Preparing Mathematics Teachers in Singapore:
The Issue of Mathematics Content Knowledge

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Abstract

There are two necessary *knowledges* for a mathematics teacher – Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge. Teacher preparation institutions normally focus on the pedagogical methods and expect the student teacher's prior university education to prepare her for the subject content. This chapter surveys the literature regarding the relationship between these two knowledges and focuses on the less understood Mathematics Content Knowledge. We also describe in some detail the responses to some inadequacies in teacher Mathematics Content Knowledge in the various programmes at the National Institute of Education, Singapore.

Introduction

Lee Shulman (1986) brought to the forefront the need to distinguish the different dimensions of teacher knowledge that will help guide teacher preparation. Instead of the prevalent view then of ensuring that the student teacher enters teacher training with adequate subject content knowledge and then equipping her with generic pedagogical knowledge, Shulman introduced the dimension at the nexus of subject content knowledge and generic pedagogical knowledge as Pedagogical Content Knowledge. We, as mathematics educators, are happy to claim Shulman as one of our own, as he based much of his research and writing on the discipline of mathematics.

Others such as Ball, Thames & Phelps (2008) have added other dimensions or sub-dimensions of teacher knowledge but in this chapter, we shall just use Shulman's original trichotomisation for its parsimony and its direct relation to the teacher education curriculum - for example in our institute, the three dimensions of Content Knowledge, Pedagogical Content Knowledge and generic pedagogical knowledge, neatly fall into our three main components of Academic Subject, Curriculum Studies and Educational Studies.

This chapter is on mathematics teacher education, so we shall use the following abbreviations for Mathematical Content Knowledge (CK) and Mathematical Pedagogical Content Knowledge (PCK), leaving out the "M" for brevity. As mathematics educators, our areas of concern are Academic Subject and Curriculum Studies, and so we will leave the area of general pedagogical knowledge to our generalist colleagues. Mathematics teacher educators also generally gear their Curriculum Studies courses along the lines of Shulman's PCK. However, the twin foci of general pedagogy and PCK quite often leave CK unattended, particularly for short PGDE programmes where the student teachers' prior university education is expected to have prepared for the subject content. Yet this expectation has not always been fulfilled because of the broadening and diversification of qualifications of the intakes over time. Before we proceed to detail why CK is lacking and how NIE has responded to make up for the deficit, we shall spend some time elaborating first on the nature of CK and its relationship with PCK, and the performance of Singapore prospective teachers in these areas in an international survey.

We first refer to the celebrated work of Liping Ma in her book *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (Ma, 2010). She makes the important assertion that even the hierarchically lowest field of mathematics, i.e., elementary mathematics, is a "field of depth, breadth, and thoroughness" (p. 122) which makes it possible (and thus, necessary for good teaching) for elementary school teachers to attain a "profound understanding" of elementary mathematics. Ma (2010) remarks that Chinese elementary school mathematics teachers seem to know more even though they do not have college degrees:

> "Chinese students typically outperform U.S. students on international comparisons of mathematics competency. Paradoxically, Chinese teachers seem far less mathematically educated than U.S. teachers. Most Chinese teachers have had 11 to 12 years of schooling – they complete ninth grade and attend normal school for two or three years. In contrast, most U.S. teachers have received between 16 and 18 years of formal schooling – a bachelor's degree in college and often one or two years of further study."
> (Ma, 2010, p.xxvi)

Ma (2010, p. xxvi) proceeds then to explain that "Chinese teachers begin their teaching careers with a better understanding of elementary mathematics than that of most U.S. elementary teachers … continues to grow throughout their professional lives."

Usiskin (2001) argues that the mathematics in CK, which he called *teachers' mathematics*, should be seen as a special kind of mathematics.

> "Even though taking more and more mathematics courses would not seem to have any down side, it can create a problem. Often the more mathematics courses a teacher takes, the wider the gap between the mathematics the teacher studies and the mathematics the teacher teaches. The result of the mismatch is that the teachers are often no better prepared in the content that they have to teach than when they were students taking that content." (Usiskin, 2001, p.86)

He proceeds in the paper to lay out three kinds of mathematics not found in typical college mathematics courses, i.e. mathematics generalisations and extensions, concept analysis, and problem analysis.

An important study by Baumert et al. (2010) showed convincingly that the CK and PCK of teachers affect the mathematical achievement of their students. The study was conducted in Germany within the COACTIV ("*Professional knowledge of teachers, cognitively activating mathematics teaching, and the development of mathematical competence*") project over a one-year period. It involved a representative sample of 4353 Grade 10 students and their 181 teachers.

The study conceptualized CK as “a profound mathematical understanding of the mathematics taught at school” (p. 142). PCK is seen as a “distinct body of instruction-[related] and student-related mathematical knowledge and skills—the knowledge that makes mathematics accessible to students” (p.142). One key claim is that the study has distinguished CK and PCK of secondary mathematics teachers conceptually and empirically. Also, PCK was reported to have explained 39% of the between-class variance in achievement at the end of Grade 10, with the implication that PCK largely determines the cognitive structure of mathematical learning opportunities. Although CK was found to be highly correlated with PCK, the results show that CK has lower predictive power for student progress.

Baumert and his colleagues’ conceptualization of CK as a “profound mathematical understanding” is certainly derived from the work of Ma (2010). Baumert et al. (2010) cite Kahan, Cooper, and Bethea’s (2003) assertion that strong CK is “a factor in recognizing and seizing teachable moments” (p. 245) and themselves add that “CK defines the possible scope for the development of PCK” (p. 166) to emphasise their view that CK is “necessary for, but not identical with, a rich repertoire of skills and methods for teaching mathematics” (p. 146). Thus, any emphasis on “subject matter knowledge” in teacher preparation should clearly explicate this conception in terms of CK and PCK, and then take into consideration that CK alone is not enough but that PCK makes greater contribution towards student progress. These need to be made clear so as to finally construct a teacher preparation curriculum that foundationally covers enough of the depth, breadth and thoroughness of the mathematics to be taught at school, and then builds in PCK modules on top of the CK foundation.

The International Association for the Evaluation of Educational Achievement (IEA) conducted the Teacher Education Study in Mathematics (TEDS-M) 2012 in 17 countries “to provide data on the knowledge that [prospective] primary and lower-secondary school teachers acquire during their mathematics teacher education … [and] to examine variations in the nature and influence of teacher education programs within and across countries” (Tatto et al., 2012, p. 17). Chapter 5 of the TEDS-M report (Tatto et al., 2012) is devoted to the CK and PCK of prospective primary and lower-secondary teachers. Items spanning four content subdomains (number and operations, algebra and functions, geometry and measurement, and data and chance) were used to assess CK. Items addressing PCK spanned three subdomains, i.e., curricular knowledge, planning for teaching and learning, and enacting teaching and learning.

