Popularizing Mathematics in Secondary Schools and Beyond

Weng Kin Ho
National Institute of Education, Singapore
wengkin.ho@nie.edu.sg

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Have you heard about ...

The student's question that math teachers fear most

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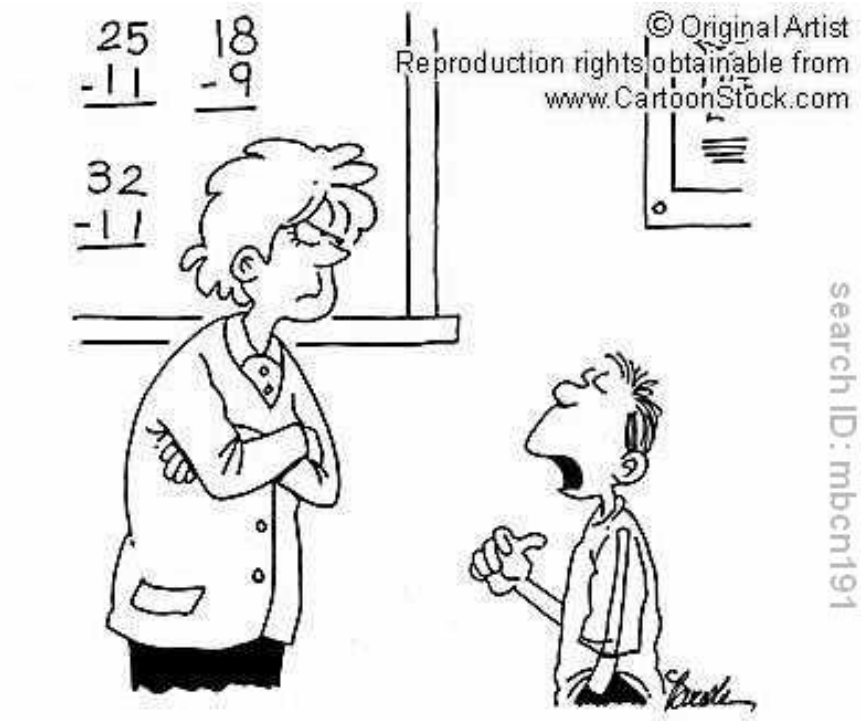
The student's question that math teachers fear most

Why must we learn math?

Students would say ...



"I don't need to learn how to subtract. I'm going to work for the government."

Have you heard about ...

The most discouraging fact to a math teacher

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The most discouraging fact to a math teacher

The most hated school subject in the world is

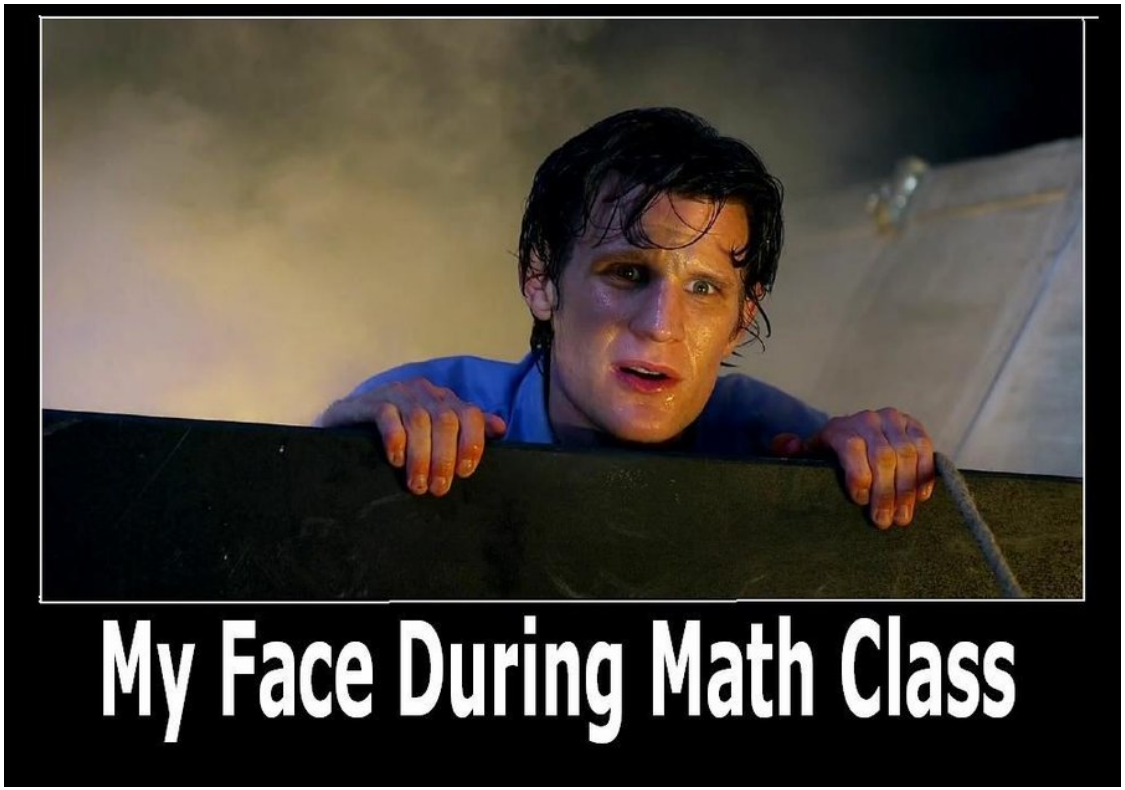
Have you heard about ...

