CHAPTER 12

LEARNING THROUGH "PLANE PUNCTUALITY"

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The prompt service of in- and out-bound flights has made Changi Airport one of the best airports in the world. This chapter showcases how modelling tasks can be woven into the rich fabric of real-life contexts, that is familiar to Singapore students.

12.1. Overview

Practitioners in applied mathematics would agree, to a large extent, that a mathematical model is a description of a system using mathematics as the vehicular language. The process of manufacturing and developing a mathematical model is termed mathematical modelling. Here, *modelling* denotes unambiguously mathematical modelling. The use of mathematical models is ubiquitous, ranging from the natural sciences to the social sciences.

Because modelling yields 'usable' representations of any existing system (Eykhoff, 1974), it is natural to ask whether it can offer a versatile platform for mathematics learners to apply a wide-ranging repertoire of mathematical skills. Pertaining to the integration of modelling into school mathematics curricula, the aforementioned question has been addressed in recent works such as Stillman *et al.* (2007), English and Watters (2004) and English (2004). These works advocate that the "modelling process is driven by the desire to obtain a mathematically productive outcome for a problem with a genuine real-world motivation" (Galbraith & Stillman, 2006, left of page 143). By engaging the students in the process of modelling, the designed activities are intended to "motivate, develop and illustrate the relevance of particular mathematical content" (Galbraith, Stillman, & Brown, 2006). In this chapter, we shall adopt this perspective of modelling.

12.2. The Singapore Scene

The importance of using "mathematical modelling as content" (Julie, 2002, top of page 3) in Singapore mathematics education has been continuously emphasised in schools since its inception in 2003. MOE's official formulation of the 4-stage cycle: (1) Mathematisation, (2) Working with mathematics, (3) Interpretation and (4) Reflection (Balakrishnan, Yen & Goh, 2010) echoes this emphasis. Subordinate to this cycle, the task designer identifies an interesting context upon which the modelling task can be constructed (Kaiser & Sriraman, 2006). Whence, the choice of a Singapore example is a necessary one: Singapore school students must first be able to readily identify with national icons, second to work through the problem, and lastly to appreciate the real-life applicability of textbook mathematics. Note that this idea has been exploited in (Wong, 2003) as a possible source of reinforcement in National Education under the label of "Homeland", albeit in somewhat the opposite direction.

12.3. This Study

The present study seeks to identify some key factors that contribute directly towards the success of mathematical modelling activities, focusing on both the process and the product, in the Singapore secondary school context. We shall explain what we mean by 'success' in our ensuing discussion.

12.4. The Method

Five groups of Secondary 2 (equivalent to Year 8 of the Australian or US education system) students are assigned a common modelling task 'Plane Punctuality' (see Figure 12.1), and they were to carry out the task over three days (1–3 June 2010). Amongst these were two mixed groups consisting of students belonging to different schools. On 3 June 2010, all groups were to present (for about 10–15 minutes) their research findings and recommendations to their peers. In those three days, two facilitators (who had received *a priori* a one-day training in facilitation of mathematical modelling activities) facilitated and monitored the students' progress. These facilitators provided minimal direct assistance.

The Singapore Changi Airport (SIN) is the main airport in Singapore and a major aviation hub in South-east Asia. SIN is currently ranked among the best 19 airports in the world. How the punctuality of arrivals and departures is managed is an important task the airport must undertake every day. How *on-time* are flights in SIN compared to other well-known international airports?

Figure 12.1. The modelling task "Plane Punctuality at Singapore Changi Airport".

The task requirement was intentionally open-ended. For instance, students were free to interpret what 'on-time' meant. Also, details such as the types of flights, time-frames of observation, and the explicit list of 'other well-known international airports' were not provided. The task design was based on the 5 + 1 design principles developed in Galbraith (2006). In particular, by applying the Didactical Principle to craft the guiding questions one derives these labels: (a) Discussion, (b) Plan, (c) Experimentation (Data organisation), (d) Representation and verification and (e) Product. These design principles guide the development of mathematisation skills, communication skills, reasoning abilities, critical thinking, skills in data representation and organisation. To a large degree, students were free to venture in their courses of investigation. This characterises the sort of learning that takes place in a modelling setting, i.e., one which focuses on the direction (reality $\rightarrow$ mathematics), where one tries to locate the appropriate piece(s) of mathematics to help one solve a real-life problem (Stillman *et al.*, 2007).

With the aim of this study in mind, field observations hoped to capture the behavior, learning outcomes, modelling processes and final product of two groups, **A** and **B**. Each group had three members. **A**'s members came from the same school, while **B** was a mixed group. A *successful* modelling experience would ideally be one in which the *meaningful application* of the aforementioned modelling *processes* results in the developing of *non-trivial insights* into the real-life situation one is trying to model. The field study is to identify those factors which directly contribute to such a success.

