Mathematics education in Singapore: Current trend and future direction

Ho Weng Kin
wengkin.ho@nie.edu.sg

National Institute of Education, Nanyang Technological University

28 May 2012

Outline

Outline

Outline

Singapore Mathematics

US sold on S'pore maths teaching techniques

23rd April 09. ST

WASHINGTON: About a decade ago, a small number of made-in-Singapore maths textbooks began circulating among frustrated American parents looking for better ways to teach the subject.

Now, these textbooks are used in about 1,200 United States elementary schools, with annual sales of about

ences in the size and culture of the countries and their education systems.

The population of Singapore, for instance, is only slightly bigger than that of Los Angeles, the largest city in the state of California.

But exploratory studies by US maths academics have found over the years that students in the US who adopt the Singapore model, together with the right training for teachers and some tweaks to fit the local context, have outperformed their peers who stuck with US textbooks.

Interest in the Singaporean textbooks grew further in late 2007, when California endorsed their use for elementary schools statewide.

Elementary schools in states such as Massachusetts, New Jersey and Wisconsin are also said to be using

Singapore Success Story

Singapore was at the top of the world in 4th and 8th grade mathematics. – Trends in International Mathematics and Science Study (TIMMS 2007).

Singapore Mathematics Curriculum

The first step in understanding the mathematics education in Singapore is to know the underlying

Singapore Mathematics Curriculum

The first step in understanding the mathematics education in Singapore is to know the underlying

Singapore Mathematics Curriculum

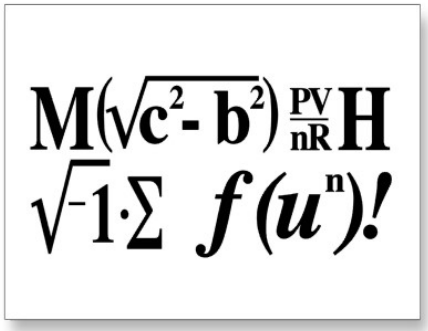
The first step in understanding the mathematics education in Singapore is to know the underlying

in its national mathematics curriculum.

Rationale

Mathematics is an excellent vehicle for the development and improvement of a person's intellectual competence in logical reasoning, spatial visualisation, analysis and abstract thought. Students develop numeracy, reasoning, thinking skills, and problem solving skills through the learning and application of mathematics. These are valued not only in science and technology, but also in everyday living and in the workplace. The development of a highly skilled scientifically- and technologically-based manpower requires a strong grounding in mathematics. An emphasis on mathematics education will ensure that we have an increasingly competitive workforce to meet the challenges of the 21st century.

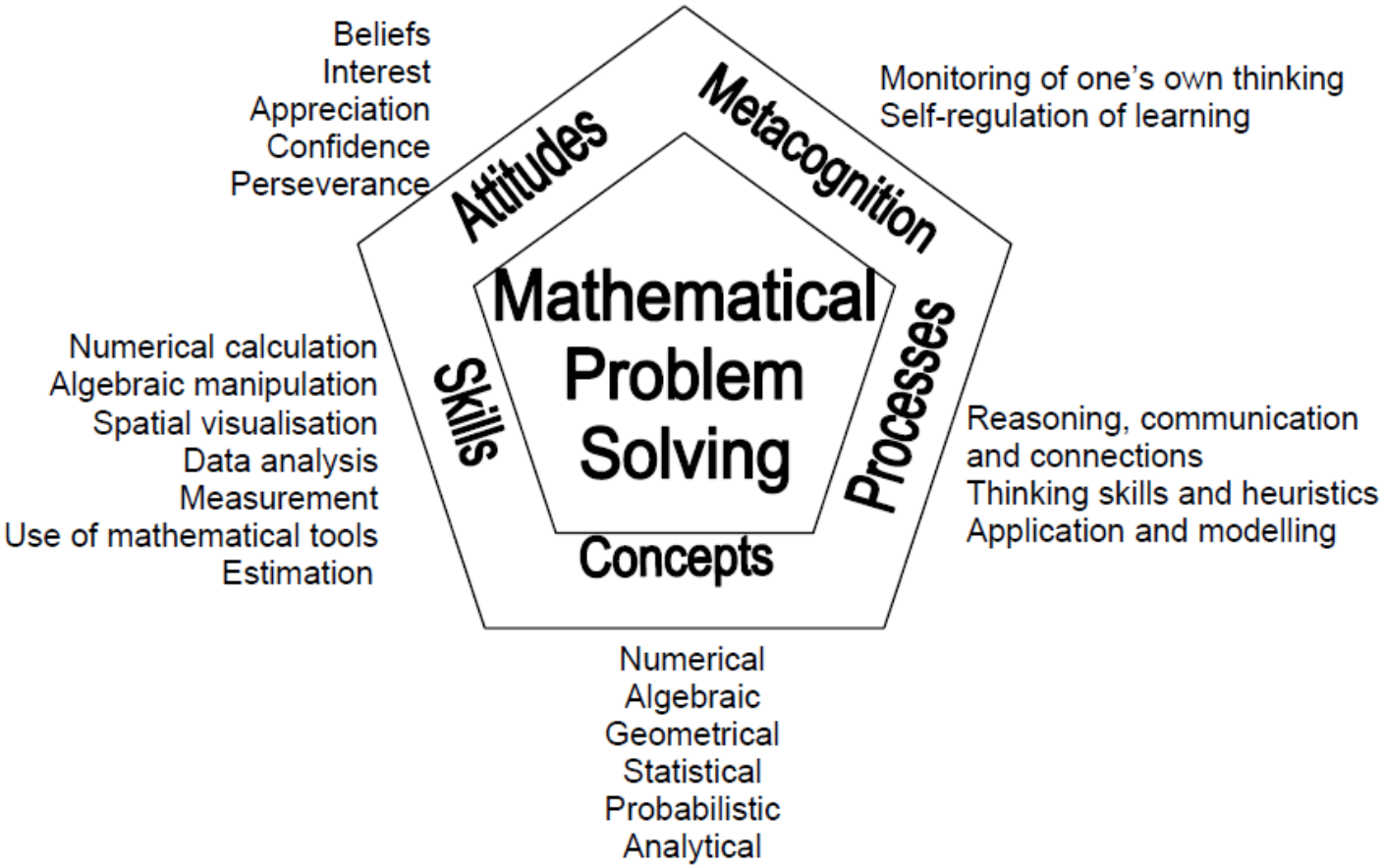
Rationale



Mathematics is also a subject of enjoyment and excitement which offers students opportunities for creative work and moments of enlightenment and joy. When ideas are discovered and insights gained, students are spurred to pursue mathematics beyond the classroom walls.

Figure: Driven by learning outcomes

Mathematics Framework



Mathematics Framework

The framework "sets the direction for teaching, learning and assessment".

The Central Theme: Problem Solving

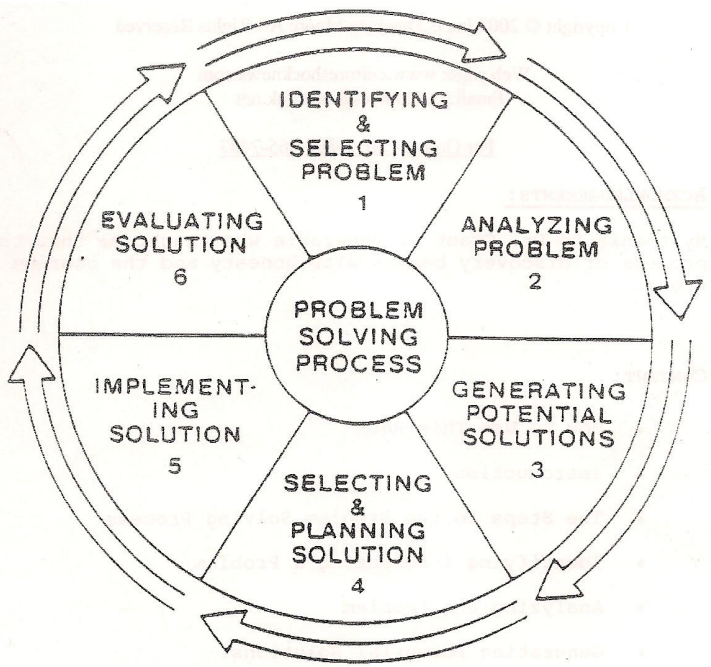
In the curriculum document, problems are defined to include "a wide range of situations" and "include non-routine, open-ended and real-world problem".

