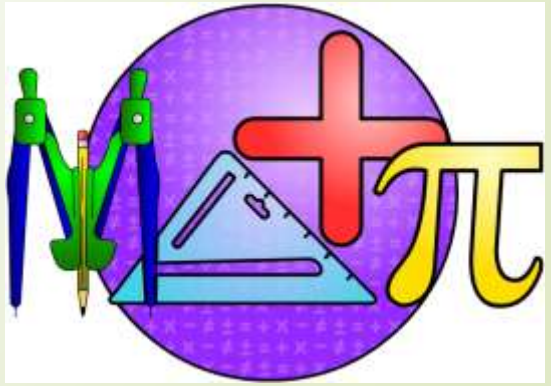
Mathematics Education in Singapore Schools

The Past, Present and Future

Berinderjeet Kaur
National Institute of Education
Singapore

Outline of Presentation

- Evolution of the intended school mathematics curriculum
- Enactment of school mathematics curriculum
- Achievement of students in mathematics
- Questions and clarifications



The White Paper (Yip et al., 1990)

In 1959 when the People's Action Party (PAP) came to power, it acted upon the **White Paper of 1956** and put in place a Five-Year Plan in education. The main features of this plan were:

- Equal treatment of the four language streams of education: Malay, Chinese, Tamil and English;
- The establishment of Malay as a national language of the new state;
- Emphasis on the study of **Mathematics**, Science and Technical Subjects.

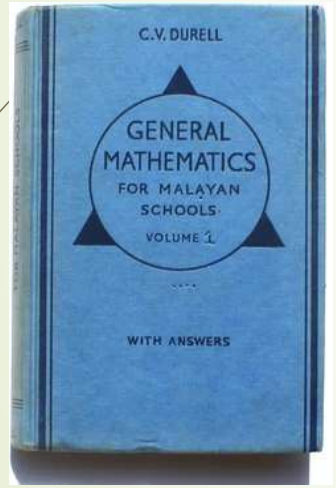
Evolution of the intended school mathematics curriculum

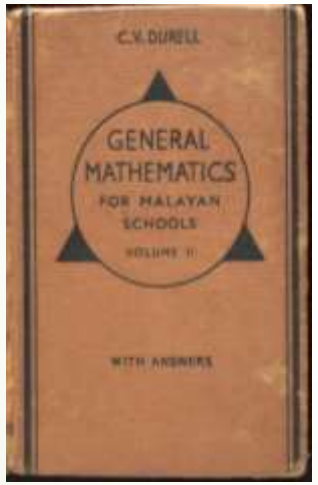
Our diverse beginnings …

- Up to the late 1950s, schools in Singapore were mainly vernacular (Chinese, Malay, Tamil and English).
- The language of instruction in Chinese schools was Chinese and their curricula were adopted from China.
- Likewise the language of instruction in English schools was English and their curricula were adopted from Britain.
- Therefore several mathematics syllabuses were in use across Singapore, with each school adopting its own.

Intended School Mathematics Curriculum

- In 1960, the first local set of syllabuses (published in English) for mathematics was made available for all primary and secondary schools irrespective of their language streams (Lee, 2008).





- A few subsequent revisions of the syllabuses led to the **1980 version for the New Education System**.

