Graphic Sequence

1. A graphic sequence is a sequence of numbers which can be the degree sequence of some graph.
2. Erdos and Gallai (1960) proved that a sequence of positive integers $\{d_1, \ldots, d_n\}$ with $d_1 \ge d_2 \ge \cdots \ge d_n$ is graphic iff $d_1 + d_2 + \cdots + d_n$ is even and the sequence obeys the property that for each integer $r \le n-1$,

$$\sum_{i=1}^{r} d_i \le r(r-1) + \sum_{i=r+1}^{n} \min(r, d_i)$$

3. Havel (1955) and Hakimi (1962) proved another characterization of graphic sequences, namely that a degree sequence with $n \ge 3$ and $d_1 \le d_2 \le \cdots \le d_n$ is graphic iff the sequence

$\{d_2, \cdots, d_{n-d_1}, d_{n-d_1+1} - 1, d_{n-d_1+2} - 1, \cdots, d_n - 1\}$ is graphical.

4. For example, we can check whether $\{3,3,3,3,4\}$ is a graphic sequence:

$$\{3,3,3,3,4\} \Downarrow \{3,2,2,3\} \Rightarrow \{2,2,3,3\} \Downarrow \{2,2,2\} \Downarrow \{1,1\}$$

Since {1,1} is a graphic sequence, the original sequence {3,3,3,3,4} is also graphic.