Problem Solving

Definition

A problem is a *non-routine* question or scenario which demands its solver to ...

Problem Solving Strategies



The Problem Solving Cycle

Figure: George Pólya's Problem Solving Cycle

Problem solving strategies

Singapore schools adopt the 4-stage version of the problem solving cycle:

Problem solving strategies

Singapore schools adopt the 4-stage version of the problem solving cycle:

1. Understand the problem

Problem solving strategies

Singapore schools adopt the 4-stage version of the problem solving cycle:

1. Understand the problem
2. Devise a plan

Problem solving strategies

Singapore schools adopt the 4-stage version of the problem solving cycle:

1. Understand the problem
2. Devise a plan
3. Carry out the plan

Problem solving strategies

Singapore schools adopt the 4-stage version of the problem solving cycle:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Evaluate the solution

Problem solving strategies

Some schools use different acronyms for the same 4-stage cycle.

Problem solving strategies

Some schools use different acronyms for the same 4-stage cycle.

Local adaptations

Problem solving strategies

Some schools use different acronyms for the same 4-stage cycle.

Local adaptations

1. **S**ee

Problem solving strategies

Some schools use different acronyms for the same 4-stage cycle.

Local adaptations

1. **S**ee
2. **T**ry

Problem solving strategies

Some schools use different acronyms for the same 4-stage cycle.

Local adaptations

1. **S**ee
2. **T**ry
3. **A**ct

Problem solving strategies

Some schools use different acronyms for the same 4-stage cycle.

Local adaptations

1. **S**ee
2. **T**ry
3. **A**ct
4. **R**e-look

Problem Solving Heuristics

School teachers and students are trained to employ 13 different problem solving heuristics.

Problem Solving Heuristics

School teachers and students are trained to employ 13 different problem solving heuristics.

Definition

A problem solving heuristic is a *rule of thumb* or *working-method* by which a problem-solving plan may be executed.

Problem Solving Heuristics

School teachers and students are trained to employ 13 different problem solving heuristics.

Definition

A problem solving heuristic is a *rule of thumb* or *working-method* by which a problem-solving plan may be executed.

We shall take a short tour to look at some of these.

Heuristics

Question 1

Sally opened a book at random. When she multiplied the page numbers of these two pages, the answer was 1640.
What were the two page numbers?

Guess and check

This type of question is first taught at Primary 2 or 3, but with smaller products.

Solution

Trying a few numbers:

$$10 \times 11 = 110$$

Guess and check

This type of question is first taught at Primary 2 or 3, but with smaller products.

Solution

Trying a few numbers:

$$10 \times 11 = 110$$

$$20 \times 21 = 420$$

Guess and check

This type of question is first taught at Primary 2 or 3, but with smaller products.

Solution

Trying a few numbers:

$$10 \times 11 = 110$$
$$20 \times 21 = 420$$
$$30 \times 31 = 930$$

Guess and check

This type of question is first taught at Primary 2 or 3, but with smaller products.

Solution

Trying a few numbers:

$$10 \times 11 = 110$$
$$20 \times 21 = 420$$
$$30 \times 31 = 930$$
$$40 \times 41 = 1640$$

Guess and check

This type of question is first taught at Primary 2 or 3, but with smaller products.

Solution

Trying a few numbers:

$$10 \times 11 = 110$$
$$20 \times 21 = 420$$
$$30 \times 31 = 930$$
$$40 \times 41 = 1640$$

The two page numbers are 40 and 41.

Heuristics

Question 2

An ice-cream stall sells 2-scoop ice-cream that looks like this:

The two flavors must be different.

Heuristics

Question 2

An ice-cream stall sells 2-scoop ice-cream that looks like this:

The two flavors must be different.

The stall has 3 flavors: chocolate, vanilla, strawberry.

Heuristics

Question 2

John wishes to buy all possible arrangements of such 2-scoop ice-cream.

Heuristics

Question 2

John wishes to buy all possible arrangements of such 2-scoop ice-cream. Assuming that John thinks, for instance, that having chocolate on top of vanilla is not the same as having vanilla on top of chocolate, how many cones of ice-cream did he buy?

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V
C	S

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V
C	S
V	C

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V
C	S
V	C
V	S

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V
C	S
V	C
V	S
S	V

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V
C	S
V	C
V	S
S	V
S	C

Making a systematic list

Targeted to Primary 1 students, such a question is solved by:

Solution

Listing out all possible outcomes:

top	bottom
C	V
C	S
V	C
V	S
S	V
S	C

John bought 6 cones of ice-cream.

Language ability

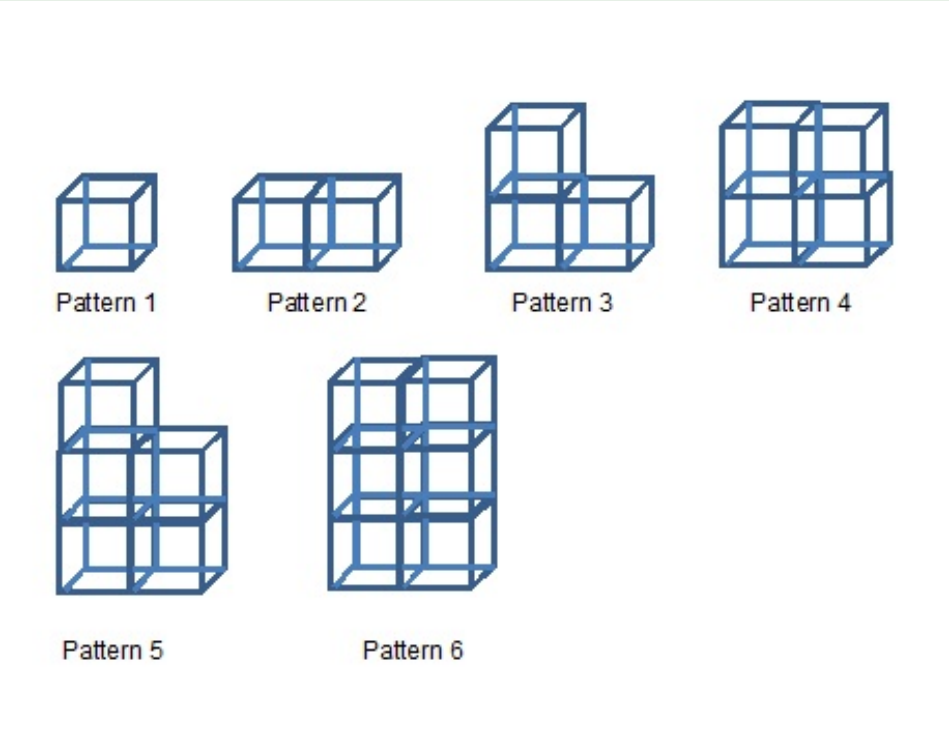
Primary school students are expected to develop a level of English Language competency sufficient for understanding the problem, as reflected in one of the aims of the mathematics curriculum:

> (2) Develop the necessary process skills for the acquisition and application of mathematical concepts and skills.

Heuristics

Question 3

Ali used 3-cm rods to build some structures. The first five structures are shown below.



Heuristics

Question 3

The table below shows the number of rods used for each structure and the height of each structure.

Structure Number	Number of rods used	Height of structure
1	12	3 cm
2	20	3 cm
3	28	6 cm
4	33	6 cm
5	41	9 cm
6		

Heuristics

Question 3

Heuristics

Question 3

(a) Complete the table for Structure 6.

Heuristics

Question 3

(a) Complete the table for Structure 6.

(b) What was the height of Structure 119?

Heuristics

Question 3

(a) Complete the table for Structure 6.

(b) What was the height of Structure 119?