The New Education System

- The ‘simple objective’ of education in Singapore,

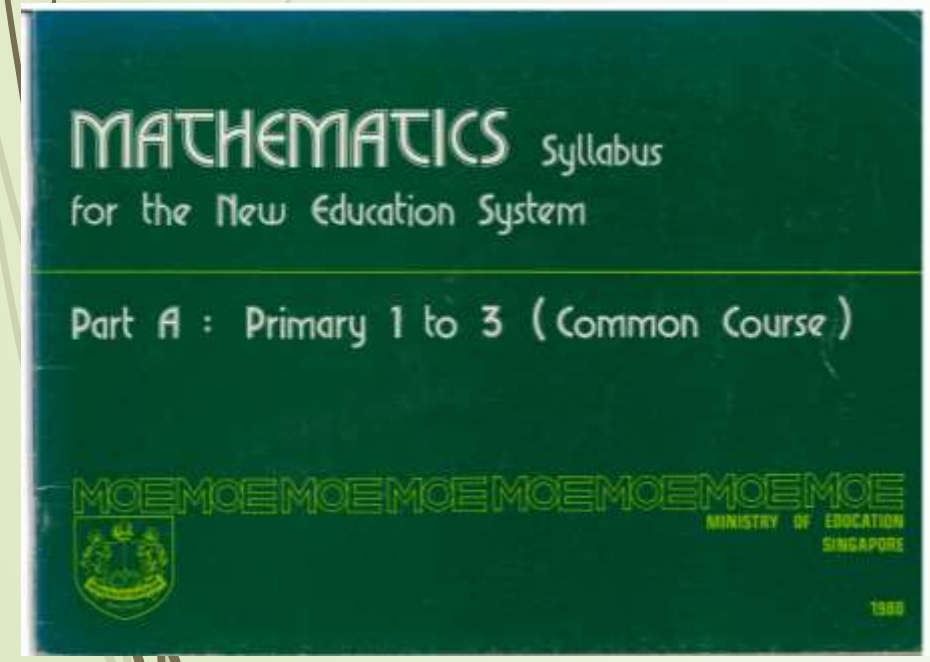
  … to educate a child to bring out his greatest potential so that he will grow into a good man and a useful citizen.

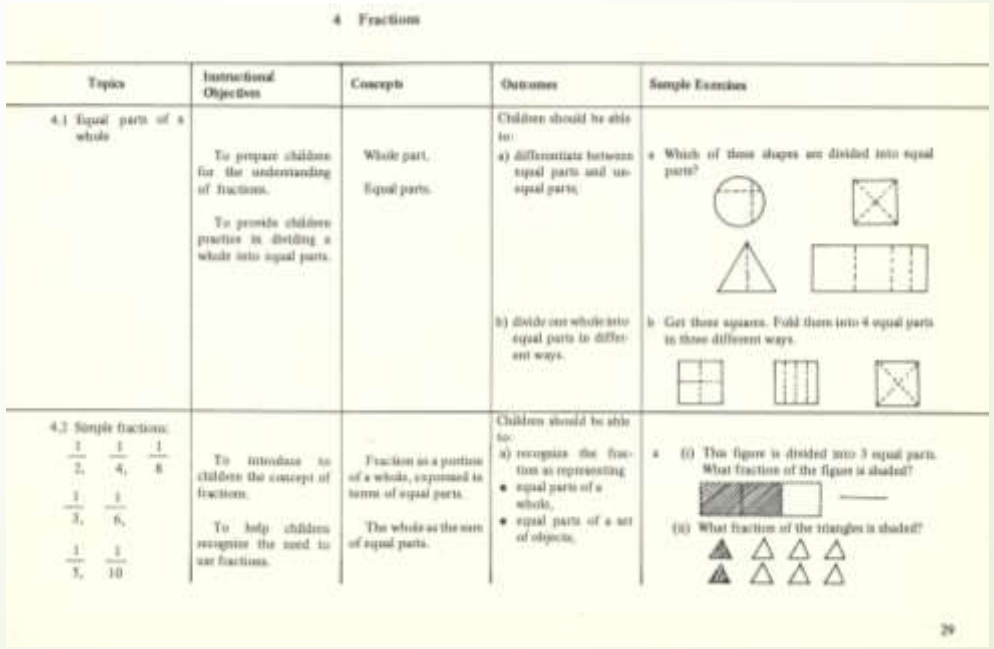
  (Lee Kuan Yew, 1979)

- The New Education System was implemented in 1981. Streaming of pupils was introduced.

1980 Primary Mathematics Curriculum

- Detailed syllabuses, textbooks, workbooks & teacher guides produced at the Curriculum Development Institute of Singapore (CDIS)
- The revised primary mathematics curriculum was implemented in 1981.




