History of Mathematics as a Teaching Strategy

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Main question

Research direction

Does *history of mathematics* have a role to play in mathematics education?

The 2000 ICMI study

An International Commission on Mathematical Instruction (ICMI) study was conducted pertaining to the **methodology** of **integrating**

History of Mathematics

in the

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- teaching, and
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of mathematics in 2000.

Coverage of ICMI study

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1. analysis of the impact of the HOM-methodology on the national curricula of several countries,
2. discussion of the philosophical, multi-cultural and inter-disciplinary issues related to this methodology,
3. archiving a spectrum of classroom implementation methods, and
4. feasibility issues with regards to such implementations.

Anything done for Singapore?

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Has a parallel study been carried out in Singapore?

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- ACMEP (Ancient Chinese Mathematics Enrichment Programme) in 2006.
- UROPS (Undergraduate Research Opportunities Programme in Science) in 2002.

Purpose of current study

The main aim is to set out to investigate the role of using HOM in teaching and learning of mathematics in Singapore.

Scope of current study

In view of the studies already carried out in primary level and lower secondary level, we chose to focus on

Pre-university level

which includes:

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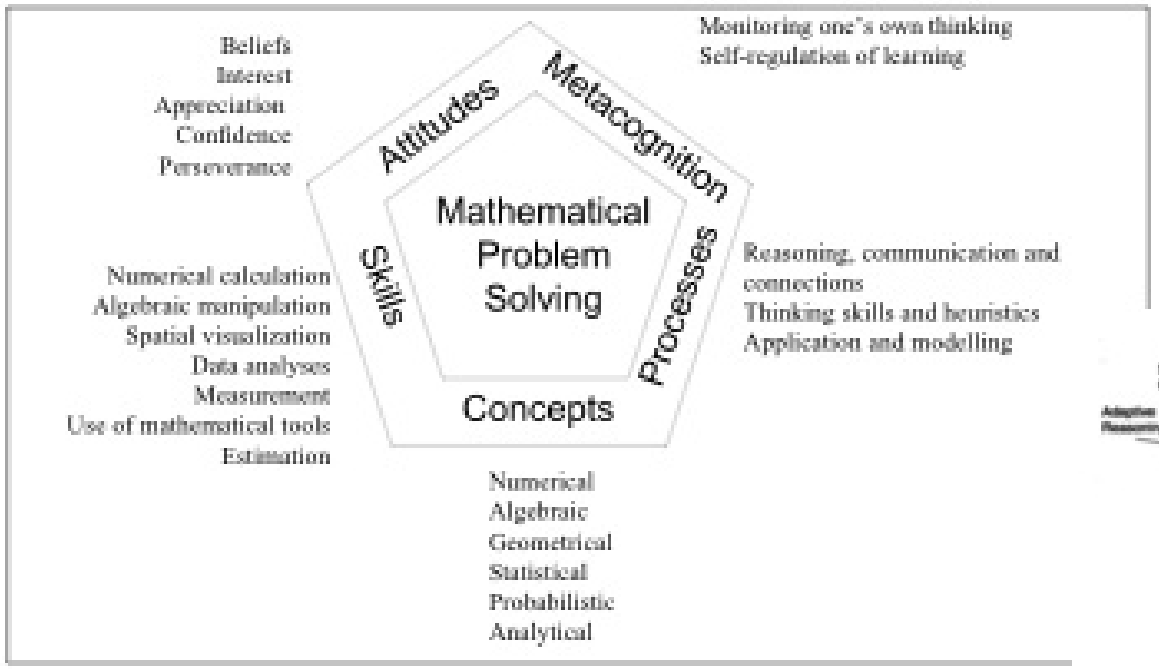
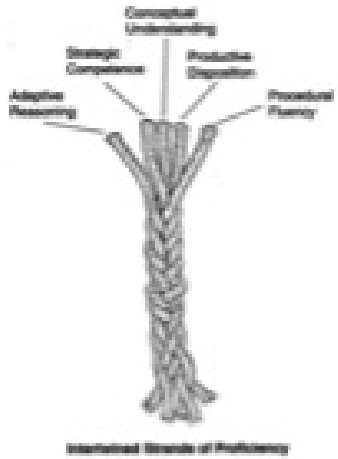
- Junior College Level
- Polytechnic Level

Aims of Mathematics Education in Singapore

Singapore Mathematics Curriculum Framework: Pentagon





Targeted objectives of HOM approach

- Increase the students' motivation and develop a *positive attitude* towards mathematics.