Prospective teachers’ CK and PCK were reported in scaled scores generated through the use of item response theory (IRT) with the mean for each of the four scales (primary CK, primary PCK, lower-secondary CK, lower-secondary PCK) at 500 and the standard deviation at 100. Tables 1 and 2 are adapted from four tables presented in Tatto et al. (2012, p. 139, 143, 147, 150). (We have no space in this chapter but it must be noted that the authors take pains to list the limitations with regard to the country coverage for each of their tables.)

Table 1: Prospective primary school teachers' mathematics CK and PCK (adapted with permission from International Association for the Evaluation of Educational Achievement (2012))

Program-Group	Country	Valid data	Mean for CK (SE) [Rank]	Mean for PCK (SE) [Rank]
Group 1: Lower Primary (to Grade 4 maximum)	Georgia	506	345 (4) [21]	345 (5) [21]
	Germany	907	501 (3) [14]	491 (5) [16]
	Poland	1799	456 (2) [17]	452 (2) [18]
	Russian Federation	2260	536 (10) [8]	512 (8) [12]
	Switzerland	121	512 (6) [12]	519 (6) [11]
Group 2: Primary (to Grade 6 maximum)	Chinese Taipei	923	623 (4) [1]	592 (2) [2]
	Philippines	592	440 (8) [19]	457 (10) [17]
	Singapore	262	586 (4) [4]	588 (4) [3]
	Spain	1093	481 (3) [16]	492 (2) [15]
	Switzerland	815	548 (2) [7]	539 (2) [9.5]
	United States	951	518 (5) [11]	544 (3) [7]
Group 3: Primary and Secondary Generalists (to Grade 10 maximum)	Botswana	86	441 (6) [18]	448 (9) [19]
	Chile	654	413 (2) [20]	425 (4) [20]
	Norway (ALU)	392	509 (4) [13]	539 (3) [9.5]
	Norway (ALU+)	159	553 (6) [6]	564 (6) [5]
Group 4: Primary Mathematics Specialists	Germany	97	555 (8) [5]	552 (7) [6]
	Malaysia	574	488 (2) [15]	503 (3) [14]
	Poland	300	614 (5) [2]	575 (4) [4]
	Singapore	117	600 (8) [3]	604 (7) [1]
	Thailand	660	528 (2) [9]	506 (2) [13]
	United States	132	520 (7) [10]	545 (6) [8]

Table 2: Prospective secondary school teachers' mathematics CK and PCK (adapted with permission from International Association for the Evaluation of Educational Achievement (2012))

Program-Group	Country	Valid data	Mean for CK (SE) [Rank]	Mean for PCK (SE) [Rank]
Group 5: Lower Secondary (to Grade 10 maximum)	Botswana	34	436 (7) [17]	436 (9) [20]
	Chile	741	354 (3) [22]	394 (4) [22]
	Germany	406	483 (5) [12]	515 (6) [10]
	Philippines	733	442 (5) [18]	450 (5) [18]
	Poland	158	529 (4) [9]	520 (5) [9]
	Singapore	142	544 (4) [7]	539 (6) [7]
	Switzerland	141	531 (4) [8]	549 (6) [4]
	Norway (ALU)	344	435 (3) [20]	455 (4) [17]
	Norway (ALU+)	148	461 (5) [16]	480 (6) [12]
	United States	121	468 (4) [15]	471 (4) [16]
Group 6: Lower and Upper Secondary (to Grade 11 and above)	Botswana	19	449 (8) [19]	409 (16) [21]
	Chinese Taipei	365	667 (4) [1]	649 (5) [1]
	Georgia	78	424 (9) [21]	443 (10) [19]
	Germany	362	585 (4) [4]	586 (7) [2]
	Malaysia	388	493 (2) [11]	472 (3) [15]
	Oman	268	472 (2) [14]	474 (4) [14]
	Poland	139	549 (4) [6]	528 (6) [8]
	Russian Federation	2139	594 (13) [2]	566 (10) [3]
	Singapore	251	587 (4) [3]	562 (6) [4]
	Thailand	652	479 (2) [13]	476 (2) [13]
	Norway (PPU & Masters)	65	503 (8) [10]	494 (16) [11]
	United States	354	553 (5) [5]	542 (6) [6]

Although not reported by Tatto et al., we did a calculation for the Pearson's product-moment correlation between the scaled scores for CK and the scaled scores for PCK and obtained a very high $r = 0.964$ for the prospective primary teachers and $r = 0.963$ for the prospective secondary teachers. The corresponding Spearman rank correlations were $\rho = 0.938$ and $\rho = 0.949$, respectively. That the correlations are so high validates the close relationship between the two kinds of knowledge, but lack of complete conformity also endorses the current understanding that CK is not sufficient by itself for classroom teaching.

The 17 countries in the survey each had prospective teachers in different teacher preparation programs and the report grouped them into 6 program-groups. With regard to Singapore, the PGDE (Primary), B.A./B.Sc. (Ed) (primary track, non-Mathematics major) and Diploma in Education cohorts were classed as Group 2 (Primary – to Grade 6 maximum), the B.Sc. (Ed) (primary track, Mathematics major) cohort was classed as Group 4 (Primary Mathematics specialists), the PGDE (Lower Secondary group) cohort was classed as Group 5 (Lower Secondary – to Grade 10 maximum), and the PGDE (Secondary group) cohort was classed as Group 6 (Lower and Upper Secondary – to Grade 11 and above). It was unfortunate that there was no B.Sc. (Ed) (secondary track, Mathematics major) graduating cohort in the year that the survey was conducted because the intakes four years earlier had only admitted the primary tracks in the B.A./B.Sc. (Ed) programmes. As it stood, the Singapore primary prospective teachers were ranked very highly in CK (4 and 3) and in PCK (3 and 1). While the PGDE (Secondary group) cohort placed well (CK 3 and PCK 4), the PGDE (Lower Secondary group) cohort was more modestly placed (CK 7 and PCK 7).

Overall then, Singapore prospective teachers ranked well among the 17 countries in the survey but more in-depth analysis of the items in the survey showed that there is still much room for improvement. For example, a rough calculation from the data given in Tatto et al. (2012, pp.144, 146, 147) shows that less than 50% of the Singapore PGDE (Lower Secondary group) prospective teachers were able to correctly answer the straightforward combinatorics question below:

> A class has 10 students. If at one time, 2 students are to be chosen, and another time 8 students are to be chosen from the class, which of the following statements is true?
>
> A. There are more ways to choose 2 students than 8 students from the class.
> B. There are more ways to choose 2 students than 8 students from the class.
> C. The number of ways to choose 2 students equals the number of ways to choose 8 students.
> D. It is not possible to determine which selection has more possibilities.