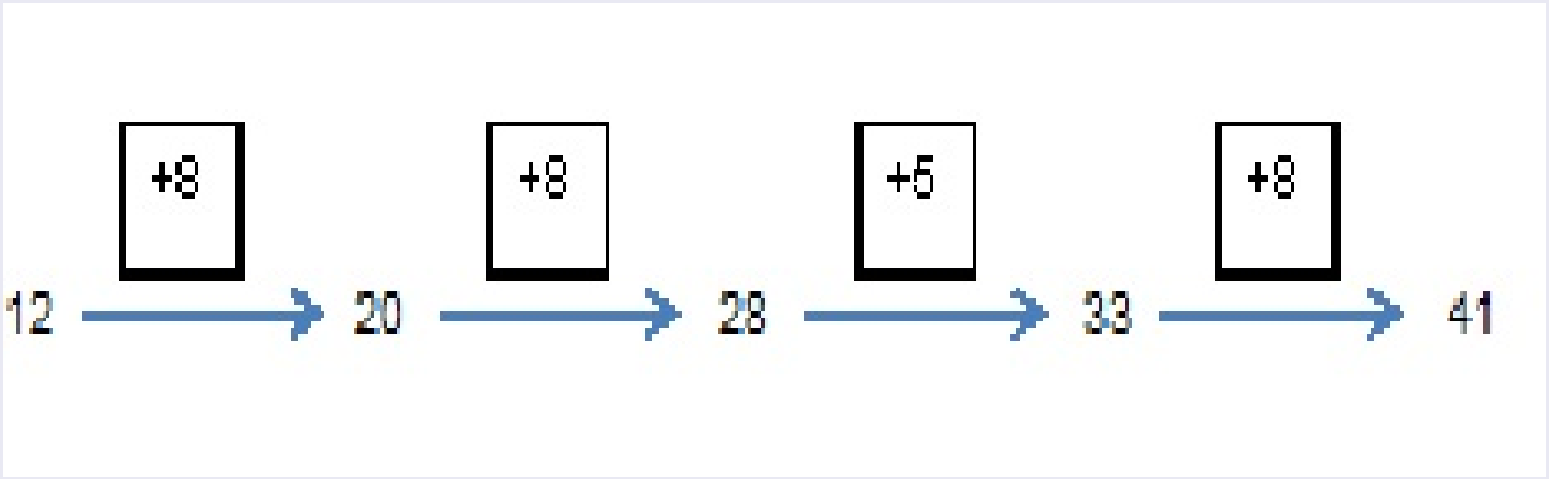
(c) How many rods were used to build Structure 119?

Looking for a pattern

This is a past year paper (PSLE) question.

Solution

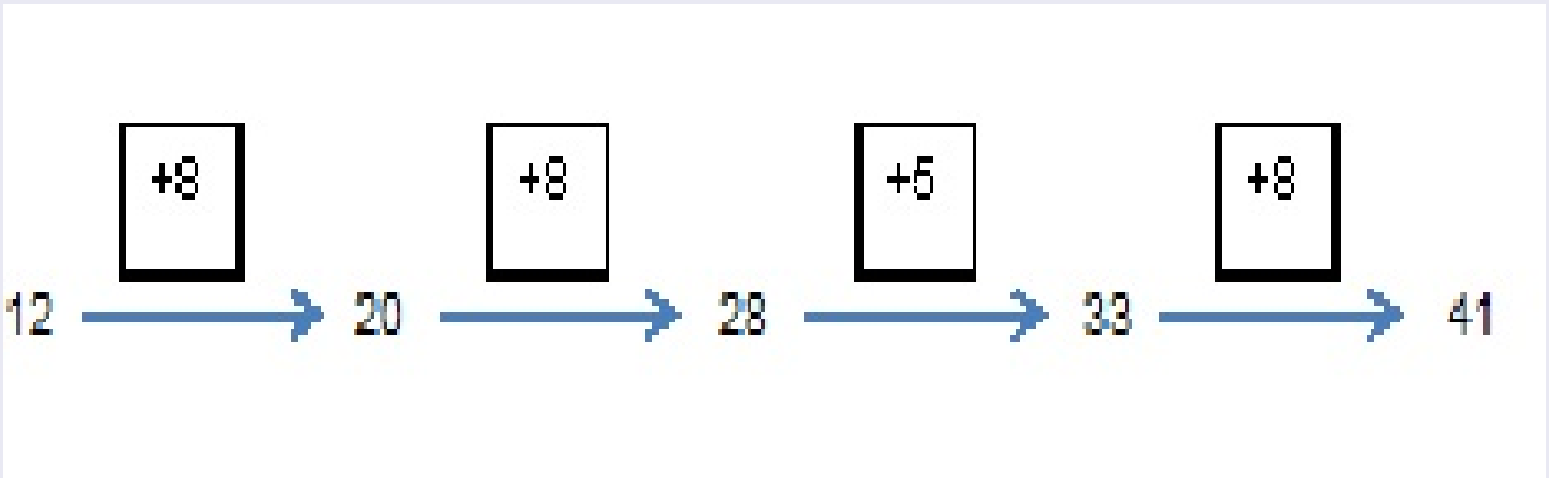
Finding the gap pattern is something typical in primary school mathematics education:



Looking for a pattern

Solution

Notice that there are some 'noise' in the first two terms:

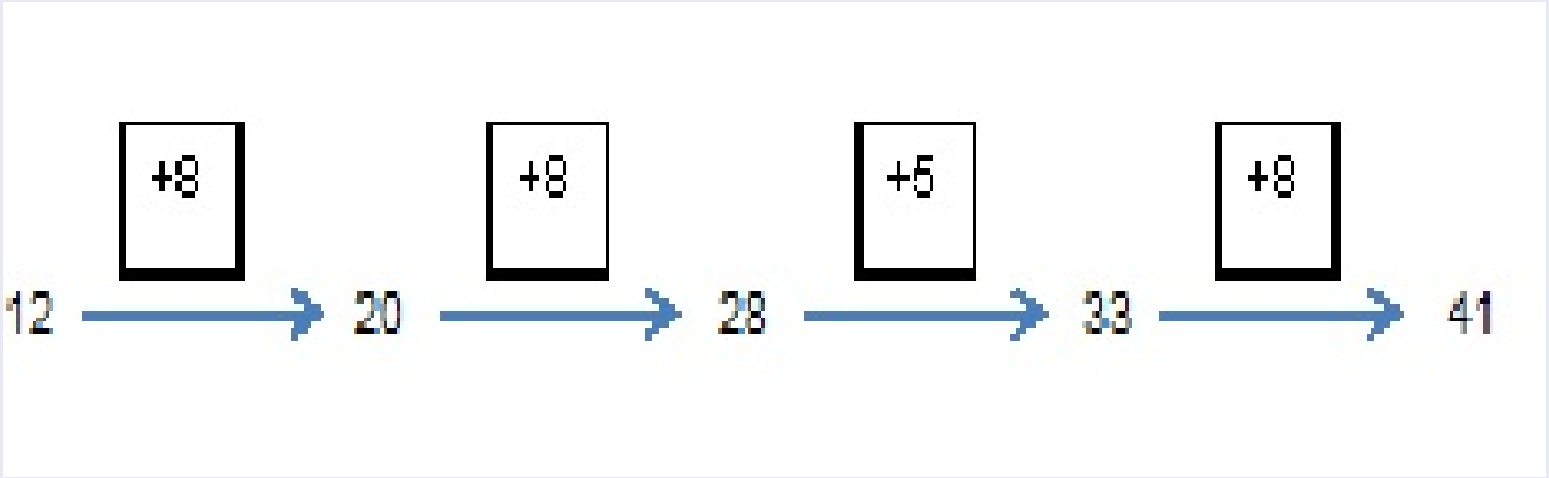


From Structure 2 onwards, the difference pattern is periodic and alternating between $+8$ and $+5$.

Looking for a pattern

Solution

Notice that there are some 'noise' in the first two terms:



From Structure 2 onwards, every occurrence of $+2$ brings about an increase in the level (i.e., an increase to the height by 3 cm).

Looking for a pattern

For Structure 6, we just need to add 5 to the number of rods used in Pattern 5 since there is no increase in the height from Structure 5 to Structure 6.

Looking for a pattern

For Structure 6, we just need to add 5 to the number of rods used in Pattern 5 since there is no increase in the height from Structure 5 to Structure 6.

Solution

Structure 6 requires $41 + 5 = 46$ rods.

Looking for a pattern

1. From Structure 3 to Structure 119, there is a difference of 116 structure numbers.

Looking for a pattern

1. From Structure 3 to Structure 119, there is a difference of 116 structure numbers.
2. Every increase of the structure number by 2 brings about an increase in the height by 3 cm and in the number of rods used by 13.