4 Fractions

Topics	Instructional Objectives	Concepts	Outcomes	Sample Exercises
4.1 Equal parts of a whole	To prepare children for the understanding of fractions. To provide children practice in dividing a whole into equal parts.	Whole part. Equal parts.	Children should be able to: a) differentiate between equal parts and unequal parts; b) divide one whole into equal parts in different ways.	a Which of these shapes are divided into equal parts? b Get these squares. Fold them into 4 equal parts in three different ways.
4.2 Simple fractions: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{10}$	To introduce to children the concept of fractions. To help children recognize the need to use fractions.	Fraction as a portion of a whole, expressed in terms of equal parts. The whole as the sum of equal parts.	Children should be able to: a) recognize the fraction as representing • equal parts of a whole, • equal parts of a set of objects;	a (i) This figure is divided into 3 equal parts. What fraction of the figure is shaded? (ii) What fraction of the triangles is shaded?

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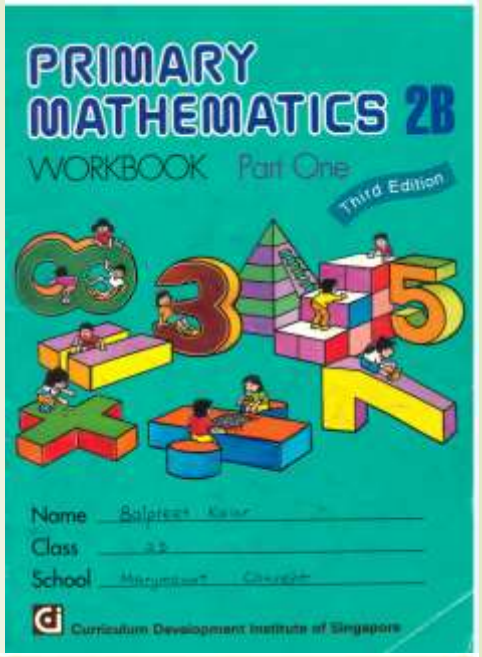
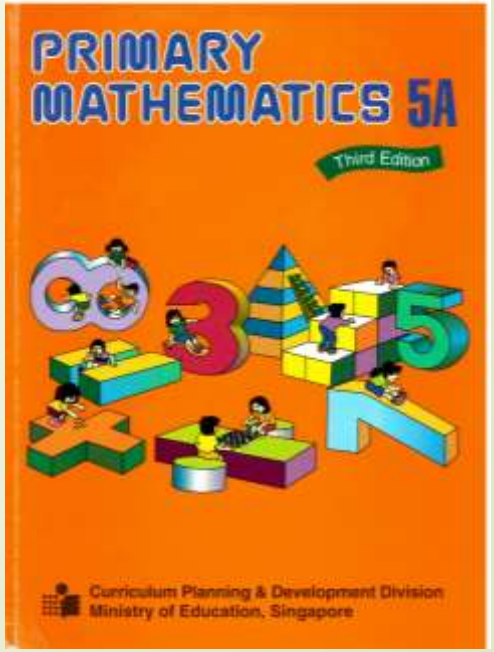
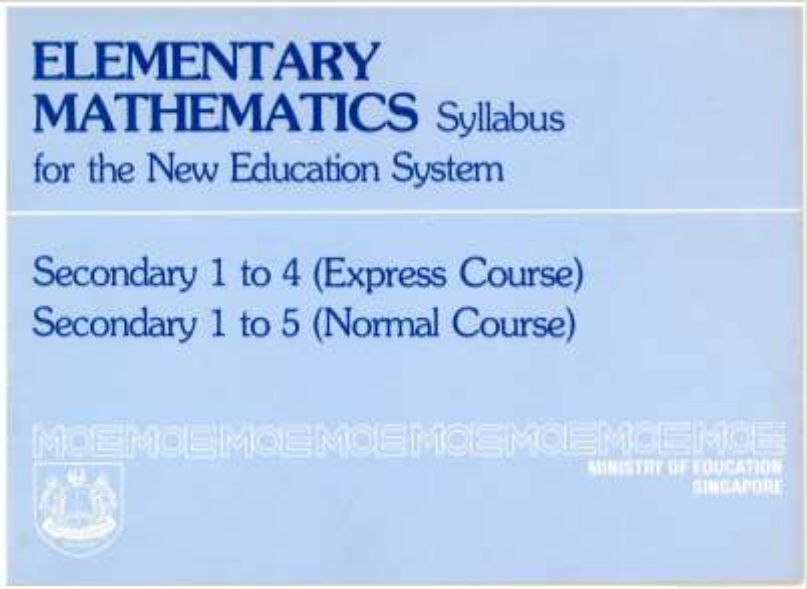




1980 Secondary Mathematics Curriculum

- The syllabuses were based on the Cambridge GCE 'O' Level Mathematics syllabus D.
- The syllabuses were implemented in 1981.
- Other than the outline of topics and brief remarks in the syllabus, teachers adopted commercially produced materials for their use.