The most discouraging fact to a math teacher

The most hated school subject in the world is

Mathematics.

Students give you this kind of look ...



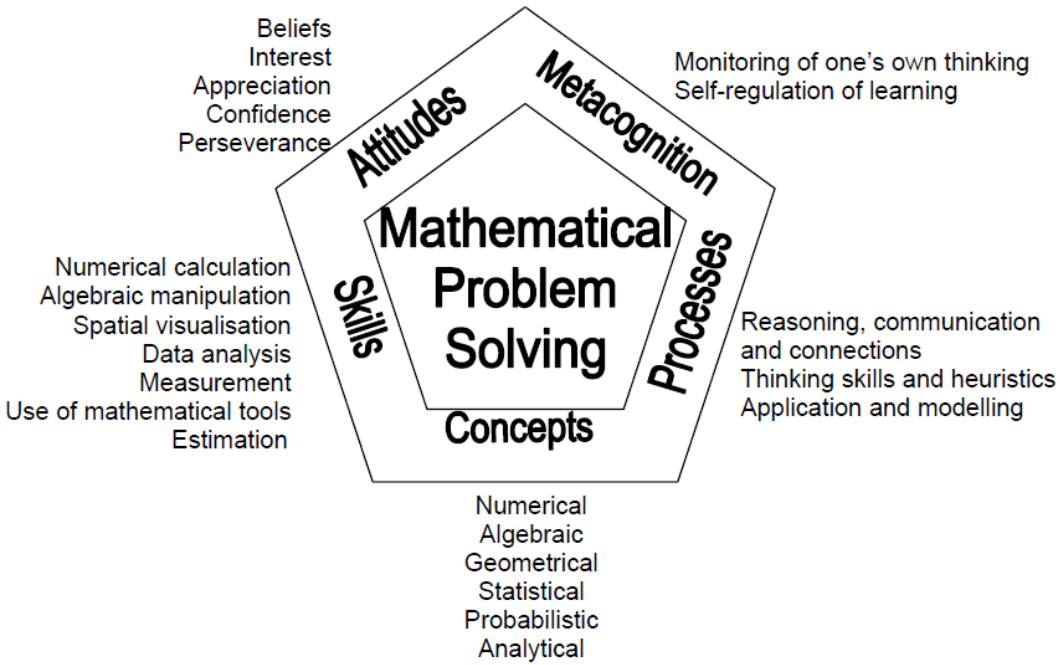
Who is to blame for this?

Attitude counts

The Singapore Mathematics Curriculum highlights

Attitude

within its famous pentagonal framework.



Attitude counts

"Many people, both pupils and adults, appear frightened of mathematics or maintain that they hate it. ... Many factors were found to influence the development of an individual's perceptions of mathematics, but school experience was found to be crucial ..."

– J. Rooney (1998)

Attitude counts

"Attitudes toward mathematics are important since there is a ... relationship between achievement in mathematics and attitudes towards mathematics."

– B. R. Evans (2007)

Selling mathematics

can be compared to selling cars ...

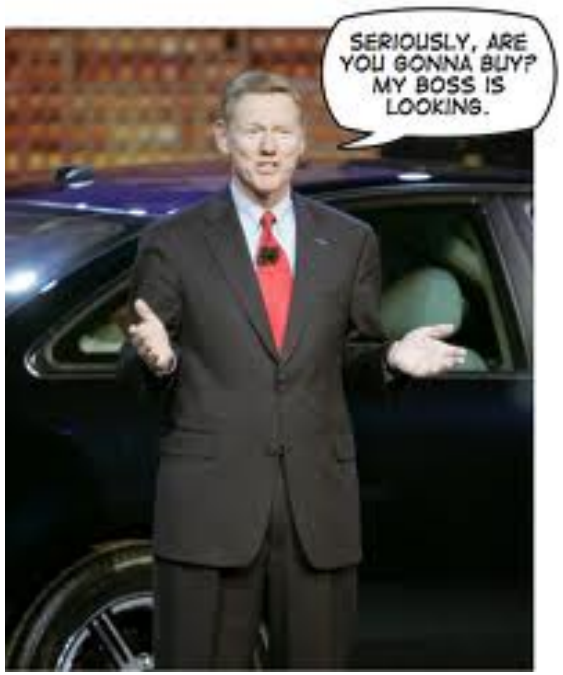


Figure: Are our students keen to buy in mathematics?