NIE faculty was aware of the inherent deficiencies in CK for some of the programmes. For all of the PGDE (Lower Secondary group) and a large proportion of the PGDE (Secondary), the prospective teachers were not graduates with mathematics majors. Applicants for the secondary programmes were however screened to ensure that they had read at least two modules of mathematics at undergraduate level before they were admitted. However, at the primary programmes, almost all the students except for those in the B.Sc. (Ed) (primary track, Mathematics major) had not done any mathematics at the undergraduate level. The next section of the chapter will describe the various curriculum initiatives taken in the NIE to develop the CK of Singapore teachers. These have contributed to Singapore's generally high standing in the TEDS-M survey but could certainly be enhanced in the light of further research into their efficacy. The first of these was the introduction of a subject knowledge component to the programmes for the preparation of primary school teachers, the second was a means to ensure content mastery in the pre-service PGDE (Secondary group) programme targeted at secondary mathematics teachers and the third was the introduction of a Master's programme for mathematics teachers in service who sought to deepen their mathematical content knowledge. Each component will be reported with regard to its motivation and development, the course design and evolution, and some results on its impact.

Some teachers have stated that they "do not teach mathematics" but that they "teach children". Correctly used, this statement draws the teacher into the realization that teaching is a human endeavor that involves human beings. Wrongly used, this statement downplays the importance of a subject matter in the education of a child. We should be "teaching mathematics to children". To this end, any self-respecting and child-respecting teacher should never shortchange her charges by being deficient in the CK that she is supposed to develop in the child. Due to her strong effect on her students, the 'caring' teacher who teaches the wrong content may cause more damage than a less influential teacher. To this end, the NIE teacher education review TE21 (for more details, see Chapter 1 of this book and NIE, 2009) has emphasised the holistic development of the student teacher in its V^3SK model which places the child/student at the centre of the teacher education mission and equally highlights that "knowledge and skills … form the basis of teachers' scholarship, which in turn informs their practice of classroom teaching." (NIE, 2009 p.44)

The Subject Knowledge (SK) Mathematics Courses

Within the primary track of the degree and diploma programmes prior to 1998, the student teachers read *two* Academic Subjects at university level and while the objective of having these subjects in the programme was to develop the CK of the student teachers, there are two reasons why the Academic Subjects did not meet their teaching needs. Firstly, the student teachers were being prepared to teach three subjects as generalists and these two Academic Subjects may not even correspond to two of the three teaching subjects. For example, most primary teachers will be teaching English and Mathematics and a third subject such as Science, Social Studies, Art or Music but the two Academic Subjects could easily be History and Literature. Secondly, even if the Academic Subject matched the teaching subject, the curriculum of the Academic Subject tended to follow normal tertiary level content topics which are not directly relevant to the knowledge necessary for a deeper understanding of the topics they would teach at primary schools.

The situation then was that many of the student teachers in the programmes would only have high school knowledge in the three teaching subjects, some of which were learnt at a superficial level and hence there was a need to not only revise but to re-learn the primary school content of these subjects from a teacher's perspective. This need motivated the introduction of a component known as Subject Knowledge (SK) Component with courses whose content would be specifically designed to be strongly linked to the primary school content in each of the subject areas, with the objectives of building up the student teachers' own understanding in the content and disciplinary processes in these subjects. In addition, these courses would be aligned with the rationale and approaches of the pedagogy courses (called Curriculum Studies (CS) courses) so that they would mutually complement and reinforce each other.

The Subject Knowledge (SK) component was first introduced into the primary track of the B.A./B.Sc (Ed) programme in 1998 and this was for all subjects taught by generalist teachers, namely, English, Mathematics, Science, Social Studies, Art and Music. While the student teachers in this track continued to read one Academic Subject of their choice, they had to do three sets of SK courses corresponding to their three Curriculum Studies areas. In most cases, the students would do English and Mathematics with one choice from Science, Social Studies, Music or Art. For the two-year Diploma in Education programme for preparation of primary generalist teachers, the SK component was introduced in 2001, replacing the Academic Subject component. Due to the short 9-month duration of the PGDE (Primary) programme, such subject knowledge courses could only be included in one of the three options made available when the programme was revised in 2005. In this option, where the potential teachers were being prepared for teaching two subjects instead of three subjects, the curriculum time freed up from the pedagogy courses for the third subject was freed up for the subject knowledge courses of the two remaining teaching subjects.

As tertiary level courses, these SK mathematics courses had to be both rigorous as well as firmly-linked to the primary school mathematics curriculum topics. The course designers were fully cognizant that most of the student teachers were not inclined towards pursuing mathematics at tertiary level and would be disinterested in abstract mathematics concepts as normally covered in pure mathematics courses in universities. For example, while a successful study of axiomatic systems, group structures and number theory may provide deep understanding of the number concepts which dominate primary mathematics, taking an abstract, logical pure mathematics approach with such topics would not only be beyond the learning capabilities of most of the student teachers at this stage in their mathematics learning journey, it would likely adversely affect their attitude towards mathematics if they cannot appreciate the relevance of what they are learning. Nevertheless, these potential teachers of mathematics need to appreciate some of the disciplinary reasoning processes of mathematics so that they do not view and teach mathematics as pure computational procedures. Thus the selection of topics from such abstract mathematics and the teaching approaches for the courses had to satisfy three basic requirements:

(a) the content and teaching approaches must clearly show how each topic is relevant to primary school curriculum and hence the teachers' professional needs
(b) the teaching approaches should role model strong pedagogy and
(c) the course content and processes should seek to develop in the potential teachers a familiarity with the practice of the disciplinary processes of mathematics

In its current form, the SK Mathematics component is structured as two courses for the diploma programme and three courses for the degree programme. For both programmes, the first course covers Number Topics and the second course deals with Topics in Geometry and Measurement. The third course in the degree programme deals with further explorations in geometry and data topics and is an optional course taken by those who intend to specialize in upper primary teaching. We will use the topic of Quadrilaterals to illustrate how the three requirements for the course are met and how the approach seeks to enable students to make sense of what they are learning.