SECONDARY EXPRESS/SPECIAL COURSE:

SECONDARY ONE (S1)

Detailed Content	Remark
Arithmetic	
1 Whole numbers and fractions (including decimals) (a) Decimal system and place value. (b) Conversion between vulgar and decimal fractions.	
2 Negative numbers, the real number line.	At this stage, the number system is extended to include negative numbers. Points on the number line represent real numbers. They are either rational or irrational. In describing natural numbers, integers, rational numbers and irrational numbers, set language and notations may be used.
3 The idea of ordering.	The number line is useful for showing the order of numbers especially negative numbers. Notations $=, \neq, >, <, \geqslant, \leqslant$ are to be used.
4 Common arithmetic processes.	This includes the four operations $(+, -, \times, \div)$ of numbers and combination of them by use of brackets. The commutative law, the associative law and the distributive law should be mentioned as occasion requires.
5 Arithmetic problems (a) Money (dollars and cents) (b) Mass, measures, densities. (c) The Celsius (centigrade) scale of temperature. (d) Speeds.	Problem solving is for the pupils to apply the Mathematical skills in real life situations; to become more familiar with the various units of measurements; to appreciate the need to give approximate answers by 'rounding off' numbers. To reinforce the pupils' computational skills, the use of calculators should not be allowed at this stage.
6 Prime numbers and factors, L.C.M. and H.C.F.	L.C.M. and H.C.F. of two or more positive integers can be found by expressing each positive integer as a product of prime numbers in index form.
7 Proportion, ratio and percentage (a) Direct and inverse proportion; proportional parts. (b) Applications of ratio and percentage.	

SECONDARY NORMAL COURSE:

SECONDARY ONE (S1N)

Detailed Content	Remark
Arithmetic	
1 Whole numbers and fractions (including decimals) (a) Decimal system and place value. (b) Conversion between vulgar and decimal fractions.	
2 Negative numbers, the real number line.	At this stage, the number system is extended to include negative numbers. Points on the number line represent real numbers. They are either rational or irrational. In describing natural numbers, integers, rational numbers and irrational numbers, set language and notations may be used.
3 The idea of ordering.	The number line is useful for showing the order of numbers especially negative numbers. Notations $=, \neq, >, <, \geqslant, \leqslant$ are to be used.
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Mensuration	
7 Length, area and volume, the metric units.	Pupils should be familiar with the following units and their usage. Length : 1mm, 1cm, 1m, 1km.