Teachers' change

"While the aim was not necessarily to change their beliefs, there was a clear agenda to unearth and explicate their beliefs about mathematics and learning mathematics so they were open to reflective consideration particularly in the light of their career of teaching mathematics."

– P. Grootenboer (2008)

A look into our inner world

As mathematics teachers and educators,

A look into our inner world

As mathematics teachers and educators,

we ask ourselves ...

What are our attitudes towards mathematics and the teaching of mathematics?

Checklist about our attitudes towards mathematics

Questions	Yes/No
Are we still passionate about mathematics?	☐
Do we still stand in awe of the beauty and power of mathematics?	☐
Are we convinced that mathematics is still useful in this 21st century?	☐
Do we still see ourselves as active mathematicians/problem solvers?	☐
Do we show, by example, the persevering spirit in us when solving a math problem?	☐

Checklist about our attitudes towards teaching mathematics

Questions	Yes/No
Are we dying to tell the person next to us a piece of beautiful mathematics?	☐
Do we believe we can make a difference in our students' perception of mathematics?	☐
Are we convinced that our students benefit from our mathematics lesson?	☐
Are we confident enough to nurture the next generation of mathematics students?	☐

Against the flow

To reverse the effects of bad vibes people get from mathematics, we must counter it:

Against the flow

To reverse the effects of bad vibes people get from mathematics, we must counter it:

Make Mathematics popular among people!

Common belief

Today, we all work on one common belief:

Common belief

Today, we all work on one common belief:

Motto

The responsibility of popularizing mathematics rests on all of us –

Mathematics Teachers!

Question

A naive equation

Mathematics as part of life =

A naive equation

Mathematics as part of life = Mathematics is a human activity

A commonly accepted view

Most people treat mathematics as ...

A commonly accepted view

Most people treat mathematics as ...

- isolated,

A commonly accepted view

Most people treat mathematics as ...

- isolated,
- timeless,

A commonly accepted view

Most people treat mathematics as ...

- isolated,
- timeless,
- ahistorical, and

A commonly accepted view

Most people treat mathematics as ...

- isolated,
- timeless,
- ahistorical, and
- inhuman.

Quite on the contrary ...

"But mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context."

– R. Hersh (1997)

The idea is ...

The way to popularizing mathematics in

The idea is ...

The way to popularizing mathematics in

- schools;

The idea is ...

The way to popularizing mathematics in

- schools;
- this country;

The idea is ...

The way to popularizing mathematics in

- schools;
- this country;
- the world; ...

The idea is ...

The way to popularizing mathematics in

- schools;
- this country;
- the world; ...

is:

The idea is ...

The way to popularizing mathematics in

- schools;
- this country;
- the world; ...

is:

Perhaps

Go back in history!

The idea is

Figure: Back in history to look for answers for the future

Popularizing mathematics

Main approach

Popularizing mathematics

Main approach

Integrate history of mathematics into mathematics teaching

Popularizing mathematics

Is it one of those things again?

Popularizing mathematics

Is it one of those things again?

Popularizing mathematics

Is it one of those things again?

The answer is: NOT JUST THIS!

Gradual change

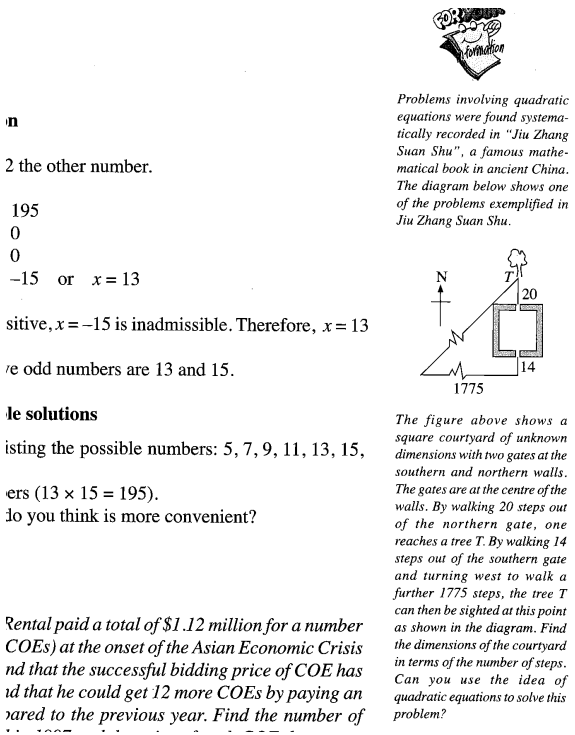
Figure: Lifestyle changes must be made slowly, one step at a time.