Looking for a pattern

Solution

$$116 \div 2 = 58$$

Looking for a pattern

Solution

$$116 \div 2 = 58$$

$$58 \times 3 = 174$$

Looking for a pattern

Solution

$$116 \div 2 = 58$$
$$58 \times 3 = 174$$
$$174 + 6 = 180$$

Looking for a pattern

Solution

$$116 \div 2 = 58$$
$$58 \times 3 = 174$$
$$174 + 6 = 180$$
$$58 \times 13 = 754$$

Looking for a pattern

Solution

$$116 \div 2 = 58$$
$$58 \times 3 = 174$$
$$174 + 6 = 180$$
$$58 \times 13 = 754$$
$$754 + 28 = 782$$

Looking for a pattern

Solution

$$116 \div 2 = 58$$
$$58 \times 3 = 174$$
$$174 + 6 = 180$$
$$58 \times 13 = 754$$
$$754 + 28 = 782$$

The height of Structure 119 is 180 cm, while the number of rods used in Structure 119 is 782.

Absence of proper statements

Deviating from traditional presentation format, primary school students are not required to write proper statements to explain what each 'horizontal' working represents.

Heuristics

Question 4

At first, the books in a library were placed on 30 shelves with an equal number of books on each shelf. 3 shelves were removed and the books on these shelves were placed on the remaining 27 shelves. Because of this, the number of books on each remaining shelf increased by 5.
What was the number of books on each shelf at first?

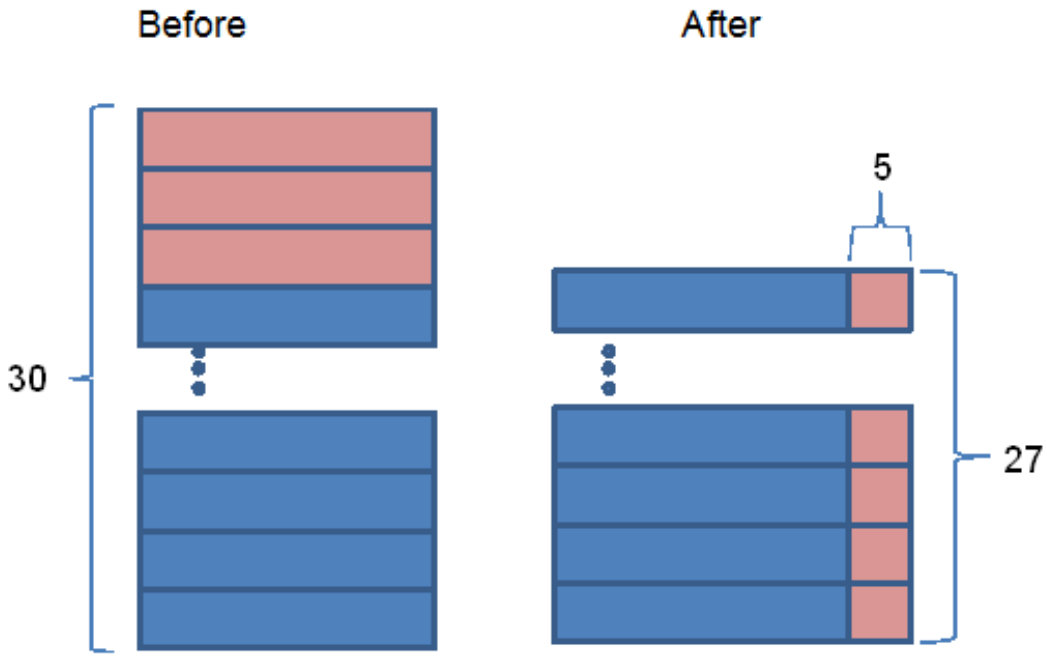
Before-After

Solution

Using a before-after approach, one compares information to find the unknown as one lists information given before and after the action.

Draw a diagram

Crucially to solve this problem, primary school students are advised to use the famous Singapore Model Method.



Draw a diagram

Solution

$$5 \times 27 = 135$$

$$135 \div 3 = 45$$

Draw a diagram

Solution

$$5 \times 27 = 135$$

$$135 \div 3 = 45$$

There were 45 books on each book shelf at first.

Use of Algebra in Primary Mathematics

Algebra is taught at Primary 6, but excludes the use of algebraic equations.

Use of Algebra in Primary Mathematics

Algebra is taught at Primary 6, but excludes the use of algebraic equations.

Why model method?

This explains why there is a need to have the model method.

Some implications

Question 5

Jim bought some chocolate and gave half of them to Ken. Ken bought some sweets and gave half of them to Jim.
Jim ate 12 sweets and Ken ate 18 chocolates. After that, the number of sweets and chocolates Jim had were in the ratio $1 : 7$ and the number of sweets and chocolates Ken had were in the ratio $1 : 4$.
How many sweets did Ken buy?

Some implications

Question 5

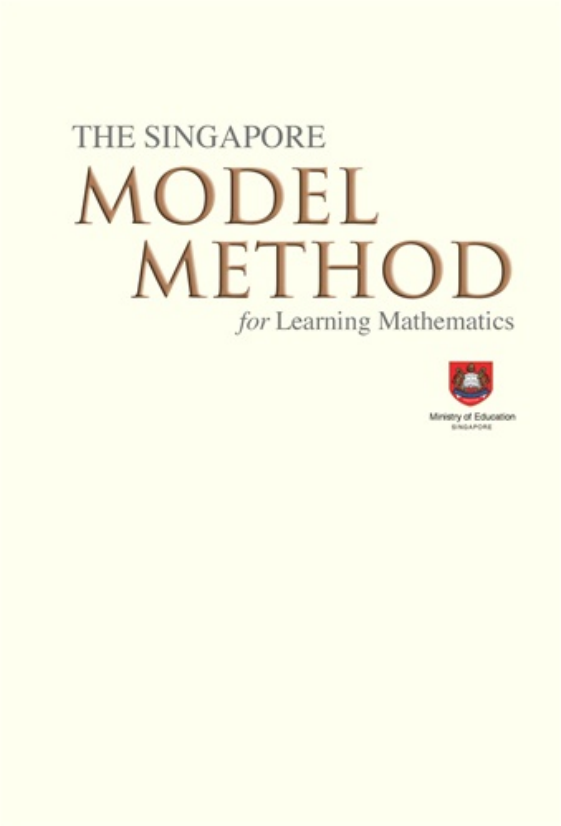
Jim bought some chocolate and gave half of them to Ken. Ken bought some sweets and gave half of them to Jim.
Jim ate 12 sweets and Ken ate 18 chocolates. After that, the number of sweets and chocolates Jim had were in the ratio $1 : 7$ and the number of sweets and chocolates Ken had were in the ratio $1 : 4$.
How many sweets did Ken buy?

Exercise

Use the model method to solve the above problem.

The Model Method

There are a number of books available in the market to help ease the anxiety ...



Use of calculators

The introduction of calculators at P5 and P6 reflects a shift to give more focus to processes such as problem solving skills. The rationale for introducing calculators at the upper primary levels is to:

Use of calculators

The introduction of calculators at P5 and P6 reflects a shift to give more focus to processes such as problem solving skills. The rationale for introducing calculators at the upper primary levels is to:

1. Achieve a better balance between the emphasis on computational skills and problem solving skills in teaching and learning and in assessment.

Use of calculators

The introduction of calculators at P5 and P6 reflects a shift to give more focus to processes such as problem solving skills. The rationale for introducing calculators at the upper primary levels is to:

1. Achieve a better balance between the emphasis on computational skills and problem solving skills in teaching and learning and in assessment.
2. Widen the repertoire of teaching and learning approaches to include investigations and problems in authentic situations.

Use of calculators

The introduction of calculators at P5 and P6 reflects a shift to give more focus to processes such as problem solving skills. The rationale for introducing calculators at the upper primary levels is to:

1. Achieve a better balance between the emphasis on computational skills and problem solving skills in teaching and learning and in assessment.
2. Widen the repertoire of teaching and learning approaches to include investigations and problems in authentic situations.
3. Help students, particularly those with difficulty learning mathematics, develop greater confidence in doing mathematics.

Use of calculators

There is a reassurance from the Ministry of Education that ...

Use of calculators

There is a reassurance from the Ministry of Education that ...

The introduction of calculators would not take away the importance of mental and manual computations. These skills are still emphasised as students need to have good number sense and estimation skills to check the reasonableness of answers obtained using the calculator.

Use of calculators

In view that students will continue to use scientific calculators in their Secondary School education, they are encouraged to own scientific calculators such as the specimen shown below:

Figure: Scientific calculators

Use of calculators

Question 6

For the first 8 months of last year, Ahmad sold an average of 50.75 m of wire per month. He did not sell any wire in the next 4 months. On the average, what was the length of wire that he sold for the 12 months?
(Give your answer correct to 2 decimal places.)