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- Help explain the difficulties and confusion that students encounter via the analysis of the development of mathematics.
- Enhance the development of students' *mathematical reasoning skills* by the use of historical problems.

Targeted objectives of HOM approach

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- Use the lives of mathematicians as a platform to introduce and inculcate good moral values such as honesty, diligence and determination, and
- Provide a guide by which teachers of mathematics may craft their lessons.

Current areas of emphasis

... focus more on

- honing on mathematical content knowledge, and

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- developing mathematical reasoning abilities:
  1. concepts,

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- honing on mathematical content knowledge, and

- developing mathematical reasoning abilities:
  1. concepts,
  2. skills, and
  3. problem solving.

Possible area to work on

... would be to inculcate a

- positive attitude towards mathematics.

Having a positive attitude in mathematics

hopes to bring about

- holding on to certain beliefs and philosophy towards mathematics,

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- invoking genuine interest and enjoying mathematics,

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- holding on to certain beliefs and philosophy towards mathematics,
- invoking genuine interest and enjoying mathematics,
- developing an appreciation of the beauty and power of mathematics,

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- building confidence in using mathematics, and

- instilling a spirit of perseverance in solving a problem as part of the mathematical training.

MOE's concern

Students' attitudes towards mathematics are shaped by their learning experiences. Making the learning of mathematics fun, meaningful and relevant goes a long way to inculcating positive attitudes towards the subject. Care and attention should be given to the design of the learning activities to build confidence in and develop appreciation for the subject.

– Ministry of Education, Singapore.

Education system in Singapore

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Compulsory Education for children of primary school age was only enforced very recently in 2000, under the **Compulsory Education Act**.

Time line of Singapore Math Education History

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5. New initiatives (1995 – 2005)

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- Introduction of Integrated Programmes

Polytechnic mathematics

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- No use of GC's

Current place of HOM in Singapore Mathematics Education

A survey was carried out in 2007 among **1000** Junior College teachers and Polytechnic lecturers to find out if HOM was integrated in the everyday mathematics lessons.