Historical snippets

Definition

Historical snippets are pieces of historical information which can be incorporated into the main text.

Historical snippets



n

2 the other number.

195
0
0
−15 or $x = 13$

sitive, $x = -15$ is inadmissible. Therefore, $x = 13$

e odd numbers are 13 and 15.

le solutions

isting the possible numbers: 5, 7, 9, 11, 13, 15,

ers ($13 \times 15 = 195$).
lo you think is more convenient?

Rental paid a total of $1.12 million for a number COEs) at the onset of the Asian Economic Crisis nd that the successful bidding price of COE has ıd that he could get 12 more COEs by paying an ıared to the previous year. Find the number of

Problems involving quadratic equations were found systematically recorded in "Jiu Zhang Suan Shu", a famous mathematical book in ancient China. The diagram below shows one of the problems exemplified in Jiu Zhang Suan Shu.

The figure above shows a square courtyard of unknown dimensions with two gates at the southern and northern walls. The gates are at the centre of the walls. By walking 20 steps out of the northern gate, one reaches a tree T. By walking 14 steps out of the southern gate and turning west to walk a further 1775 steps, the tree T can then be sighted at this point as shown in the diagram. Find the dimensions of the courtyard in terms of the number of steps. Can you use the idea of quadratic equations to solve this problem?

Figure: New Syllabus Mathematics 3, 6th Edition, Teh, K.S. & Loh, C.Y. (2007), p.14

Historical snippets

Historical snippets can appear in different forms:

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1. Biographies

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4. Attribution of authorship and priorities
5. Dates and chronologies
6. Pictures of mechanical instruments, architectural, artistic and cultural designs

Historical snippets

Prior to the teaching of the volume of the cone, the following can often be invoked.

Historical snippets

Pop quiz

Who is he?

Historical snippets

Pop quiz

Who is he?
(1) Isaac Newton

Historical snippets

Pop quiz

Who is he?
(1) Isaac Newton (2) Socrates

Historical snippets

Pop quiz

Who is he?
(1) Isaac Newton (2) Socrates
(3) Democritus, The Laughing Philosopher

Historical snippets

Pop quiz

Who is he?
(1) Isaac Newton (2) Socrates
(3) Democritus, The Laughing Philosopher
(4) Budai, The Laughing Buddha

Historical snippets

A fun way to create a lesson induction on the volume of a cone is by introducing the (probably) first person in history who discovered the formula $V = \frac{1}{3}\pi r^2 h$, i.e.,

Photograph or portrait of mathematicians

Figure: Democritus (460 BC – 370 BC)

Mechanical instruments

Definition

By a mechanical instrument, we mean a historical tool that can be employed to perform mathematical calculations or carry out geometrical constructions (which are beyond the abilities of the straightedge and compass).

Ancient geometrical problem

Use of mechanical instruments

The (unmarked) straightedge and the compass together define the Euclidean geometry.

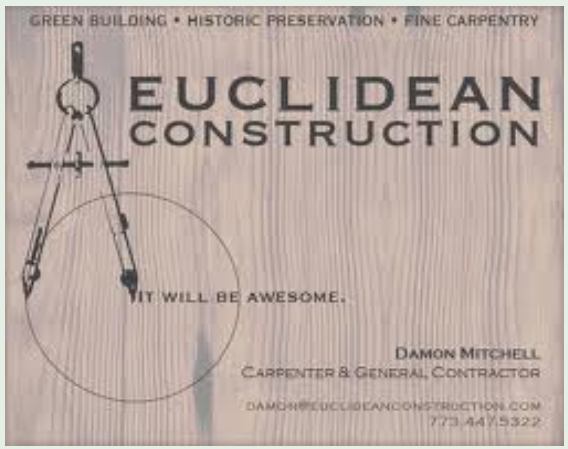

Figure: Ruler and compass

Ancient geometrical problem

Use of mechanical instruments

In 1597, Galileo invented a geometric compass, a scientific instrument with two arms that can be used for making calculations and geometric measurements.

Ancient geometrical problem

Use of mechanical instruments

The job of bisecting any given angle using only straightedge and compass is an essential part of the training in secondary school geometry.

`http://www.allmathwords.org/en/b/bisect.html`

Ancient geometrical problem

Question

Can we, using only straightedge and compass, trisect any given angle (i.e., divide a given angle into three equal parts)?

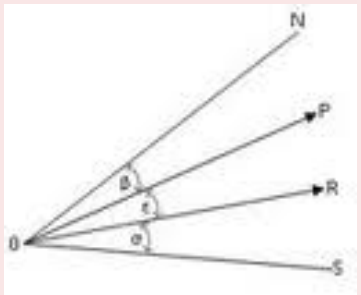

Figure: Ancient Greek Trisection Problem

Ancient geometrical problem

Answer

It takes the power of modern algebra to prove that trisection using unmarked straightedge and compass is impossible.