For the topic of quadrilaterals, Singapore teachers are expected to teach properties of special quadrilaterals such as parallelograms, rectangles and squares in the primary school syllabus and the student teachers are made aware of this fact when the topic is taught. The topic is covered in greater depth and scope to include quadrilaterals and properties beyond what is taught in the primary curriculum. The approach taken does not merely deal with properties of each type of quadrilateral in isolation (as tends to be the case at primary schools) but emphasizes the relation between the different types of quadrilaterals through the exploration of their properties. These explorations are often carried out using manipulatives or dynamic geometry software followed by the use of short deductive proofs. It was felt that short deductive proofs were necessary to develop in the student teachers an understanding of two disciplinary attributes of mathematics: (a) the concept of definitions and inclusions and (b) the logical reasoning method of establishing mathematical truths. At an inter-topic level, the topic of quadrilaterals is well connected to the prior topic of triangles and there is constant use of logical deduction to connect the topics. Using this approach, the course seeks to build up the teachers' own reasoning skills as well as bring home the importance of having a holistic overview of geometry through connections between various geometrical entities as well as across various geometry topics. This principle of making mathematical connections between topics and within topics with the theme of mathematical consistency running through the various topics undergirds all the courses in the SK component.

The mathematics pedagogy courses at NIE advocate the principle that teachers should not be dispensers of knowledge but instead try to facilitate sense-making by their pupils in the constructivist sense. However, although the student teachers appreciate the pedagogical principles in their pedagogy classes, they were unable to see such teaching methods actually put into practice as the content of what they are learning are theories and methods of teaching and not mathematics content. One important objective of the SK Mathematics courses, therefore, is for student teachers to experience for themselves such sense-making from the perspective of learners of Mathematics. As mentioned above, geometrical properties were established through initial exploration followed by deductive proofs. Such an approach models the learning process in schools where children learn properties through exploration and experimental means and where logical reasoning is taught at a later stage as proposed by the van Hiele theory. Student teachers are given opportunities to experience as learners the pedagogical approaches learned in their PCK courses.

Using such approaches throughout the SK Mathematics courses poses quite a few challenges for those teaching the courses since the approaches need to be very different from the more didactic ways of teaching tertiary mathematics and yet somewhat different from the constructivist concrete methods suitable for teaching younger students. The Mathematics teacher educators are however convinced that such approaches are necessary and positive feedback has been received. For example, the use of multi-base blocks to illustrate the re-grouping processes in unfamiliar bases alerted the student teachers to difficulties in learning place value and re-naming and how such teaching approaches could help in developing the concept. Some feedback from students shows that they are appreciative of the use of concrete materials to bridge the gap from experiential reality to abstract concepts and reasoning.

The student teachers' mathematical backgrounds also span a wide achievement range and they lack the motivation of the Academic Subject mathematics students who generally have stronger mathematical ability. Not having faced mathematics classroom situations, while the student teachers find their pedagogy courses useful, they need some convincing that their mathematics CK need to go beyond procedural knowledge and skills and that the content topics of SK mathematics are truly relevant and useful. Not surprisingly, those stronger in mathematics tend to realize these needs more quickly and thus appreciate the objectives and content of the courses more than those who find the learning more heavy going. The duration of the course was also a constraint against deeper learning by the less mathematically-capable student teachers who needed more time to internalize the mathematical process which were unfamiliar to them.

Despite the fact that the SK and pedagogy courses are taught by colleagues from the same academic group, the connection between the SK Mathematics courses and the mathematics pedagogy courses could be much stronger. Logistical and resource constraints as well as programme structures and curriculum space required the SK and CS courses for any particular class of student teachers to be taught by different academic staff across different semesters. More details pertaining to this issue is discussed in Lim-Teo (2010).

The effectiveness and impact of these courses on the PCK of the student teachers is difficult to ascertain since there are many variables which affect a teacher's PCK and comparisons between groups are not possible because SK Mathematics is a core component taken by all student teachers in the degree and diploma programme who are being prepared to teach elementary mathematics. Nevertheless, there is one piece of evidence from another local study on the PCK of primary mathematics teachers of a PGDE (Primary) cohort where the subgroup which did SK mathematics performed significantly better in the mathematics PCK test than those who did not have SK mathematics (Cheang et al, 2007).[1] Matthews, Rech and Grandgenett (2010) also provided evidence from a study in the United States that student teachers who took specialized content courses meant for teaching at elementary school level had significantly higher mathematical content knowledge than student teachers who took more general mathematics courses.

Similar SK courses have also been offered as in-service courses across the past fifteen years and feedback from practising teachers has been very positive particularly when they found mathematical justification for mathematics results which they had always accepted at face value and simply re-conveyed to their students. The feedback from those who had taught the SK courses to both pre-service and practising teachers is that the latter were more appreciative because they had experience with real difficulties faced in mathematics classrooms which were addressed by the courses whereas the pre-service student teachers treated the courses as just programme requirements without real appreciation of their relevance to their future teaching.

While it seems that the SK courses may be more effective for practising teachers than for prospective teachers, it will be impossible to ensure that all primary mathematics teachers would come back for such content courses once they complete their pre-service education. Thus, in view of the SK courses' role in developing teachers' content knowledge, the SK Mathematics courses will continue to form an essential component of our pre-service programmes for primary teachers. If would be even more ideal should teachers be able to return to re-visit and build on such knowledge after some years of teaching experience.

The School Mathematics Mastery Test for Secondary Mathematics Teachers

A brief developmental history

The PGDE (Secondary) programme which spans a little more than 9 months has been the oldest and most economical way of training secondary teachers in Singapore. The programme focuses

[1] Due to the shortness of the PGDE (Primary) programme, curriculum time was only available for the inclusion of SK courses in the upper primary track where the student teachers were trained to teach two subjects instead of three subjects in the general track. However, this track was only operational for two cohorts in 2004 and 2005 because the Ministry of Education decided that all subsequent student teachers entering the programme would be channeled to the general track.

on pedagogical courses and general education courses. Up to around 1990, most of the student teachers in this programme would have acquired the content knowledge for their two teaching subjects through the corresponding subjects at undergraduate levels. For example, to be a Mathematics teacher in the 1980s, the recruits would have been Mathematics majors. Therefore, in those days, the PGDE (Secondary) programme had always assumed that the recruits had the requisite CK.

However, since the mid-1990s, the requirements to be admitted for the two teaching subjects have been broadened tremendously, due to the Ministry of Education's need to increase the teaching force very substantially. In the area of Mathematics, large numbers of non-mathematics majors could be assigned to teach mathematics even if their highest mathematics qualification was 'A' level mathematics. For example, in the July 2001 cohort of 374 pre-service mathematics teachers, 31% were engineering graduates and 13% were business graduates; only 28% were mathematics majors. Moreover, with the broadening of university curriculum at the local universities, even those who had taken some modules in mathematics may not have read the relevant modules necessary for a solid understanding of the content needed at the secondary school levels. It is not surprising that the anecdotal experience of those teaching the mathematics pedagogy course as well as from school mathematics heads was that the general level of CK of the PGDE (Secondary) teachers had declined.