Use of calculators

Question 6

For the first 8 months of last year, Ahmad sold an average of 50.75 m of wire per month. He did not sell any wire in the next 4 months. On the average, what was the length of wire that he sold for the 12 months?
(Give your answer correct to 2 decimal places.)

Solution

$$50.75 \text{ m} \times 8 = 406 \text{ m}$$

$$406 \text{ m} \div 12 = 33.83 \text{ m} \quad (2 \text{ d.p.})$$

Thinking Schools, Learning Nation

MOE's Vision

Ministry of Education's vision of "Thinking Schools, Learning Nation" (TSLN) was first announced by Prime Minister Goh Chok Tong in 1997.

Thinking Schools, Learning Nation

MOE's Vision

Ministry of Education's vision of "Thinking Schools, Learning Nation" (TSLN) was first announced by Prime Minister Goh Chok Tong in 1997.

This vision describes a nation of thinking and committed citizens capable of meeting the challenges of the future, and an education system geared to the needs of the 21st century.

Thinking Schools, Learning Nation

MOE's Vision

Thinking schools will be learning organisations in every sense, constantly challenging assumptions, and seeking better ways of doing things through participation, creativity and innovation. Thinking Schools will be the cradle of thinking students as well as thinking adults and this spirit of learning should accompany our students even after they leave school.

Thinking Schools, Learning Nation

MOE's Vision

A Learning Nation envisions a national culture and social environment that promotes lifelong learning in our people. The capacity of Singaporeans to continually learn, both for professional development and for personal enrichment, will determine our collective tolerance for change.

Higher Order Thinking

The focus is to train school students to develop higher order thinking skills, in the sense of Bloom's taxonomy:

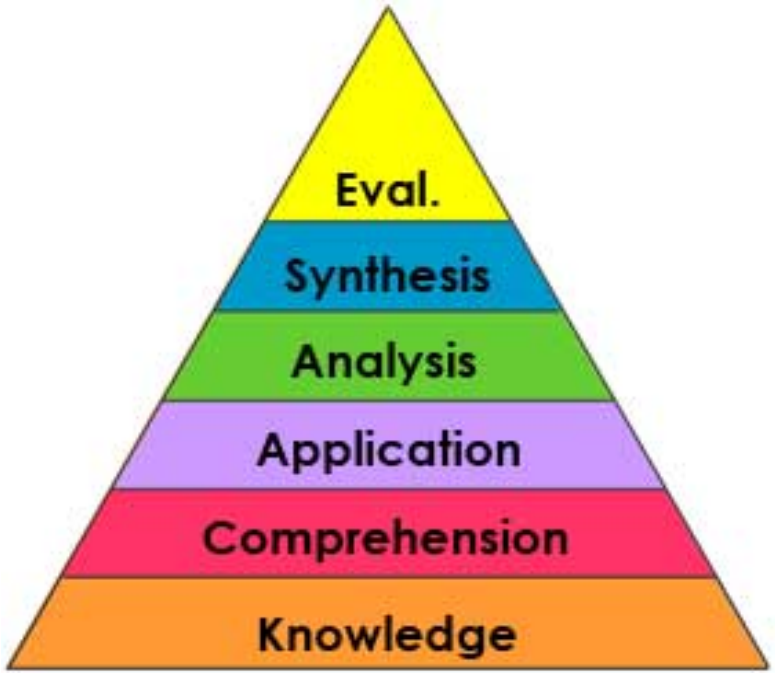
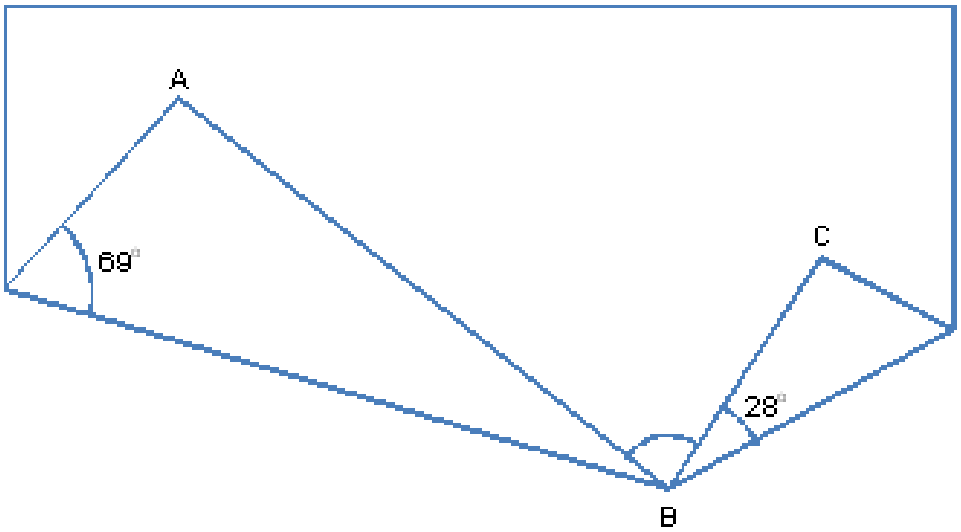


Figure: Bloom's taxonomy

Higher Order Thinking

Question 7

In the figure below, a rectangular piece of paper is folded at two of its corners A and C as shown. Find $\angle ABC$.



Teach Less, Learn More

MOE's initiative

TLLM is about teaching better, engaging the learners, building their character and preparing them for life, rather than teaching a barrage of set formulae for tests and examinations. It revives the passion, mission and aspirations we possessed the day we assumed our roles as educators.

Teach Less, Learn More

Shift from Quantity to Quality in Education

TLLM is also about shifting the focus from "quantity" to "quality" in education. More "quality" in terms of classroom interaction, opportunities for expression, the learning of life-long skills and the building of character through innovative and effective teaching approaches and strategies. Less "quantity" in terms of rote-learning, repetitive tests and following prescribed answers and set formulae.

Teach Less, Learn More

Shift from Quantity to Quality in Education

As part of the blueprint on holistic education laid out by Mr Tharman Shanmugaratnam, Minister for Education, in a Parliamentary speech, we "will seek to cut back on quantity ... so as to provide more 'white space' in the curriculum, space which gives schools and teachers the room to introduce their own programmes, to inject more quality in teaching, or give students themselves the room to exercise initiative and shape more of their own learning."

Implications on Mathematics Education

TLLM has important implications on Singapore's Mathematics Education:

Implications on Mathematics Education

TLLM has important implications on Singapore's Mathematics Education:

- Movement of topics from Primary to Secondary

Implications on Mathematics Education

TLLM has important implications on Singapore's Mathematics Education:

- Movement of topics from Primary to Secondary
- Movement of topics from Elementary to Additional Mathematics

Implications on Mathematics Education

TLLM has important implications on Singapore's Mathematics Education:

- Movement of topics from Primary to Secondary
- Movement of topics from Elementary to Additional Mathematics
- Movement of topics from A-Level to O-Level

Implications on Mathematics Education

TLLM has important implications on Singapore's Mathematics Education:

- Movement of topics from Primary to Secondary
- Movement of topics from Elementary to Additional Mathematics
- Movement of topics from A-Level to O-Level
- Removal of topics

Movement from Primary to Secondary

- Square roots, Cube roots

Movement from Primary to Secondary

- Square roots, Cube roots
- Algebraic equations

Movement from Primary to Secondary

- Square roots, Cube roots
- Algebraic equations
- Geometrical construction of bisectors and figures

New additions to Secondary Mathematics

- New statistical representations such as box-and-whisker diagram and dots plot

New additions to Secondary Mathematics

- New statistical representations such as box-and-whisker diagram and dots plot
- Number patterns with closed formulae

Movement from Elementary to Additional Mathematics

- Application of inverse matrices in solving simultaneous linear equations in 2 variables

Notably, plane geometry questions requiring geometrical proofs are introduced in Additional Mathematics.

Movement from A-Level to O-Level

- Partial fractions

Removal of topics

- Matrices as geometrical transformation from Elementary Mathematics
- Trigonometry (3D-trigonometry, small angles, complicated trigonometrical identities) from A-Level Mathematics
- Removal of Mechanics option, only left with Pure Math and Statistics
- Entire subject of Further Mathematics, with selected topics put into H2 Mathematics

Sample questions from Secondary Mathematics

Following from Primary Mathematics, Secondary Mathematics cover in a more in-depth manner:

- number and algebra,

Sample questions from Secondary Mathematics

Following from Primary Mathematics, Secondary Mathematics cover in a more in-depth manner:

- number and algebra,
- geometry and measurement, as well as

Sample questions from Secondary Mathematics

Following from Primary Mathematics, Secondary Mathematics cover in a more in-depth manner:

- number and algebra,
- geometry and measurement, as well as
- statistics and probability units.

