed with

- ① 21 senior doctors (only these can be senior attending surgeons)

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## Personnel management system

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- ② 44 junior doctors
- ③ 60 senior nurses (only these can be operating nurses)
- ④ 40 junior nurses
- ⑤ 30 anesthesiologists

Design an efficient and effective personnel management system for daily operations.

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- Its behavior is very complex, woven together as a fabric of several variables.
- Based on the user's need, certain important behavioral aspects are formulated to a theoretical model.
- Interpretation to the real world situation is not guaranteed to be accurate.
- Cyclically improving the model, with every added availability of subjective information.

# Complexity

Model complexity involves a trade-off between

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of the model.

# Complexity

## Occam's razor

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As for what is relevant, it is up to the user and the model builder!

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- What opportunities are there to turn real world problems into mathematical modelling experiences?

# Implications

- What can be considered a mathematical modelling task?
- How can we design more meaningful mathematical modelling task?
- What opportunities are there to turn real world problems into mathematical modelling experiences?
- Are we mindful of the applications of Information Technology in mathematical modelling?

## References and resources

- ➊ Plus teacher and student package: Mathematical Modelling at <http://plus.maths.org/content/os/issue44/package/index>
- ➋ McLaughlin, Michael P. (1999) 'A Tutorial on Mathematical Modeling'.
- ➌ Patrone, F. Introduction to modeling via differential equations, with critical remarks.