Ancient geometrical problem

Answer

It takes the power of modern algebra to prove that trisection using unmarked straightedge and compass is impossible.
However, this fact does not forbid us from inventing instruments that can do the job.

Ancient geometrical problem

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Figure: Creative violence: Tomahawk

Ancient geometrical problem

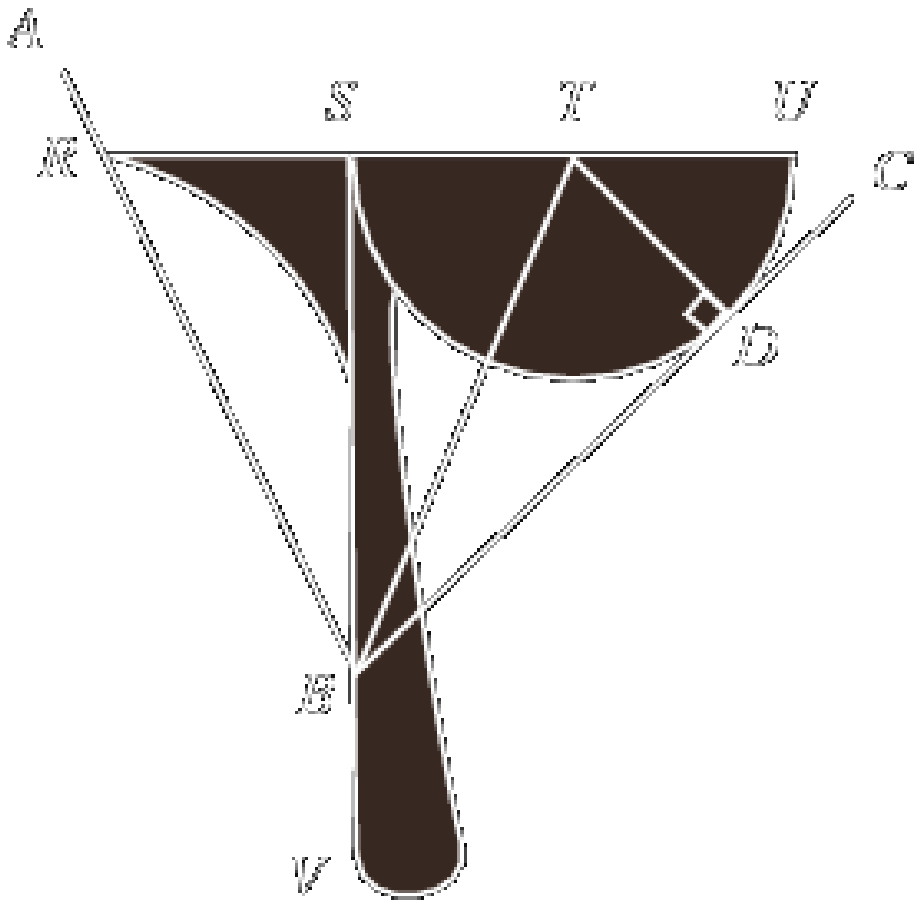

Figure: Extracted from a 19th century textbook, author unknown

Ancient geometrical problem

By this time, it is not uncommon for students to have this kind of imagery in their heads:

Ancient geometrical problem

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Ancient geometrical problem

Life demonstration

It is easy to replicate one such tomahawk, a handy 'weapon' for our geometry lessons.

Ancient geometrical problem

A follow-up exercise on geometry

Give a geometrical proof that this trisecting tomahawk works.

Ancient geometrical problem

A follow-up exercise on geometry

Give a geometrical proof that this trisecting tomahawk works.

Hint: Congruent triangles.

Taking advantage of errors

Definition

Mathematical *errors* and *misconceptions* that actually took place in history can be constructively used in teaching and learning of mathematics.

Taking advantage of errors

Egyptian mathematics

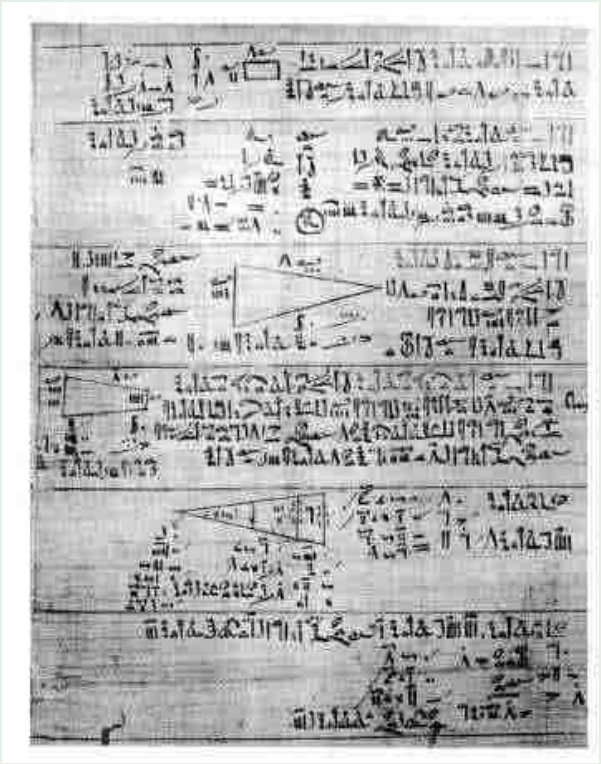
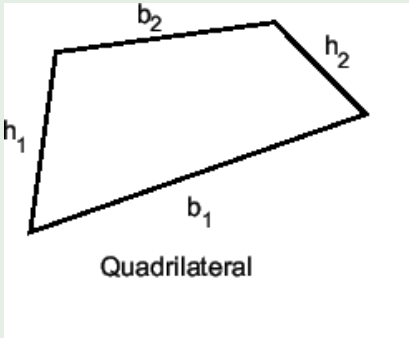