In 2003, in response to this apparent lack in CK in beginning Mathematics teachers, the NIE teacher educators for Secondary Mathematics decided to pay close attention to the mastery of the content of secondary mathematics. The team had already collected data from the 2001 cohort which showed that a quarter of the potential mathematics teachers failed to score half the maximum score for a competency test based on 'O' level mathematics content.

Conceptualization of the Content Upgrading

The idea of having PGDE (Secondary) CS mathematics prospective teachers sit for a Mathematics Proficiency Test (MPT) was mooted and four models of content upgrading were considered:

- Model 1. Use MPT as a qualifying test to disqualify and reject candidates who fail.
- Model 2. Use MPT to identify prospective teachers who need to take a separate module on mathematics content during their training at NIE.
- Model 3. Use MPT to inform prospective teachers and school administrators on the prospective teachers' proficiency level of secondary school mathematics. The grade for the test can be separately reflected in an 'appendix' to the prospective teacher's academic transcript.
- Model 4. Use the passing of MPT as a pre-requisite to passing the PGDE (Secondary) CS mathematics course.

There were reservations for the first three models. For Model 1, having a qualifying test may turn away those who want to teach mathematics. The reduced intake may not meet the high demand from the schools for mathematics teachers at that time. For Model 2, running a separate module for prospective teachers would mean a strain on the existing mathematics staff. Moreover, the content level would be below what is expected at a tertiary institution. For Model 3, a grade on an 'appendix' to the prospective teacher's transcript was deemed as not sufficient motivation for prospective teachers to want to pass the MPT well. In addition, if a prospective teacher passes the CS mathematics module but has a poor grade for the MPT, it may send an ambiguous signal with respect to his/her qualification to teach secondary mathematics. Model 4 was finally adopted in 2004 as most colleagues agreed that basic proficiency for secondary school mathematics is essential for teachers and failure of the MPT should rightfully render the prospective teacher not qualified to teach secondary mathematics. Eventually, the MPT was renamed as School Mathematics Mastery Test (SMMT). For more details, readers may refer to Toh, Chua & Yap (2007).

The guiding principle for developing SMMT was to make teachers aware of the mathematics content relevant to their immediate teaching needs and their readiness in this respect. This CK would be the "teachers' mathematics" of Usiskin (2001), including "explanation of new ideas, alternative ways of approaching problems" (p. 96). Hence, the mathematics content to be tested for the SMMT only includes O-level Mathematics and Additional Mathematics knowledge, albeit from a higher perspective. The prospective teachers must already have acquired mathematical knowledge of the O-level Mathematics when they were students. However, this extensive knowledge is largely limited because it is based mainly on their experience as students (Jaworski & Gellert, 2003). Hence, the SMMT can serve as a "reflection" component for the prospective teachers on the secondary school mathematics content.

The goal of the SMMT is to provide a mechanism that will motivate prospective teachers to revise, self-study and build up their secondary school mathematics content up to a mastery level adequate to meet the demands in teaching mathematics in the secondary schools. For a prospective teacher who has not mastered secondary school mathematics content, it is expected that, the learning will take time and so any short intensive module will unlikely be pedagogically effective in helping them attain the desirable proficiency level. However, it is also our belief that the prospective teacher, being a university graduate, will certainly be able to achieve mastery through their own self-study given sufficient time and awareness of the importance of school mathematics content knowledge.

SMMT was not used to assess the performance of the prospective teachers in the PGDE (Secondary) CS courses. Instead, a "pass" in the SMMT, is seen as indicative of the proficiency of the O-level mathematics content knowledge. SMMT "mastery level" is equivalent to a distinction according to the O-level standard. Merely scoring more than 50% would not indicate that the candidate has sufficient CK for teaching, and hence does not warrant a "pass".

For the PGDE (Secondary) cohorts beginning in July 2003 up to January 2005, SMMT was required to be 'mastered' within three attempts as part of the course requirement for CS mathematics. This is essentially Model 4 discussed above. For those who failed to clear the SMMT by Attempt Two, online help in the form of self-paced learning was provided. A series of three lectures was also conducted to clarify the errors and misconceptions common to secondary school mathematics. Eventually, all prospective teachers were expected to pass the SMMT by the third attempt. It was observed that, indeed as believed, many prospective teachers picked up the content through self-study and also through sufficient exposure to the course materials presented in the CS mathematics module.

In July 2005, after an NIE curriculum review conducted by the Office of Teacher Education to streamline the different content upgrading modules across subjects, it was decided that content upgrading be moved before the PGDE Programme proper. Effectively, this meant de-linking the SMMT and the PGDE (Secondary) mathematics course and that the performance in SMMT would no longer have any bearing on the awarding of the postgraduate diploma. Other than that, the entire structure of the SMMT remained.

Format of SMMT

The SMMT is a two-hour paper consisting of about 18 to 20 questions of varying length. For each round of testing, two sets of SMMT papers (Secondary and Lower Secondary) are generated.

SMMT (Lower Secondary) test content covers the entire Mathematics syllabus while SMMT (Secondary) test content includes also all the topics from Additional Mathematics (see http://seab.gov.sg for the detailed mathematics syllabi). It should also be noted that the questions of SMMT are not directly similar to the typical O-Level examination questions. Most of the questions test the candidates' understanding at a deeper level of mathematical concepts that are taught in O-Levels. Two sample questions are shown in Figure 1 below.

4. Give two triangles that satisfy the SSA (or ASS) property but are not congruent, hence demonstrating that SSA is not a congruency test for triangles. Under what conditions do two triangles satisfy SSA property and are congruent? (Note that in fact RHS is a "special" condition for SSA).

5. Use the congruency test for triangles to prove that any point on the perpendicular bisector of line segment AB is equidistant from A and B.

Figure 1. Two SMMT sample questions

For more sample SMMT questions for other topics, readers can refer to http://math.nie.edu.sg/pgde/smmt.html. A collection of typical SMMT questions with complete solutions and the underlying rationale behind these questions can be found in Toh (2009).

Prospective teachers' Performance in SMMT

We shall next report the performance of the PGDE (Secondary) prospective teachers from four recent cohorts of the students, for both the Secondary (S) and Lower Secondary (LS) groups.

Table 3. Performance of two cohorts of PGDE (Secondary) prospective teachers in SMMT.