Showcase of non-routine questions

Question 8

Written as the product of its prime factors

$$4900 = 2^2 \times 5^2 \times 7^2.$$

Showcase of non-routine questions

Question 8

Written as the product of its prime factors

$$4900 = 2^2 \times 5^2 \times 7^2.$$

(a) Express 168 as the product of its prime factors.

Showcase of non-routine questions

Question 8

Written as the product of its prime factors

$$4900 = 2^2 \times 5^2 \times 7^2.$$

(a) Express 168 as the product of its prime factors.

(b) Hence write down

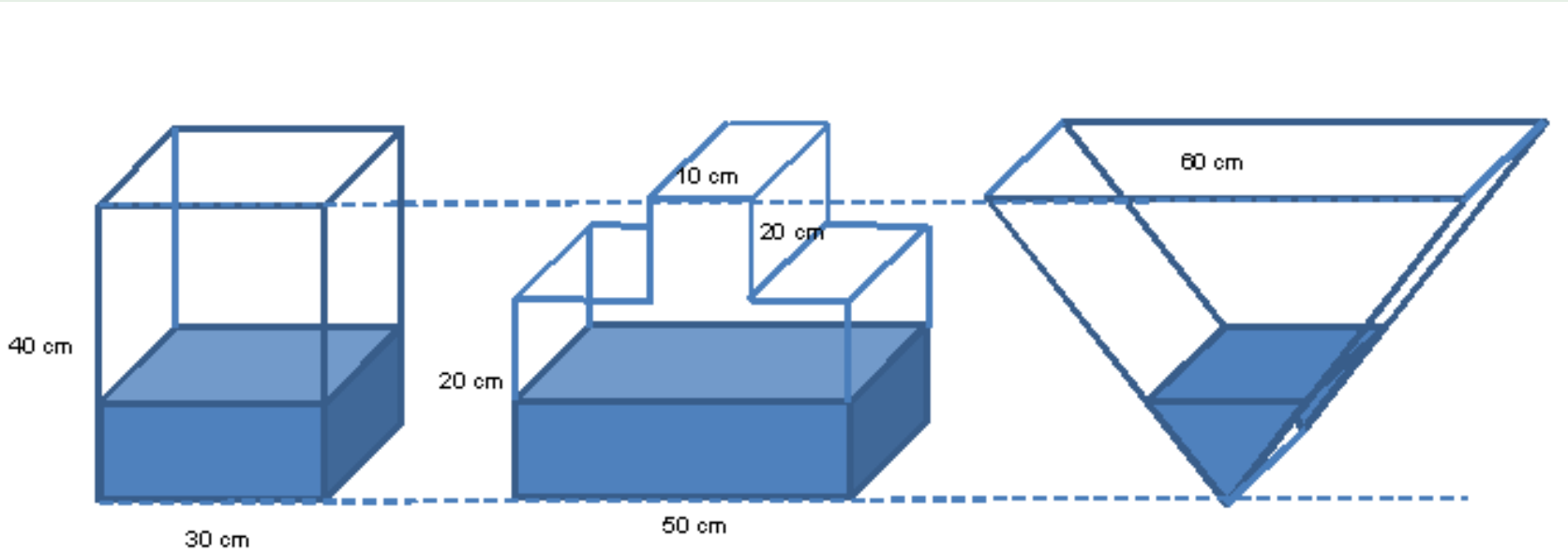
(i) the LCM of 4900 and 168, giving your answer as the product of its prime factors.

(ii) the greatest integer that will divide both 4900 and 168 exactly.

Showcase of non-routine questions

Question 9

Each of the containers shown in the diagram has a height of 40 cm. Their other dimensions are as shown.



Showcase of non-routine questions

Question 9

All three containers have uniform cross-sections.
The containers are being filled to the brim with water which flows into each one at some constant rate.
It takes 12 minutes to fill each container. Sketch the graphs for all three containers showing the relationship between the depth of water, d cm, and the time, t minutes, as each container is being filled.

Showcase of non-routine questions

Question 10

Three integers, a, b and c, are such that $a < b < c$. The three integers are said to form a Pythagorean Triple if

$$a^2 + b^2 = c^2.$$

(a) Show that 7, 24 and 25 form a Pythagorean Triple.

Showcase of non-routine questions

Question 10

Three integers, a, b and c, are such that $a < b < c$. The three integers are said to form a Pythagorean Triple if

$$a^2 + b^2 = c^2.$$

(a) Show that 7, 24 and 25 form a Pythagorean Triple.

(b) Form a Pythagorean Triple

(i) in which the last two integers are 40 and 41,

(ii) in which the first integer is 11.

Showcase of non-routine questions

Question 11

Given that n is an integer,

(a) write down expression for the next two odd numbers after $2n - 1$,

Showcase of non-routine questions

Question 11

Given that n is an integer,

(a) write down expression for the next two odd numbers after $2n-1$,

(b) (i) find, in its simplest form, the expression for the sum of these three odd numbers,

Showcase of non-routine questions

Question 11

Given that n is an integer,

(a) write down expression for the next two odd numbers after $2n - 1$,

(b) (i) find, in its simplest form, the expression for the sum of these three odd numbers,

(ii) explain why the sum is a multiple of 3,

Showcase of non-routine questions

Question 11

Given that n is an integer,

(a) write down expression for the next two odd numbers after $2n - 1$,

(b) (i) find, in its simplest form, the expression for the sum of these three odd numbers,

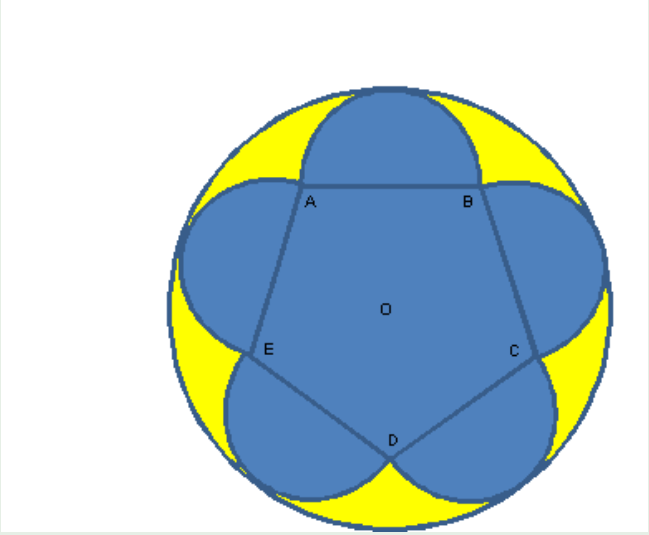
(ii) explain why the sum is a multiple of 3,

(c) find, in its simplest form, an expression for the sum of the squares of these three odd numbers.

Showcase of non-routine questions

Question 11

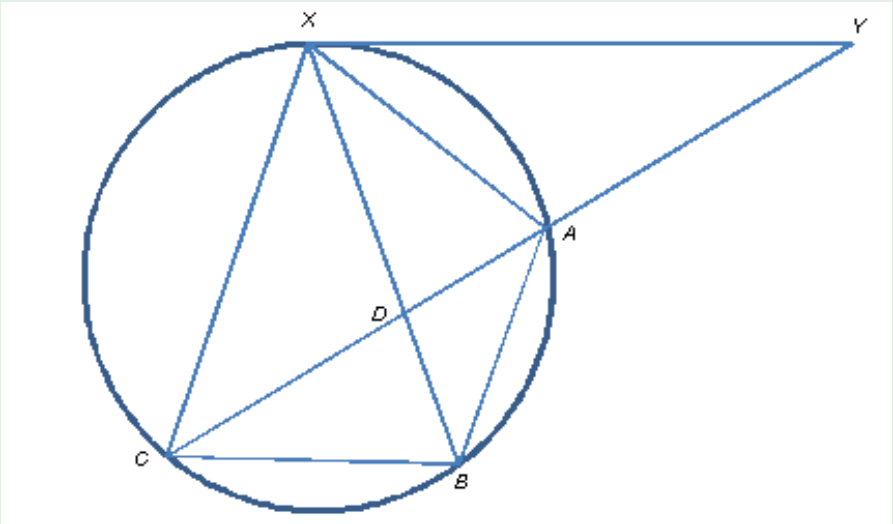
The diagram shows an ornament which is made of wire.
It consists of a regular pentagon, $ABCDE$, of side 4 cm, five equal semi-circles with diameter 4 cm and an outer circle, centre O.
Each semi-circle touches the outer circle as shown.