Figure: The Ahmes Papyrus

Taking advantage of errors

Egyptian mathematics

A surviving deed from Edfu in Egypt, dating back to the 2nd century BC, gave the area of a quadrilateral as the product of the pairs of arithmetic means of opposite sides.



$$A = \left(\frac{b_1 + b_2}{2}\right)\left(\frac{h_1 + h_2}{2}\right).$$

Taking advantage of errors

Egyptian mathematics

Ask students to investigate

Taking advantage of errors

Egyptian mathematics

Ask students to investigate

- how good this formula is;

Taking advantage of errors

Egyptian mathematics

Ask students to investigate

- how good this formula is;
- when it will give the correct answer;

Taking advantage of errors

Egyptian mathematics

Ask students to investigate

- how good this formula is;
- when it will give the correct answer;
- what some special cases yield;

Taking advantage of errors

Egyptian mathematics

Ask students to investigate

- how good this formula is;
- when it will give the correct answer;
- what some special cases yield;
- whether the Egyptians were aware that it is wrong.

Historical packages

Definition

Historical packages are a collection of materials narrowly focused on a small topic, with strong ties to the curriculum, suitable for two or three class periods, ready for use by teachers in their classrooms.

Historical packages

A historical package

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- can be built around short fragments of primary sources, e.g., usually a 3-4 line quotation.

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- constitutes of teacher's instruction and students' activities.
- comes in the form of a folder including the detailed activity, historical and didactical background, guidelines for classroom implementation, expected student reactions (based on previous classroom trials), and the illustrative material needed in the form of slides, ICT-based worksheets, etc.

Historical packages

Pythagoras Theorem

"The oldest, shortest words - "yes' and "no" - are those which require the most thought."

– Pythagoras

Historical packages

Pythagoras Theorem

There are several 'origins' of this theorem arising from different cultures and civilizations:

Historical packages

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Historical packages

Pythagoras Theorem

There are several 'origins' of this theorem arising from different cultures and civilizations:

- Chinese
- Egyptian
- Greek
- Babylonian

Historical packages

Pythagoras Theorem

There are more than 140 proofs of this theorem. In the course of teaching the theorem, present 4 of them.

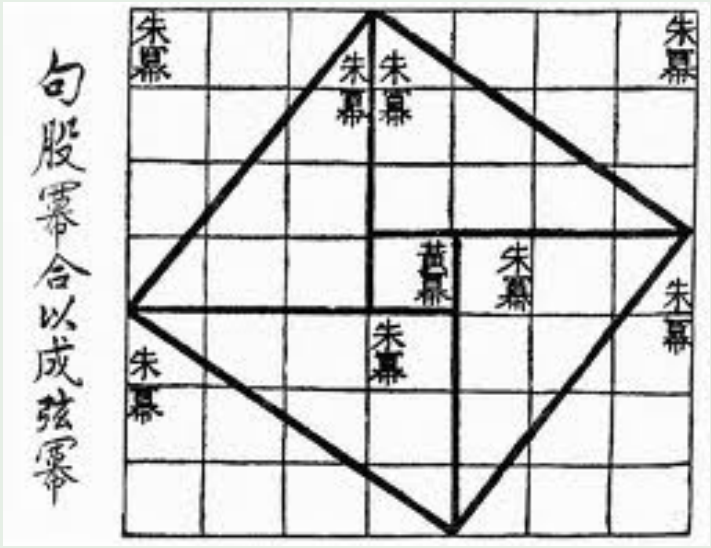

Figure: Different proofs

Historical packages

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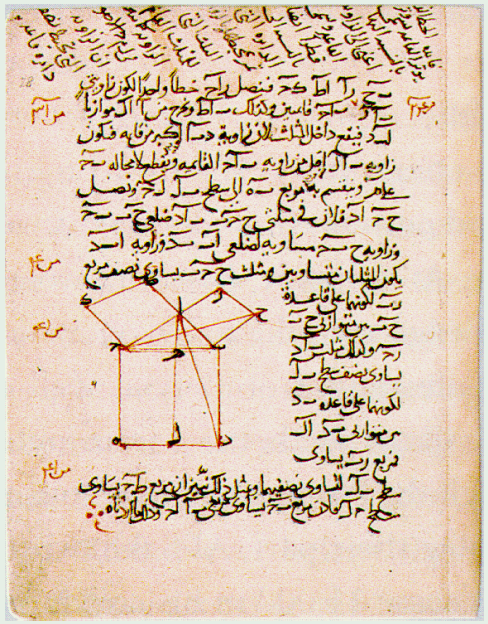