12.5. Students' Mathematical Modelling Experience

12.5.1. *Discussion*

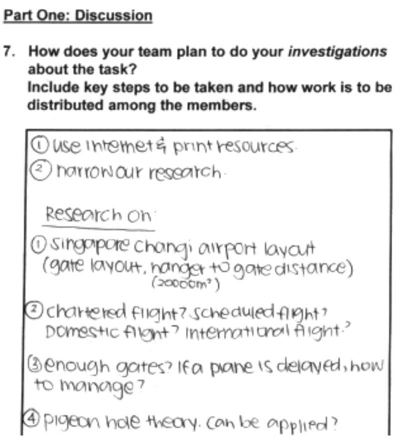
Flight punctuality, defined by **A**, was "being on time or with a maximum of 15–30 minutes delay". **B** defined flight punctuality as flights being able to arrive or depart an airport within the desired period of time, where 'desired period of time' was qualified to be 15 minutes from the schedule timing. **B** allowed the possibility of a plane arriving or departing earlier than scheduled, and also identified punctuality as a concept tied to frequency. Hence **B** demanded punctuality to entail a minimum of 90% of the total flights meeting the above requirement.

The groups' goals were different. While **A** set out to maximise punctuality in terms of ground operations, **B** sought for a comparison between Singapore and other international airports. The 'discussion' aspect of the task design provided evidence of the groups' attempts to understand and simplify the problem, e.g., **B** decided to carry out their study focusing only on arrivals. Decision-making like this constantly occurred in the entire modelling endeavour. Both the groups identified the variables affecting the chosen goal. **A** singled out crew effectiveness and airport layout, while **B** looked at location of the peak periods in a day. The process of identifying the relevant variables affecting the chosen goal moved the students towards mathematising the problem they were trying to solve. This was then followed by formulation of underlying assumptions, such as "the passengers are cooperative" and "there were no terrorists" for **A**, and "the weather is fine", "the (political) condition of the country is stable" for **B**. Seino (2005) advocates that the awareness of assumptions "plays the role of a bridge that connects the real world and the mathematical world" (Seino, 2005, top of page 665) and serves as an effective teaching principle. **A** articulated such awareness in their log-book: *"(in) modelling ... the answers are neither wrong nor right, quite a lot of things we need to assume it, and when the answer is out we still need to check whether it is reasonable for us, but we like challenging."*

12.5.2. *Plan*

The next phase was planning. Question 7 in the handout facilitated this change of phase by probing "How does your team plan to do their investigations about the task?"

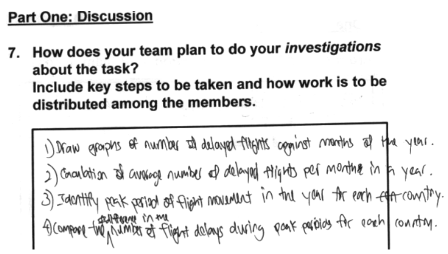
Changes made in plans subsequently took place for both groups: **B** moved the step of identifying the peak period of air traffic in a day from (3) to (1), while **A** made a much sharper refinement (See Figure 12.2). The action of 'fine-tuning' and 'adjusting' their plans were evident of the non-linearity and cyclical nature of the key modelling activities (Doerr, 1997). Keywords used by the students, such as "graphs", "number of delayed


Part One: Discussion

7. How does your team plan to do your *investigations* about the task?
Include key steps to be taken and how work is to be distributed among the members.

① use internet & print resources.
② narrow our research.

Research on:
① Singapore Changi airport layout (gate layout, hangar to gate distance) (20000m²)
② chartered flight? scheduled flight? Domestic flight? international flight?
③ enough gates? If a plane is delayed, how to manage?
④ pigeon hole theory. Can be applied?


Part One: Discussion

7. How does your team plan to do your *investigations* about the task?
Include key steps to be taken and how work is to be distributed among the members.

1) Draw graphs of number of delayed flights against months of the year.
2) Calculation of average number of delayed flights per month in a year.
3) Identify peak period of flight movement in the year for each country.
4) Compare the difference in the number of flight delays during peak periods for each country.

S/No.	Items	Remarks
1.	Research on reasons why planes may have been delayed	• No place for planes to park • Ineffective crew (not guiding the planes where and how to land) • Drawing graphs of number of delayed flights is not enough.
2.	Efficiency of other terminals	• Put all statistics into a table format • Just the average number of delayed flights is insufficient for statistical inferences
3.	Consequences of planes being late	
4.	How to solve problems	• Speed flow density relationship • Queuing theory, Pigeonhole theory

Figure 12.2. Group A's plan refinement (top left: before 2^{nd} meeting; bottom: after 2^{nd} meeting); Group B's plan (top right: before 2^{nd} meeting)

flights", "difference", "speed flow density relationship", "statistics", "queuing theory" and "pigeonhole theory" were also evident of the students' attempt to mathematise.

12.5.3. *Experimentation, data organization, representation and verification*

Experimentation, data organisation, representation and verification observed in **A** and **B** became very much intertwined and cyclical. Here, one focuses on the *diversity* of the outcomes evolved.