Find the area of one of the yellow regions in the above diagram.

Showcase of non-routine questions

Question 12



The diagram shows a point X on a circle and XY is a tangent to the circle. Points A, B and C lie on the circle such that XA bisects the angle YXB and YAC is a straight line. The lines YC and XB intersect at D.

Showcase of non-routine questions

Question 12

(i) Prove that $AX = AB$.

(ii) Prove that CD bisects angle XCB.

(iii) Prove that triangles CDX and CDA are similar.

A-Level Mathematics

This is now offered at three levels:

A-Level Mathematics

This is now offered at three levels:

1. H1

A-Level Mathematics

This is now offered at three levels:

1. H1
2. H2

A-Level Mathematics

This is now offered at three levels:

1. H1
2. H2
3. H3 (only for high ability H2 Math students)

Singapore A-Level Mathematics

There are three levels of assessment objectives for the examination. The assessment will test candidates' abilities to:

Singapore A-Level Mathematics

There are three levels of assessment objectives for the examination. The assessment will test candidates' abilities to:

AO1 understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphic calculators;

Singapore A-Level Mathematics

There are three levels of assessment objectives for the examination. The assessment will test candidates' abilities to:

AO1 understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphic calculators;

AO2 reason and communicate mathematically through writing mathematical explanation, arguments and proofs, and inferences;

Singapore A-Level Mathematics

There are three levels of assessment objectives for the examination. The assessment will test candidates' abilities to:

AO1 understand and apply mathematical concepts and skills in a variety of contexts, including the manipulation of mathematical expressions and use of graphic calculators;

AO2 reason and communicate mathematically through writing mathematical explanation, arguments and proofs, and inferences;

AO3 solve unfamiliar problems; translate common realistic contexts into mathematics; interpret and evaluate mathematical results, and use the results to make predictions, or comment on the context.

Use of Graphic Calculator (GC)

- The use of GC without computer algebra system will be expected.

Use of Graphic Calculator (GC)

- The use of GC without computer algebra system will be expected.
- The examination papers will be set with the assumption that candidates will have access to GC.

Use of Graphic Calculator (GC)

Figure: Commonly Used Models of Graphic Calculators

GC in action: Investigative work

Question 13

The sequence x_n is given by the recurrence relation

$$x_{n+1} = 4x_n(1 - x_n).$$

Using a graphic calculator, study the behaviour of the sequence in terms of its convergence for each of the following cases:

(i) $x_0 = 1$

(ii) $x_0 = 0.75$

(iii) $x_0 = 0.5$

(iv) $x_0 = 0.25$

Use of Graphic Calculator (GC)

Question 14

(i) In a group of 20 persons, each person has a different birthday. Find the total number of arrangements in which this is possible.

Use of Graphic Calculator (GC)

Question 14

(i) In a group of 20 persons, each person has a different birthday. Find the total number of arrangements in which this is possible.

(ii) In a group of N persons, each person has a different birthday. Show that the total number of possible arrangements is

$$\frac{365!}{(365-N)!}$$

Using a graphic calculator, find the least integer value of N for which the probability that there are at least two persons in a group of N having the same birthday exceeds 0.5.

Use of ICT

ICT stands for Information and Communication Technology. Its usage in classroom teaching has been initiated by IT Master Plans 1-3.

Use of ICT

ICT stands for Information and Communication Technology. Its usage in classroom teaching has been initiated by IT Master Plans 1-3.

IT1 Introduce ICT infrastructure and lessen content to free up time for ICT

Use of ICT

ICT stands for Information and Communication Technology. Its usage in classroom teaching has been initiated by IT Master Plans 1-3.

IT1 Introduce ICT infrastructure and lessen content to free up time for ICT

IT2 Integrate ICT into lessons

Use of ICT

ICT stands for Information and Communication Technology. Its usage in classroom teaching has been initiated by IT Master Plans 1-3.

IT1 Introduce ICT infrastructure and lessen content to free up time for ICT

IT2 Integrate ICT into lessons

IT3 Student independence in learning via ICT

Assessment modes

Assessment modes are both formative and summative.

Formative assessment

Formative assessment takes the form of

Formative assessment

Formative assessment takes the form of

- classroom questions and answers

Formative assessment

Formative assessment takes the form of

- classroom questions and answers
- students' consultation

Formative assessment

Formative assessment takes the form of

- classroom questions and answers
- students' consultation
- daily classwork, homework and assignments

Summative assessment

Summative assessment takes the form of

Summative assessment

Summative assessment takes the form of

- continual assessment in the form of class tests, common tests, etc.

Summative assessment

Summative assessment takes the form of

- continual assessment in the form of class tests, common tests, etc.
- semestral assessment (mid-year examinations)

Summative assessment

Summative assessment takes the form of

- continual assessment in the form of class tests, common tests, etc.
- semestral assessment (mid-year examinations)
- end-of-year examination

National Examinations

The major national examinations are as follows:

National Examinations

The major national examinations are as follows:

1. Primary Six Leaving Examination (PSLE)

National Examinations

The major national examinations are as follows:

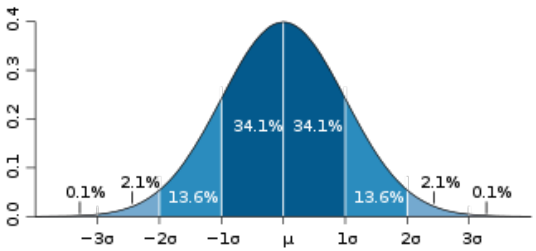
1. Primary Six Leaving Examination (PSLE)
2. GCE 'O' Level Examination

National Examinations

The major national examinations are as follows:

1. Primary Six Leaving Examination (PSLE)
2. GCE 'O' Level Examination
3. GCE 'A' Level Examination

Alternative assessment



"The normal curve is a distribution most appropriate to chance and random activity. Education is a purposeful activity and we seek to have students learn what we would teach. Therefore, if we are effective, the distribution of grades will be anything but a normal curve. In fact, a normal curve is evidence of our failure to teach." – Benjamin Bloom

Alternative assessment

Apart from the traditional test-examination modes, there are alternatives:

Alternative assessment

Apart from the traditional test-examination modes, there are alternatives:

- performance assessment

Alternative assessment

Apart from the traditional test-examination modes, there are alternatives:

- performance assessment
- presentation

Alternative assessment

Apart from the traditional test-examination modes, there are alternatives:

- performance assessment
- presentation
- demonstration/exhibition

Alternative assessment

Apart from the traditional test-examination modes, there are alternatives:

- performance assessment
- presentation
- demonstration/exhibition
- portforlio assessment

Learning experience

Learning experience refers to activities organized for the purpose of experiencing the usage of mathematics in real-life applications.

Learning experience

Exemplar

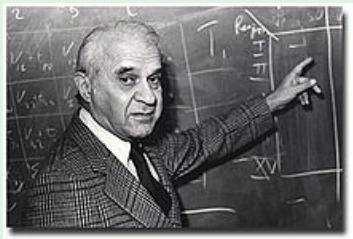


Figure: Wassily Wassilyovich Leontief

Wassily Wassilyovich Leontief (Russian: August 5, 1906, Munich, Germany February 5, 1999, New York), was a Russian-American economist notable for his research on how changes in one economic sector may have an effect on other sectors.