Percentage of usage of HOM

Assuming mutual exclusion of the different usages,

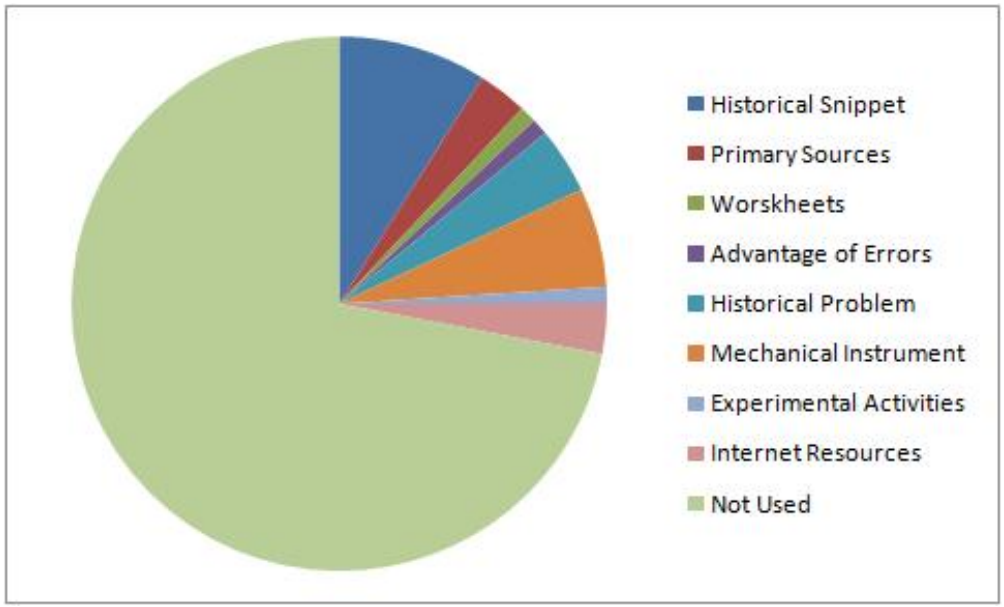

Figure: Percentage of usage of HOM

at most 18% used HOM.

Current place of HOM in Singapore Mathematics Education

The survey revealed that in actuality less than

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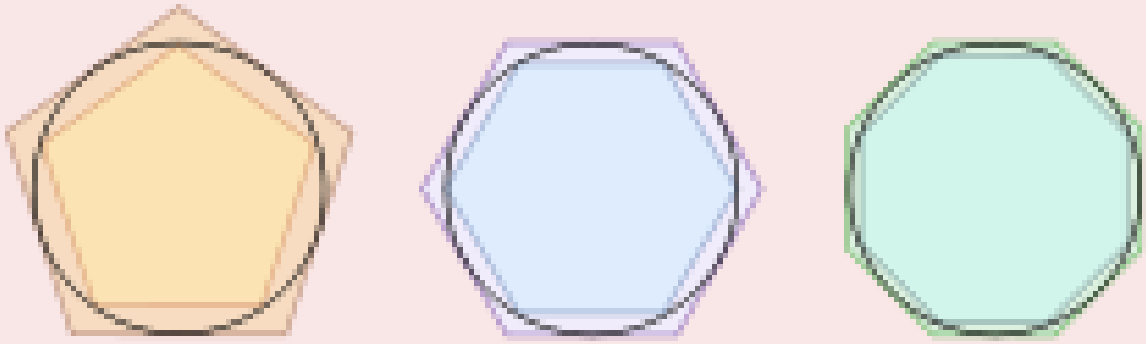
10 %

made use of the HOM in the teaching of Pre-U mathematics.

Ask yourself...

Question



What did Archimedes have in mind when he made rational approximations to π?

Two problems

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2. One is too enthusiastic to re-enact historical moments because one believes in *psychological recapitulations*.

What is the problem here?

Falsely believing that:

What is the problem here?

Falsely believing that:

There is *only one* course that historical development just had to take!

Unhistorical reading of history

The consequence

Accepting textbook development as the actual historical development.

Unhistorical reading of history

Example (The definition of the dot product)

Try recalling which is the first definition (you encountered in high school) of

the **Dot Product** of 3D-vectors?

Unhistorical reading of history

Example (The definition of the dot product)

Is it

(A) $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or

(B) $\mathbf{a} \cdot \mathbf{b} = \mathbf{a_1 b_1} + \mathbf{a_2 b_2} + \mathbf{a_3 b_3}$, where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ with respect to the usual orthonormal basis?

Unhistorical reading of history

It turns out that the more natural way (B) is indeed historically the original one, first defined as a by-product of the *quaternion product* – an independent invention of Josiah Willard Gibbs and Oliver Heaviside.

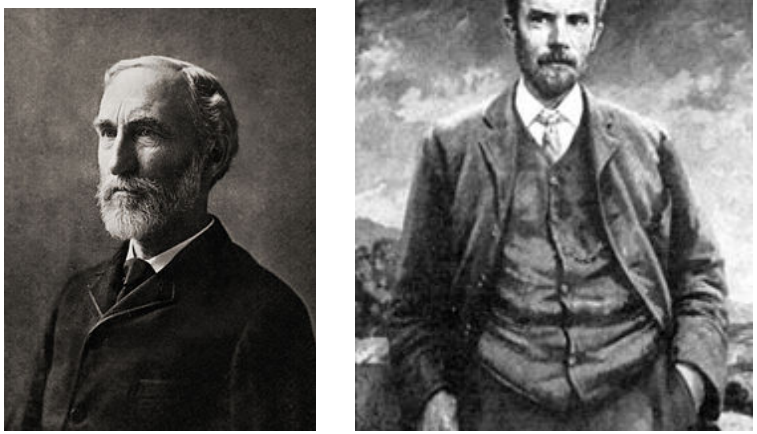

Figure: J.W. Gibbs and O. Heaviside

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Falsely believing that:

The individual development of the learner is just a brief repetition of the evolution of mankind.