Cohort	Pass Attempt 1	Pass Attempt 2	Cat A: Pass Attempt 1+2	Cat B: Pass Attempt 3	Cat C: Fail Attempt 3	Total
Jan11(LS)	13	0	13	5	3	23
Jan11 (S)	47	0	47	6	1	54
Jul11 (LS)	46	14	60	7	4	71
Jul11 (S)	85	26	111	5	2	118
Jan12 (LS)	6	2	8	2	0	10
Jan12 (S)	52	5	57	12	1	70
Jul12 (LS)	26	15	41	18	3	62
Jul12 (S)	69	46	115	7	1	123

We adopt the classification of the prospective teachers in Toh, Chua and Yap (2007) using three categories: Cat A, Cat B and Cat C. Cat A reflects the prospective teachers who passed their SMMT in the first two attempts. They were not given additional help by their NIE tutors on additional secondary school mathematics content knowledge. Given sufficient time for their revision and raising their awareness of school mathematics, they were able to beef up their content knowledge and clear the mastery test. Cat B prospective teachers did not manage to pass the SMMT in the first two attempts in which independent self-study was expected. They attended a series of content upgrading lectures on secondary school mathematics and managed to pass the SMMT in the third attempt. In other words, they needed additional help to pass their content upgrading. Cat C prospective teachers failed the third attempt, that is, they were not able to reach the "mastery level" of school mathematics content despite the additional help provided by the NIE.

It can be seen that a very high percentage of prospective teachers in these four cohorts were able to reach the "mastery level" by their own effort (Cat A), while a relatively small portion of them needed additional help to build up their school mathematics content knowledge (Cat B). Very few teachers (Cat C) were not able to reach the content "mastery level" by the end of their PGDE programme.

After every cohort, the Ministry of Education would be informed about the prospective teachers who were not able to clear the SMMT by the third Attempt. These teachers would then be required to attend a series of intensive mathematics content upgrading lectures as (in-service professional development courses) stipulated by the NIE after their graduation from the PGDE programme.

Prospective teachers' feedback about the SMMT

A course feedback was conducted at the end of the PGDE Programme for every cohort of the PGDE (Secondary) prospective teachers. The following two questions about the SMMT were included among the feedback questions:

E1. The SMMT motivates me to revise secondary school mathematics.

E2. I am able to make adequate preparation for SMMT.

A five-point scale (Strongly Agree, Degree, Neutral, Agree, Strongly Agree) was used to obtain their opinion. The summary of the feedback for those who responded from the July 12 cohort (for both S and LS) is shown in Table 4.

Table 4. Feedback about the SMMT from the Jul 12 (S and LS) prospective teachers.

	N	SD	D	N	A	SA
E1	164 (100%)	7 (4%)	16 (10%)	19 (12%)	68 (41%)	54 (33%)
E2	163 (100%)	20 (12%)	34 (21%)	26 (16%)	62 (38%)	21 (13%)

Generally, most prospective teachers (74%) agreed that SMMT motivated them to revise secondary school mathematics content. A slightly lower percentage (51%) agreed that they were able to make adequate preparation for SMMT.

In anticipation of this feedback, the SMMT team has always included information about SMMT before the prospective teachers begin their PGDE programme during the e-briefing. In response to a relatively low positive response to E2, the SMMT team has decided to do a face-to-face briefing about SMMT prior to the commencement of the PGDE programme.

Master of Science (Mathematics for Educators)

A brief developmental history

The Ministry of Education in Singapore has, since the turn of the 21st century, repeatedly emphasized on the importance of maintaining high quality of our teaching force, and developing teachers professionally. During meetings with schools, the Ministry often cited Finland's success in teacher education: one of her secrets being high standards for teachers' qualifications. In particular, all Finnish teachers hold masters' degree: primary school teachers major in education, while upper grade teachers concentrate their studies in a particular subject, e.g., mathematics, as well as didactics (Sahlberg 2010).

For Singapore, although all mathematics teachers in public schools are, by requirement of the Ministry, graduates from the Postgraduate Diploma in Education and thus have received training in pedagogical matters concerning their teaching subjects, not all of them are mathematics graduates. Recent years saw an increasing emergence of mathematics teachers who held undergraduate degrees in engineering, or majored in a mathematics-related discipline (e.g., computer science) other than mathematics. Therefore many of such mathematics teachers acknowledged that their university education had not adequately prepared them for the teaching profession. They saw the need to deepen their CK so as to become more confident mathematics teachers. In addition, in-service mathematics teachers would want to refresh their CK and meet greater demands (e.g., gifted education, curriculum planning, etc.).

The Master of Science (Mathematics for Educators) (or M.Sc. (MAE), for short) is a graduate studies programme designed to meet the aforementioned needs. Designed as a coursework programme to provide rigorous training in advanced mathematics for mathematics teachers, this programme differentiates itself from other competing courses offered elsewhere in that the acquisition of wide and in-depth knowledge in mathematics is emphasized along with its connection with mathematics teaching. The program design is founded on the belief that a strong mastery of mathematics will enable a mathematics teacher teach better and promote higher-order thinking among mathematics learners.
We now give some examples of questions, contributed by teachers as well as NIE mathematics staff, which pose difficulties with regards to the mathematics content of the school mathematics taught in Singapore.

- Does $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ always have a *t*-distribution with n–1 (and why not n) degrees of freedom? [A-level H2 Mathematics]
- When expressing $\frac{1}{x^2-1}$ as a sum of partial fractions, one student wrote
  "Multiplying both sides by $x^2 - 1$, one gets
  $$1 = A(x + 1) + B(x - 1).$$
  Substituting x =1 and $x = -1$, we obtain respectively $1 = 2A$ and $1 = -2B$. Therefore, $A = 1/2$ and $B = -1/2$."
  Is there anything wrong with his answer? Does this method lead to problems and how does one overcome them? [O-level Additional Mathematics]

One of the desired outcomes of the MAE programme is that graduates are equipped with the knowledge to answer these questions. Mathematics teachers who have a "profound understanding" of the school mathematics they teach are less likely to (i) convey wrong concepts to students, (ii) make wrong sequencing decisions when planning for lessons and schemes of work, and (iii) create noise that obstructs accuracy in students' assessment.

Course design
To build the connection between advanced mathematics learning at a master's level and teaching of (comparatively simpler) mathematics in schools, this programme offers a wide range of Level 1 courses that specifically highlight the deeper mathematical structures underlying the topics of both elementary and additional mathematics listed in the secondary school mathematics syllabus. As mentioned earlier, some amount of mathematics at the undergraduate or postgraduate level is

required to tackle content-related questions, such as those stated in the preceding subsection. Level 1 courses are intentionally designed to meet this requirement. For illustration purpose, we show in tabular form how the course content for MSM815 Discrete Mathematics and Problem Solving is designed, together with some design rationale/principle:

Table 5. Structure of a Level 1 course MSM815

Topic	Contents	Rationale
Counting – its principles and techniques	Addition principle, multiplication principle	Permutation and combination (A level mathematics) Elementary probability (O & A level mathematics)
	divisors of natural numbers	Number topics in primary and secondary school mathematics
	subsets and arrangements, bijection principle, principle of inclusion and exclusion	Event space, mutually exclusive events, elementary probabilty
	binomial expansion, Pascal's triangle,	Binomial theorem (O level Additional Mathematics)
Graph theory and applications	graphs, travelling salesman problem, graph colouring, the Konigsberg bridge problem, the Chinese postman problem	Mathematical modeling in secondary schools Real life applications of matrices (O level Additional Mathematics)

The Level 2 courses will then further develop expertise in a number of mathematical fields. Intended to be more abstract and sophisticated in nature, Level 2 courses allow the course participants to deepen their roots, and securing them to firmer grounds in various mathematical disciplines. These courses range over both pure and applied mathematics. The coverage of topics in each course is chosen by practising mathematicians in the respective fields with an emphasis on advanced concepts related to most recent developments. Level 2 courses are intentionally pegged at a much higher level than those of Level 1. Table 6 below displays some courses at Level 1 and their corresponding Level 2 courses offered in this programme.