Group A. First, this group set out to verify that SIN was indeed ranked among the top in terms of its flight punctuality. From a reliable internet source, they obtained the punctuality rates of the top-5 major international airports, and compared these with SIN (See Table 12.1):

An easy computation of the average yields 75.82%, thus confirming the claim. Aware that the maximum data point is always higher than the mean (hence making this calculation an overkill), what the students really wanted was to see how much SIN deviated above the average. With the aim to find out the possible causes of flight delay, **A** then deduced from the available data that about 5000 arrivals and departures in SIN occurred every week, i.e., yielding an average of 714 flights per day. **A** then collected data from Flight statistics (n. d.) on 1 June 2010, reporting 375 departures and 369 arrivals. They wrote: *"Out of these flights, 50% of the departing flights were punctual, while 88% of the arriving flights were on time. This totals up to 159 delays in departure and 41 delays in arrivals, and gives us an average of 15 flights each for both departure and arrival terminals per hour."*

Then **A** deduced that "30 gates will be in use" in each hour. Then crucially, they realised that while being parked at a gateway, planes required some turnaround time which involved "disembarking the passengers,

Table 12.1. *Punctuality rates of top-5 international airports in 2010.*

Country	Singapore	Tokyo	Rome	UK	India
Puntuality Rate	92%	90%	83.1%	65%	49%

unloading and reloading baggage, re-fuelling and cleaning the plane before passengers could board again and set off to their destination".

Delays, as **A** perceived, were caused by the violation of a simple rule: at anytime, the number of available gates must be at least equal to the number of incoming planes that need the gates. **A** recognised this to be an instance of the *pigeonhole principle*. Motivated by this principle, **A** worked out (using a layout diagram of the 92 gateways in SIN) the maximum turnaround time for just enough gateways available to park/service the planes. Since the airport is populated by 30 planes in each hour, it would have taken about 3 hours to have all the gateways occupied if one assumes that all the airplanes which were parked at the gateways had not left yet. By track of reasoning, **A** deduced that the turnaround time could at most be 2 hours.

Group B. **B** went on to identify the peak periods for arrival for 5 airports: (1) Hong Kong International, (2) Suvaranabhumi International, (3) Changi International Airport, (4) Kuala Lumpur International Airport and (5) Beijing Capital Airport. In addition, **B** compared the data from SIN with that from the Hong Kong International Airport: "The 5.7% of airplane delays at the SIN could be caused by unexpected weather conditions and also the timing of departure from its origin country. The low 2.2% of delayed flights at HK Airport might also be due to luck and good weather conditions on that day".

12.5.4. *Product*

Group A. **A** extended their scheduling argument to the seating schedule for the passengers. Guided by queuing theory, **A** proposed to seat the passengers efficiently and quickly by first seating the people situated furthest from the doors, and the people in the window seats, before proceeding 'outwards'. Two seating arrangements were proposed (see Figure. 12.3). Also, baggage due for transit should be placed near the door of the cargo compartment.

Group B. For their product, **B** gave qualitative suggestions:

1. Airport personnel should be properly trained.
2. Reduce waiting time for passengers.
3. Increase the airport's customer service satisfactory level.

Back of plane

1	8	15
2	9	16
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21

Front of plane

Back of plane

1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21

Front of plane

Figure 12.3. Group A: Two proposed seating arrangements based on queuing theory.

12.6. Findings and Implications

The keen reader must have noticed by now the presence of a significantly wide gap in the sophistication of modelling tools used by **A** as compared to **B**. A probe into **A**'s background knowledge revealed that they were mathematically gifted students trained for the Singapore Mathematics Olympiad. Also, prior to this activity, one particular member of **A** had completed a mathematics project (where she used queuing theory to model traffic congestion near her school). In this respect, **B** lacked advanced mathematical training/exposure.

It is clear that of the two groups, **A** had done a better job. While it would be easier to account for **A**'s success by simply appealing to their larger mathematical expertise, one should perhaps relook at the aspect of *mathematical modelling competency*. Here, mathematical modelling competency refers to one's ability to identify relevant questions, variables, relations or assumptions in a given real-world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem in relation to the given situation, as well as the ability to analyse or compare given models by investigating the assumptions being made, checking properties and scope of a given model (Niss, Blum, & Galbraith, 2007).

A made it a habit to check the properties and scope of a chosen model from time to time, while the same was not observed of **B**. For example, having decided queuing theory as a mathematical model, **A** looked for more data, i.e., the data of the timing between the passengers' loading and unloading, to confirm the appropriateness of such a model. Recorded in their daily log-book was this: *"First, we need to obtain more data for more accurate analysis. Next we can cater to a specific peak period where there are more airplanes arriving and departing. Also we should cater directly to the different airplane sizes so specific gates can be allocated to different models of planes, and thus, the appropriate equipments and technicians would be on hand."*

Mathematical modelling is a versatile approach to meaningful learning of mathematics, where modelling tasks and questions can be readily crafted around day-to-day experience in any given culture and country. The flight punctuality problem described above serves as one such exemplar. Though localised to a small sample of 6 students, our study seems to indicate that mathematical modelling competency is probably the key factor that determines the diversity in behaviours and learning outcomes derived from a single mathematical modelling activity. Thus, a classroom teacher who wishes to exploit mathematical modelling should certainly bear in mind the current modelling competencies of the students he or she is teaching.

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