Psychological recapitulation

The consequence

Historical approach = Direct import of historical movements in classroom experience

Psychological recapitulation

Example (Emergence of i)

As a teacher/learner of higher mathematics, how do you motivate the definition of the unit imaginary number $i = \sqrt{-1}$?

Psychological recapitulation

Example (Emergence of i)

Do you think it historically emerged from

(A) the study of quadratic equations

OR

(B) the study of cubic equations?

Psychological recapitulation

Early mathematicians such as Niccolo Fontana Tartaglia and Gerolamo Cardano found that it is inevitable to use imaginary numbers and rules to manipulate them to solve even the real roots of a cubic or quartic equation.

Figure: N.F. Tartaglia and G. Cardano

Two leaps

Two leaps

- Intellectual leaps from a lower psychogenetic stage to a higher one.

Two leaps

- Intellectual leaps from a lower psychogenetic stage to a higher one.
- Historical leaps from a lower level to a more advanced level in the human knowledge paradigm.

Warning

We should not need to find a direct relationship between the *intellectual leap* and the *historical leap*!

Justification for the proposed framework

Instead, identify the respective mechanisms that trigger the leaps, and find a link between these.

Psychogenetic leap and underlying mechanism

To exploit the powers of the HOM approach, one inevitably needs to

(1) Identify the *learning points* along the syllabus or curriculum where the learner needs to experience a psychogenetic leap (e.g., when presented with difficulty or confusion).

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(3) Explore the psychogenetic mechanisms that can be used to promote a smooth/effective transition over these learning points.

Historical leap and underlying mechanism

Simultaneously, one needs to

(1') Identify the *historical mechanisms* corresponding/associated to those identified psychogenetic mechanisms.

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(3') Explore the *historical points* (e.g., historical events or elements where these problems arose).

Proposed didactic framework

This triggers a continuous interplay of the following processes:

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This triggers a continuous interplay of the following processes:

- Forward implementation

Proposed didactic framework

This triggers a continuous interplay of the following processes:

- Forward implementation
- Backward sourcing

Aims

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- belief
- confidence
- perseverance

Student group targeted

- Academic Year 2007–2008

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- Linear Algebra and Vectors module

HOM approach

- History of mathematics intentionally integrated into the lesson

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- 1st 6 weeks: no obvious incorporation of HOM, only historical snippets

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- Next 6 weeks: full-fledged approach

HOM approach

Week No.	**Topic**	**Historical Element**	**Item**
6	Gaussian Elimination	Ancient Instrument	Ancient Chinese Rod Numerals
7	Matrices	Historical Snippets	Life story of *forgetful* J. Sylvester
9,10	Eigenvectors and eigenvalues	Historical Problem	The Invariant Subspace Problem of D. Hilbert
11,12	Vectors	Primary Sources	Descartes' dream Descartes' notebook

Experiential learning using ancient instruments/artefacts

The modern Gaussian elimination method has long been exploited by the ancient Chinese using the Ancient Chinese Rod Numerals:

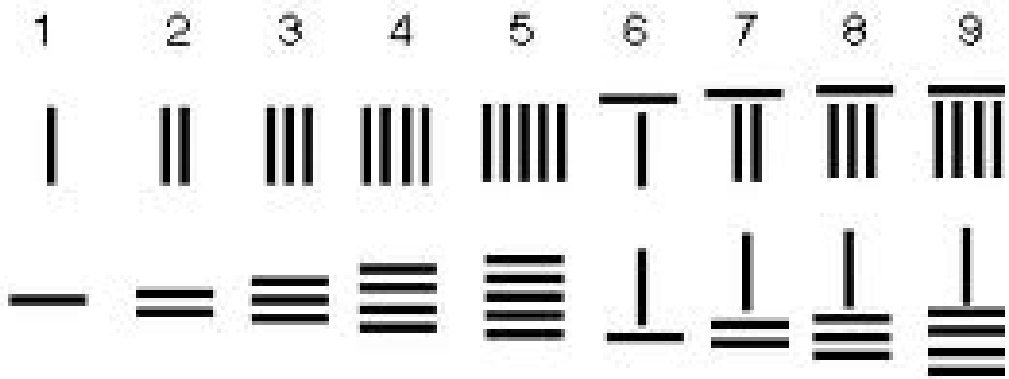


Figure: Chinese Rod Numerals

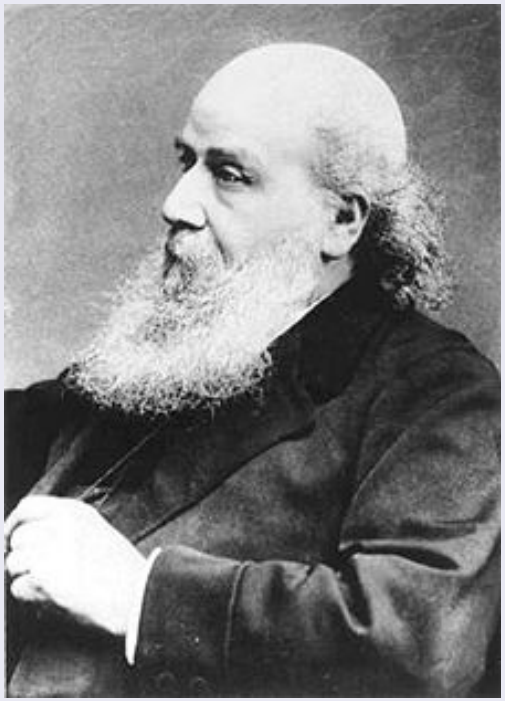
Historical snippets

Historical snippets are pieces of historical information incorporated into the main text or presentation.

Historical snippets

James Joseph Sylvester

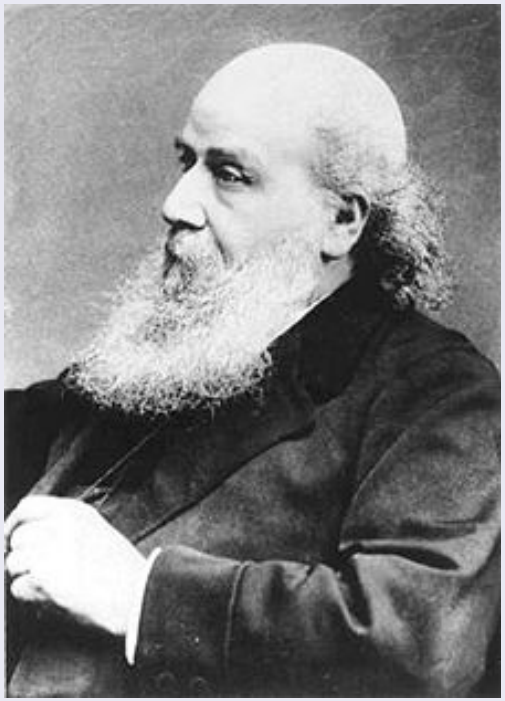
James Joseph Sylvester (3 September 1814 - 15 March 1897) was an English mathematician. He made fundamental contributions to matrix theory, invariant theory, number theory, partition theory and combinatorics.



Historical snippets

James Joseph Sylvester

He played a leadership role in American mathematics in the later half of the 19th century as a professor at the Johns Hopkins University and as founder of the American Journal of Mathematics. At his death, he was professor at Oxford.