Examples of Level 1 courses and corresponding Level 2 courses offered in M.Sc. (MAE)

Level 1	Level 2
Advanced Calculus and Applications for Educators; Elements of Mathematical Analysis with Applications in the Teaching of Calculus	Real Analysis; Functional Analysis
Abstract Algebra for Educators	Commutative and Non-commutative Algebra
Discrete Mathematics and Problem Solving	Directed Graphs: Theory. Algorithms and Applications; Vertex Colouring and Chromatic Polynomials
Statistical Reasoning for Educators	Statistical Methods

The core course, Mathematical Inquiry, is a capstone course, i.e., the only mandatory course in this programme which is aimed at providing students with an excellent opportunity to examine current research in a chosen area of pure or applied mathematics. For this course, a student is supervised by an academic staff member who is a practising mathematician in a specific field, and required to perform independent desk study on selected research journal articles or chapters of specific graduate textbooks. A proposed MSM800 project, for instance, may require a student to read recent mathematics journal articles on say, partial fractions (Bradley & Cook, 2012) or exponential functions (Ho et al., 2012), and present his or her understanding of the paper in the form of a concise report of no more than 30 pages. Most of these projects are crafted to help students appreciate the deep mathematical theories associated to the mathematics they teach at school, and thus enhance both their CK and PCK.

Course requirement

Guided by the aforementioned course design rationale, it is mandated that the award of the M.Sc. (MAE) degree takes place upon the successful completion of 10 courses consisting of (i) MSM800 Mathematical Inquiry, (ii) at least two and no more than five Level 1 courses, and (iii) at least four Level 2 courses. While it is important for teachers to equip themselves with a deep understanding of the content knowledge related to school mathematics, there must be a certain level of expectation that master's degree graduates in mathematics demonstrate a respectable level of familiarity and competency with proof and proving, mathematical rigour and abstractness in mathematics. The condition that at least four Level 2 courses be taken is enforced to produce this desired outcome.

Students' feedback

Quality management surveys were conducted from 2010 to 2013 to obtain feedback from students to provide us with some indication of the effects of the course on the students' professional competency and also areas of concern which need address. We shall now employ a qualitative treatment of the survey data to highlight these effects and concerns.

Not surprisingly, the survey returns indicate that the programme has a positive effect on the *students' professional competency* in teaching mathematics. There is a general consensus that the programme has been successful in increasing the content knowledge and understanding of the teachers who graduated:

> "Good programme to revise and **strengthen mathematical concepts and knowledge** …"
>
> "This course has been very beneficial because it gave me a **deeper conceptual understanding** of many topics that I am currently teaching at JC."

Graduates were aware of the intentional connections made with the mathematics topics taught in schools and also the manner in which the knowledge imparting was carried out:

> "It **stimulates the mind** and **adds breadth to the knowledge we are imparting** to our students in school."
>
> "The lecturers also **made links** to the teaching of the topics (where applicable) in schools."

The courses in this programme promote the affective components of learning mathematics:

> "I began to **appreciate the connections** between various areas of mathematics which I was not able to see during my under-graduate course. In particular, I was amazed at the links between two seemingly unrelated areas

of mathematics, like statistics and matrix algebra, graph theory and topology, and even topology and abstract algebra. This programme has further **stimulated my interest and passion for mathematics**."

There seems to be substantial indication that the knowledge upgrade acquired and the training received by these graduates has translated into positive outcomes in students' learning in the classrooms:

"The content taught also connects what I am currently teaching to a higher level which is a very good platform for me to **stretch my students' ability** in Mathematics."

A number of comments were regarding programme matters, and these shed some insight on the *quality of the programme* offered on the whole. The first thing to notice that is relevant to the programme design is that there is a wide spectrum of students' background and standards in mathematics. A small number of students expressed that the courses were easy:

"Overall I felt the courses that I took had a low difficulty and expectation level. … But in general many of the courses felt like they were similar difficulty or even easier than undergrad courses (MSM 813, 815, 826, 829)."

while there were a sizeable number who thought otherwise:

"…most **challenging** as the topic was an area that is totally new to me, but I enjoyed the course thoroughly."

"The analysis and topology courses were plenty **difficult** though!"

Concerning the capstone course MSM800 Mathematical Inquiry, graduates agreed that though it was challenging because it required them to perform independent desk study, the experience of having gone through it is nonetheless rewarding:

"I also really **enjoyed** the Inquiry course, though it was the one I was **most worried** about."

"The most **difficult** period was when I was doing my mathematical inquiry where I had to **do self-study** and **depended a lot on my own**."

"I find particularly useful the course requirement on Mathematical Inquiry as it is thorough research, one is forced to understand and **think more critically** about the topic."

The programme was perceived to be one of high quality and value-addedness as summarized by the following remark:

"The course is **rigorous, engaging and fulfilling** ..."

"I became a more **confident, competent and independent learner**."

One of the aims of the NIE is to train and nurture a high quality teaching force in Singapore. To do so would mean that the faculty does what it preaches and be *exemplary in its own teaching practices*. Guided by their research in both content and pedagogy, instructors ensure high standards in their lessons. The following feedback confirmed the above:

"Luckily, I had my supervisor, who helped me along the way. He gave me an **insight of how mathematics can be seen as beautiful and interesting.** Though, I was just an average student, he has given me an opportunity to learn from him in depth and breadth of Category Theory. I really appreciated his effort and time taken for me during that period."

"The **lecturers are understanding and patient**, and build a good rapport with the students ..."

"It was nice that the **program and faculty take into account that we are current teachers and thus busy**."

"… and very inspiring professors who not only are well-versed in their content but also being a **role model in teaching**."