Figure: Different proofs

Historical packages

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Historical packages

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Figure: Different proofs

Historical packages

Pythagoras Theorem

Historically motivated activities such as:

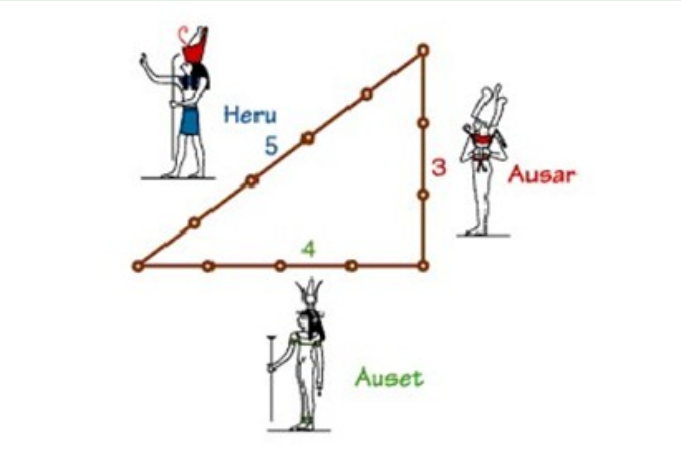

Figure: Making a right-angled triangle from a string of 11 knots

Films

Definition

There are a number of good movies

- about the lives of mathematicians, or
- involving mathematicians,

that can highlight the human, cultural and social context of mathematics and mathematicians.

Films

Top 5 "Must-watch"

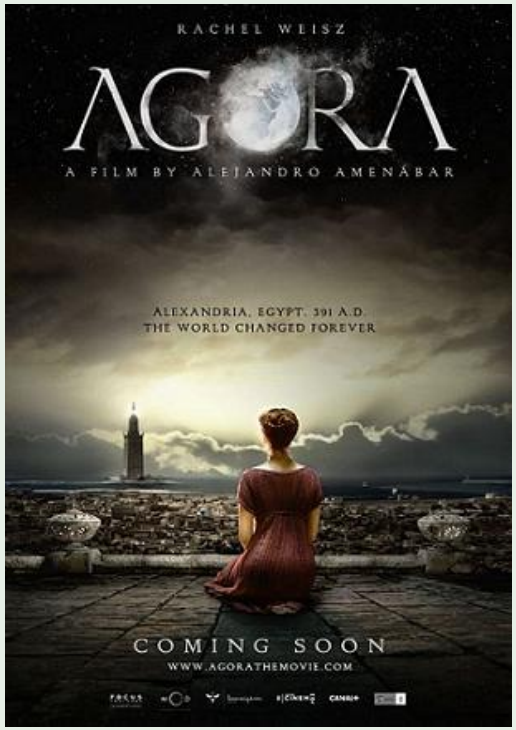

Figure: Agora (2009) - The life of Hypatia

Films

Top 5 "Must-watch"

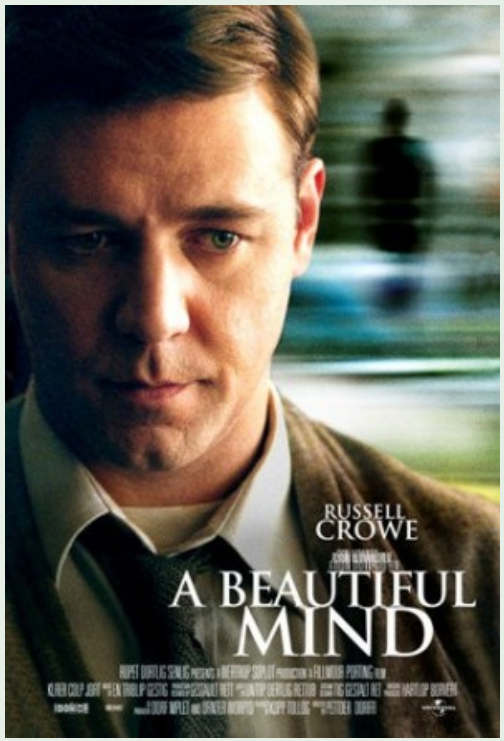

Figure: A Beautiful Mind (2001) - A fictional depiction of John Nash

Films

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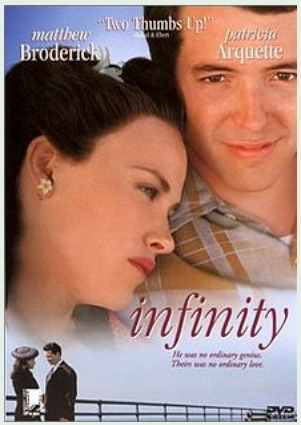