Historical problem and worksheets

Invariant subspace problem

After the students engaged in a worksheet activity about transformation of vectors by matrices, they were told of the 1900 list of important mathematical problems posed by David Hilbert – one of which is the invariant subspace problem.

Historical problem and worksheets

Invariant subspace problem

Let H be a complex Banach space of dimension greater than 1. Does every bounded linear operator $T : H \longrightarrow H$ have a non-trivial closed T-invariant subspace, i.e., a closed linear subspace W of H which is different from $\{0\}$ and H such that $T(W) \subseteq W$.

This problem provides a lead to teaching of eigenvectors and eigenvalues.

Primary source

It appears to me that if one wants to make progress in mathematics one should study the masters. – Niels Henrik Abel (1802 – 1829)

Primary source

Gottfried Leibniz's copy of Descartes' secret notebook

The activity was situated to reinforce on the relationship between geometry and algebra. It was based on a copy of Leibniz's handwritten copy of Descartes' secrete notebook, where he wrote two number sequences

$$4,\ 6,\ 8,\ 12,\ 20$$

and

$$4,\ 8,\ 6,\ 20,\ 12.$$

Primary source

Gottfried Leibniz's copy of Descartes' secret notebook

The students were guided to formulate an equation

$$F + V - E = 2$$

linking the two number sequences, introducing to them a topological invariant.

Primary source

Gottfried Leibniz's copy of Descartes' secret notebook

- Deeper connection between geometry and algebra

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- Appreciate the cultural differences reflected in the mathematical writing

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Gottfried Leibniz's copy of Descartes' secret notebook

- Deeper connection between geometry and algebra
- Appreciate the cultural differences reflected in the mathematical writing
- Bring out the spirit of determination as exemplified by Leibniz in his quest of Descartes' secret notebook.

Students' and teacher's log

- Student logbook

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- Student logbook
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Students' and teacher's log

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- Detailed lesson plans

Students' and teacher's log

- Student logbook
- Teacher logbook
- Detailed lesson plans
- Lesson observations

Students' sample comments

- I am motivated by Rene Descartes because every science derives from maths.

Students' sample comments

- I am motivated by Rene Descartes because every science derives from maths.
- Besides mathematics, I learn something special: I have learned to be passionate and love something and put it into action.

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- I am motivated by Rene Descartes because every science derives from maths.
- Besides mathematics, I learn something special: I have learned to be passionate and love something and put it into action.
- History of mathematics very interesting.

Students' sample comments

- I am motivated by the way the lecturer presents with the interesting things about past mathematicians and eigenvectors and eigenvalues.

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Students' sample comments

- I am motivated by the way the lecturer presents with the interesting things about past mathematicians and eigenvectors and eigenvalues.
- I look forward to the next lesson because I am motivated and want to learn more.
- This is a mental picture of my impression of today's lesson: Very motivational. I do not feel any barriers between students and lecturers.

Students' survey

- Find out about their changes in attitudes.

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- Survey targeted to cover 4 aspects:
  1. Belief

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  4. Perseverance

Sample survey questions: Belief

1. I believe that linear algebra is useful.

Sample survey questions: Belief

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2. I believe, as a result, that mathematics is useful.

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4. I believe that mathematical theorems of linear algebra are universally true.

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6. I believe, independent of my experience in linear algebra module, that mathematical theorems are universally true.

Sample survey questions: Perseverance

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3. In group discussions, when challenged with a difficult problem, I advise the group to evade the problem.
4. When challenged with a question by the teacher, I put in some thought to the problem and try my best to answer.

Hypothesis testing

- 5% level of significance
- Wilcoxon-Mann-Whitney test
- Alternative hypothesis: Median score for treatment group (TM) is significantly higher or lower than that of control group (CM)
- Treatment group size: 17 (TM)
- Control group size: 57 (CM)
- Median calculated over 4 categories: Belief, Interest, Confidence, Perseverance

Research findings

The historical approach is produces significantly higher median scores for the treatment group as compared against the control group in terms of the components:

Belief and Perseverance

End of talk

Thank you for your patience!