The way forward

The three innovations described above were our response to improving teachers' CK. What type of CK is actually needed or appreciated by the teachers remains not completely understood. Subject Knowledge taught in NIE is an attempt to have a better understanding of mathematics *related* to elementary mathematics as conceptualized by Ma (2010). We feel that it can be improved with a sharper focus of "better understanding elementary mathematics" (Ma, 2010, p. xxvi). To this end, prospective and practising teachers can be made to see the relationship of what they are learning to their teaching by actually motivating the course with actual elementary school problems and concepts and building the course materials directly around these.

The M. Sc. (MAE) programme will also benefit from a review of its courses from a perspective of Usiskin's *teachers' mathematics*, which we see as a generalisation of Ma's *profound understanding of fundamental mathematics*. In particular, the original notion of courses to be taught with a view to their direct application to school mathematics ought to be explicitly refreshed for faculty teaching in the programme.

As an example of what content would attract teachers to improve their CK, we would like to refer to a workshop entitled "Touching on infinity in the secondary mathematics syllabus" conducted by one of the authors early in 2013. Teacher participants were asked if they agreed with the statement "0.999… is *exactly* equal to 1". Table 8 below shows the number of responses (ranging from 1 (Strongly Disagree) to 5 (Strongly Agree)) before and after an explanation of the concepts of representations, sequences, and ε-N definition of convergence which was finally brought to bear on answering the question. The dramatic increase in the number of 'correct' answers shows that the teachers were able to follow a quite mathematically rigorous explanation for a secondary mathematics issue.

Table 8. Agreement with the statement "0.999… is *exactly* equal to 1"

	1	2	3	4	5	Mean
Before	8	3	4	2	3	2.45
After	0	0	1	3	16	4.75

The feedback for the workshop also showed that teachers would appreciate CK courses with direct application to their teaching. Post-workshop comments were positive and some comments are reproduced below.

- The strengths of the workshop are: *interesting way of explaining limits; explanation of 0.9999… = 1; clear explanation of concepts to maths theory; relevant content, insightful, challenging presumptions; concepts were explained very clearly with appropriate and relevant experiences; it goes to the very fundamentals of maths … I really enjoyed as it helps me recall the maths learnt in university; interesting and engaging; provides formal proof of certain concepts involving infinity.*
- I will use the knowledge and skills I have learnt in: *Additional maths - exponential functions; there is scope for infinity in the recurring numbers and representation; to explain and prove*

concepts in maths; teaching high ability students; recurring numbers and representation; self-development; teaching of numbers; explaining to my students on the concept of infinity.

- I would like to participate in workshops on the following areas: *more thought provoking maths (eg. symbols representing numbers); similar workshop; areas related to mathematics on research and problem solving; more on conceptual understanding; similar to this workshop (content based).*

Thus in all our reviews, we will probably need more feedback from practising teachers to help shape the course content for CK development in the future.

Besides constantly reviewing that the CK in the programmes are really relevant to the mathematics in the school curriculum, enough time must be allocated for sufficient coverage of CK. One immediate solution in the Singapore context would be to have teachers specialize in only one subject at the secondary level. Table 9 shows the number of mathematics teachers and the number of teachers teaching only mathematics from a convenience survey of three mainstream secondary schools conducted this year (2013).

Table 9. Specialisation of secondary mathematics teachers

School	Mathematics teachers	Only teaching maths	Percentage
A	21	14	67
B	21	9	43
C	20	9	45
Total	62	32	52

Since the survey did not include junior colleges where teachers specialize in only one subject, it would not be unfair to estimate that at least half of all mathematics teachers finally teach only mathematics in the school. In this light, the current policy of training all prospective secondary teachers in NIE to teach two subjects would seem to be wasteful with regard to the minor teaching subject and the loss of curriculum time to enhance the preparation of the teacher for the major subject. The main reason for having two teaching subjects for secondary teachers in Singapore was that, from the logistical point of view, the principal of a school would find it easier to deploy four halves than two wholes within the school timetable. On the whole, this comes at an educational cost because the teacher who is trained in two subjects within a given time will necessarily have learnt less in his major. Both CK and PCK would be affected. However, prospective secondary mathematics teachers in Korea (Park, 2010), another TIMSS high performing nation, take only mathematics as their academic subject when in the university and are trained to teach mathematics only. The way forward for Singapore and NIE is to loosen the requirement that secondary teachers be trained for two subjects.

Certainly, a case can also be made for extending the PGDE programme for secondary teachers. Singapore did not fare as well in the TEDS-M survey for the secondary track as in the primary track. One possible reason is that the secondary teachers were all prepared in a one-year PGDE programme and all the time allocated for mathematics was spent on PCK. There was only the SMMT tool that was used as somewhat of a last resort to at least show the student teachers where they stood with regard to CK. We would recommend extending the programme for another semester for content upgrading to ensure that beginning teachers will at least be ready content-wise when they step into the schools.

A similar argument on specialization could be made for the primary school teachers to specialize in teaching just one or two subjects, particularly at the upper primary levels. The TEDS-M study had showed that our B.Sc. (Ed) (mathematics major) group were ranked first for PCK and third for CK and, as mentioned in the section on SK Mathematics, the PGDE (Primary) sub-group which had specialized in two subjects and hence undergone the SK Mathematics courses outperformed their cohort-mates who were trained for three teaching subjects and had no curriculum time for taking any SK courses. Since it is difficult for any single person to have strong content mastery in three subjects, specialization would not only allow more curriculum time to develop relevant CK but also build on relative subject strengths of the student teachers, particularly for the one-year PGDE (Primary) programme.

For those with no intention to take on a postgraduate content programme, in-service courses are currently the only avenue for CK development. However, there is often a mismatch between what the teachers think they need and the courses that are available. In addition, teachers sometimes do not know what they need – this can be seen from the pre-workshop responses to the statement "0.999… is *exactly* equal to 1". Since any top-down measure where teachers are recommended to attend in-service courses in identified areas of weaknesses is likely to be demoralizing and hence de-motivating, a possible solution would be to post a self-assessment for teachers in a widely viewed site, for example the Ministry of Education's Curriculum website or the Academy of Singapore Teachers website. In this self-assessment, teachers may try to solve a number of problems and view their results anonymously. The feedback would direct them to courses which would be suitable to develop that deficient part of their CK.

This chapter has discussed the importance of relevant mathematical content knowledge for teachers and the measures which NIE has taken to ensure and/or develop such knowledge in prospective as well as practising teachers. With varying teacher profiles in future, the curriculum content of such courses to enhance teachers' content knowledge will continue to evolve over time and determination of what content is most useful will need to be informed by further research. While teacher educators can focus on determining such curriculum details, given time and resource constraints of teacher preparation courses and the myriad demands on practising teachers, further proposed measures to meet the needs of teachers in the area of their own subject content knowledge will only be possible provided the crucial importance of teachers' subject content knowledge is strongly recognized by the educational leaders in the system.

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