Learning experience

Exemplar

A open economy has three industries producing respectively Water, Electricity and Coal. The number of units of output of industry S_i needed to produce 1 unit of output of industry S_j is recorded in the matrix below

$$\mathbf{A} = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.5 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.1 \end{pmatrix}$$

Learning experience

Exemplar

Let the number of units of output of industry S_i be x_i, and the external demand for industry S_i be b_i. Then

Learning experience

Exemplar

Let the number of units of output of industry S_i be x_i, and the external demand for industry S_i be b_i. Then

$$\begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.5 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

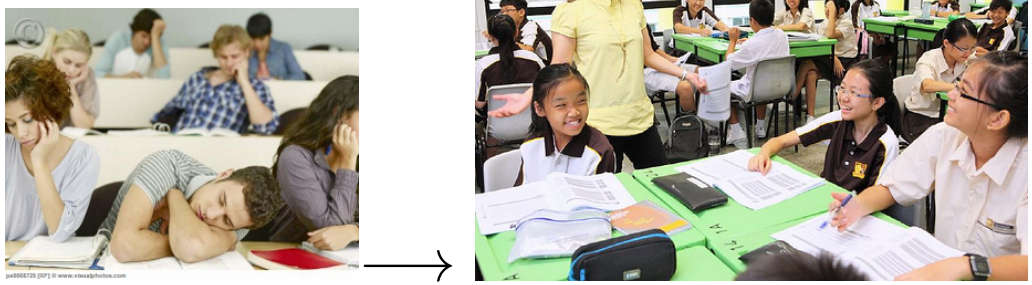
Learning experience

Exemplar

Given $\mathbf{b}$, one can solve the system of linear equations

$$\mathbf{A}\mathbf{x} + \mathbf{b} = \mathbf{x}$$
$$(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{b}$$
$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}$$

Student-centred learning



The future trend will be a continued movement:

Teacher-centred learning $\longrightarrow$ Student-centred learning

Position in current syllabus

Currently, mathematically modelling is already officially stated as one of the skills expected to be taught and learnt in Singapore mathematics classrooms:

> Mathematical modelling is the process of formulating and improving a mathematical model to represent and solve real-world problems. Through mathematical modelling, students learn to use a variety of representations of data, and to select and apply appropriate mathematical methods and tools in solving real-world problems. The opportunity to deal with empirical data and use mathematical tools for data analysis should be part of the learning at all levels.

Mathematical modelling in schools

There is a pilot project with several secondary schools, in promoting mathematical modelling as a means of learning mathematics through meaningful contexts.

Mathematical modelling

What is mathematical modelling?

A *mathematical model* is a description of a system using mathematical language and concepts.

Mathematical modelling

What is mathematical modelling?

Mathematical modelling refers to the business of manufacturing or building a mathematical model.

Modelling tasks

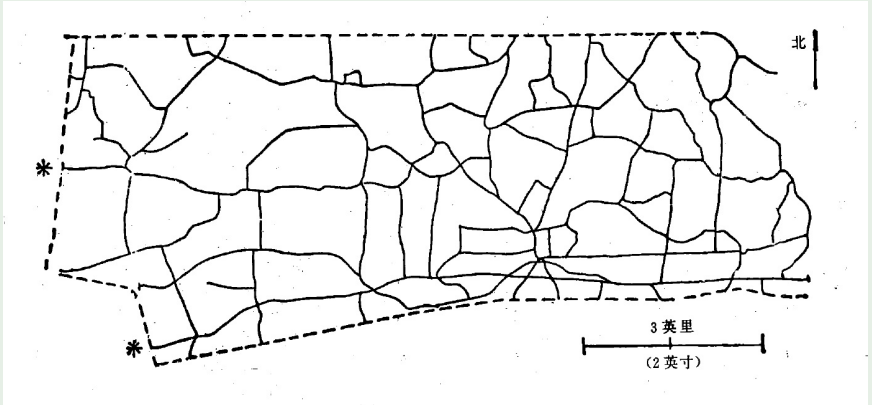
Snowplow deployment in Maryland

In the county of Wicomico in the state of Maryland, snowplows are deployed to clear snow after heavy snowstorms.

Modelling tasks

Snowplow deployment in Maryland

In a particular scenario, two snowplows are dispatched respectively from a garage about 4 miles west of each of the two points demarcated by $*$'s on the dual-carriageway layout map below:



Propose an efficient deployment scheme for the snowplows in clearing the snow?

Modelling tasks

Vending machine

A typical can-drinks vending machine accepts coins of different denominations, and upon receiving the user-supplied choice of soft drinks, together with the correct payment, dispenses the correct product.

There are several different products, not necessarily equally priced.

Modelling tasks

Vending machine

- A soft-drink company organizes a charity activity in which each participant is given a set of N coins with randomly distributed denominations.

Modelling tasks

Vending machine

- A soft-drink company organizes a charity activity in which each participant is given a set of N coins with randomly distributed denominations.
- The coins are then inserted into a prototypical vending machine.

Modelling tasks

Vending machine

- If the amount inserted is sufficient, the participant gets to choose the product he or she desires. Otherwise, the participants tops up with his or her own money for the desired product. This top-up is then donated to charity.

Modelling tasks

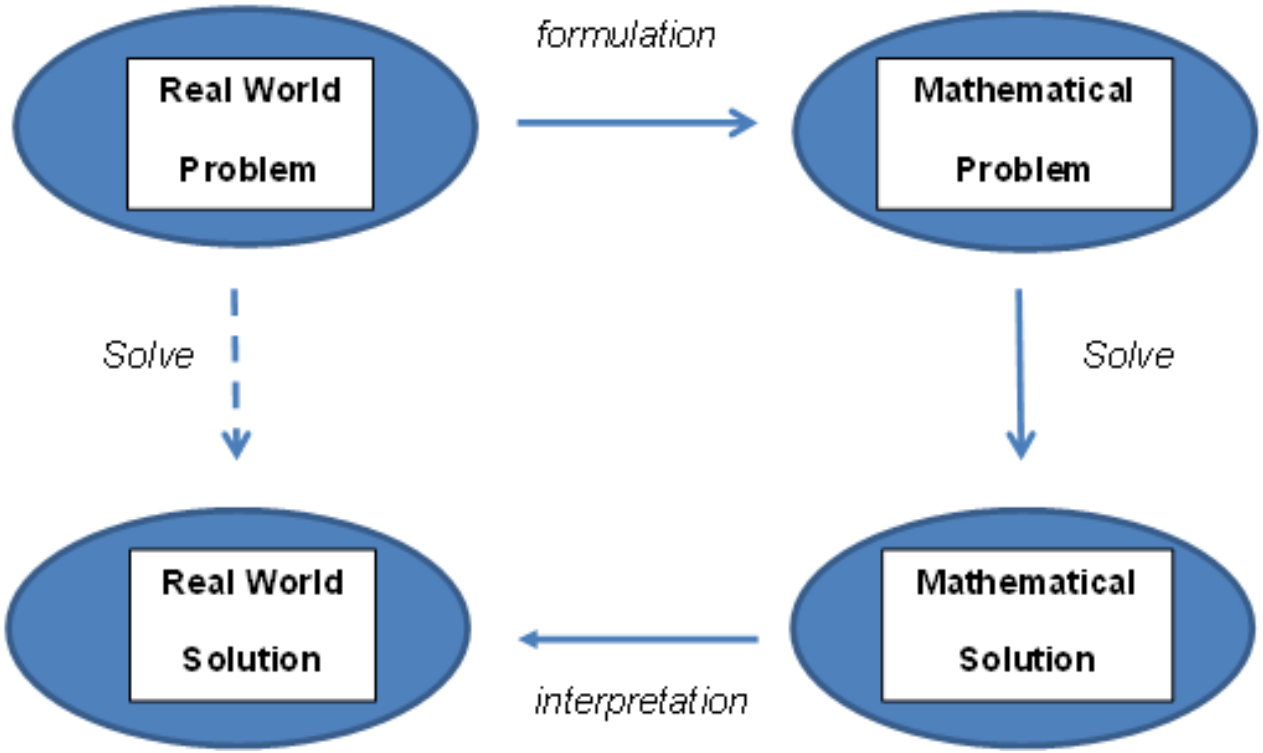
Vending machine

- If the amount inserted is sufficient, the participant gets to choose the product he or she desires. Otherwise, the participants tops up with his or her own money for the desired product. This top-up is then donated to charity.

Design a feasible distribution of the denominations for each participant, giving a suitable fixed number N.

Mathematical modelling cycle

Mathematical modelling is characterized by the following cycle:



Mathematical modelling outreach

A mathematical modelling outreach is a purposeful and concerted effort to organize a two to three-days program to promote mathematical modelling as a meaningful channel or platform to acquire mathematical knowledge and put it to good practice.

MMO and Lee Peng Yee Symposium 2010

For a good example of such a kind of outreach program, you may wish to visit the website:
`http://www.nie.edu.sg/nienews/dec10/17-01.html`.

The End

"The point is to make math intrinsically interesting to children. We should not have to sell mathematics by pointing to its usefulness in other subject areas, which, of course, is real. Love for math will not come about by trying to convince a child that it happens to be a handy tool for life; it grows when a good teacher can draw out a child's curiosity about how numbers and mathematical principles work." – Alfred Posamentier, Professor of Mathematics Education at the City College of New York, 2002.