Figure: Infinity (1996) - A story about Nobel Prize-winning physicist Richard Feynman

Films

Top 5 "Must-watch"

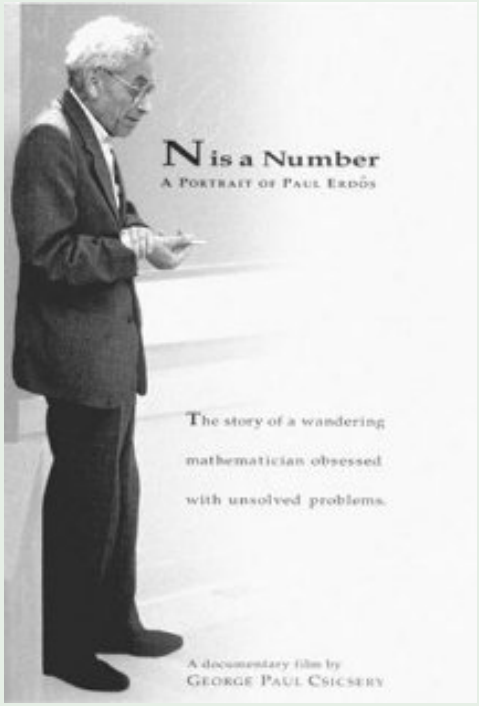

Figure: *N* Is a Number: A Portrait of Paul Erdos (1993) – the life of Hungarian mathematician, Paul Erdös.

Films

We really must watch this:

`http://www.youtube.com/watch?v=zRNGV85kPbI`

Outdoor experiences

Definition

The mathematics of outdoor experiences refers to the identification of forms and shapes, patterns in nature, in architecture (past and present) and in art.

Outdoor experiences

Museum/Science Center Trips

Students and teachers should have at least one outdoor experience at:

Mathematics Everywhere & Everyday

exhibition at Hall F, Singapore Science Center.

Outdoor experiences

Temple mathematics

An overseas trip to Japan should include an excursion to the temples, where San-Gaku used to thrive.

Figure: Mathematics puzzles (San-Gaku) at Japanese temples

Feasibility: potentialities, limits and risks

Potentialities:

Feasibility: potentialities, limits and risks

Potentialities:

- Result in better understanding of the topic

Feasibility: potentialities, limits and risks

Potentialities:

- Result in better understanding of the topic
- Create a learning environment different from traditional chalk-and-talk

Feasibility: potentialities, limits and risks

Potentialities:

- Result in better understanding of the topic
- Create a learning environment different from traditional chalk-and-talk
- Inculcate better attitudes of the learners as well as their teachers

Feasibility: potentialities, limits and risks

Limits:

Feasibility: potentialities, limits and risks

Limits:

- Short of teachers training in history of mathematics

Feasibility: potentialities, limits and risks

Limits:

- Short of teachers training in history of mathematics
- Short of curriculum time

Feasibility: potentialities, limits and risks

Limits:

- Short of teachers training in history of mathematics
- Short of curriculum time
- Short of suitable assessment rubrics

Feasibility: potentialities, limits and risks

Risks:

Feasibility: potentialities, limits and risks

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- Going too far back in history, cannot make relevant connection with topic within a short time

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- Overt emphasis on historical elements versus mathematical elements
- Unfamiliarity on the part of the students because of differences in cultural, sociological, political and historical background

Feasibility: potentialities, limits and risks

Risks:

- Going too far back in history, cannot make relevant connection with topic within a short time
- Overt emphasis on historical elements versus mathematical elements
- Unfamiliarity on the part of the students because of differences in cultural, sociological, political and historical background
- Science-stream students feel uncomfortable with 'humanities-nature' of this approach

Conclusion

Dare you look into the past?

The End

Thank you!

References

1. Evans, B.R. (2007). Student attitudes, conceptions, and achievement in introductory undergraduate college statistics. *The mathematics Educator, 17*(2), 24–30.
2. *History in Mathematics Education – The ICMI Study.*, Fauvel, J. and van Maanen, J. (Eds), Kluwer Academic Publishers (2000).
3. Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 11*, 479–497.
4. Hersh, R. (1997). *What is Mathematics Really?*, Oxford University Press.
5. Rooney, J. (1998). Teaching Influence on Life-long Perceptions of Mathematics. *Teaching Mathematics and its Applications, 17*(1), 12–18.