1990 School Mathematics Curriculum

Key features

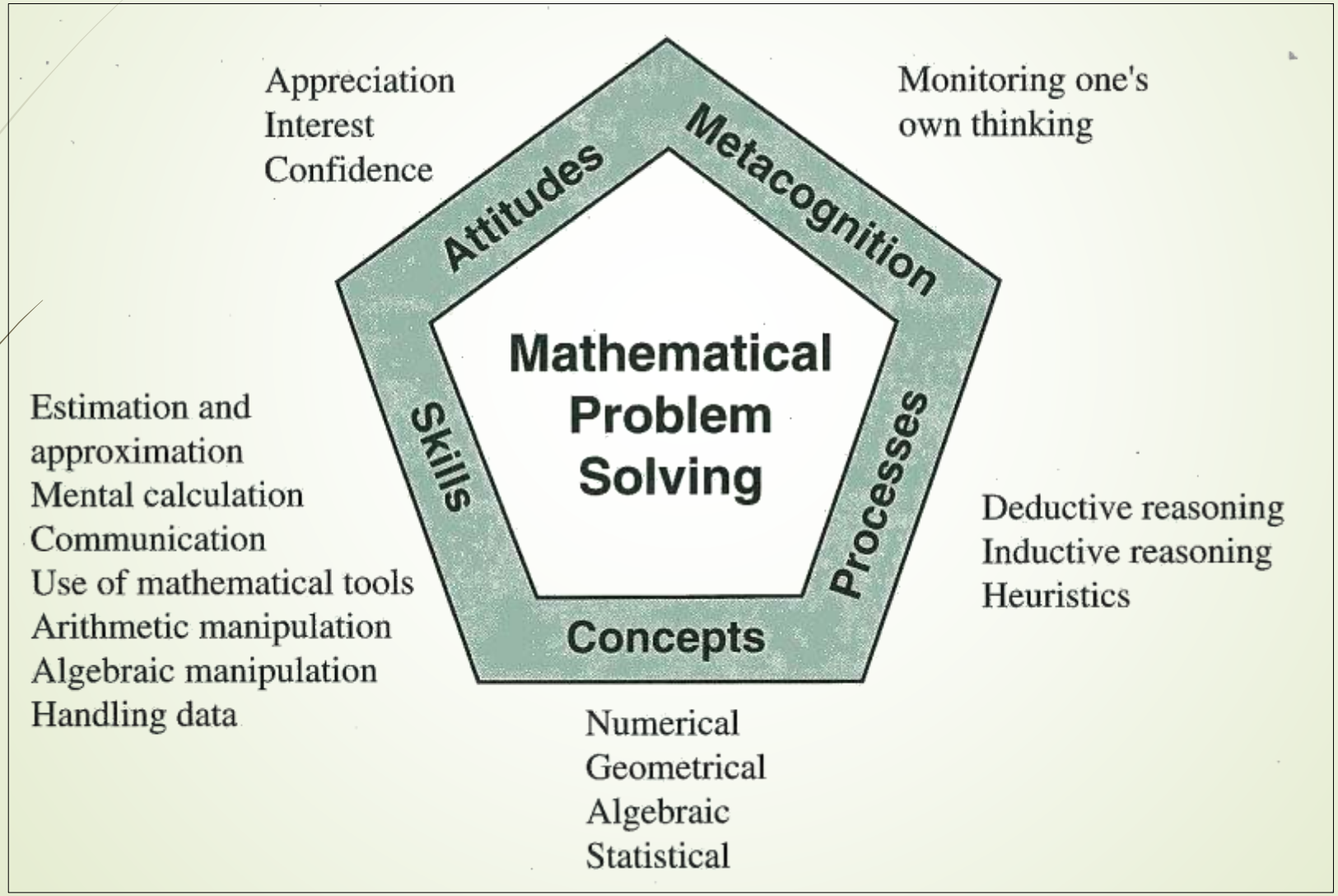
- A set of aims from primary to secondary
- A framework for the problem solving curriculum
- Detailed syllabuses – content charts / topics – outcomes - notes
- Teaching approaches
- Assessment

Aims of Mathematics Education in Singapore

Mathematics education for the primary and secondary schools aims to enable pupils to

- Acquire the necessary **mathematical knowledge** and **skills**, develop **thinking processes** and **apply** them in mathematical situations that they will meet in life;
- Use **mathematics** as a means of **communication**;
- Develop **positive attitudes** and a sense of **personal achievement** in mathematics; and
- Appreciate the **importance** and **power** of **mathematics** in the **world** around them.

Framework of the Mathematics Curriculum



Detailed Syllabuses – Content charts & topics – outcomes - notes

CONTENT CHART - PRIMARY 1 TO PRIMARY 4

	WHOLE NUMBERS	MONEY AND MEASURES	STATISTICS	FRACTIONS	GEOMETRY
P1	1 Numbers up to 100 a) cardinal and ordinal numbers b) number notation and place values c) addition and subtraction d) multiplication and division	1 Measurement of a) length b) weight in non-standard units 2 Time (12-h clock) o'clock, half past 3 Money dollars ($) and cents (¢)	1 Picture graphs	1 Half and quarter	1 Shapes 2 Patterns
P2	1 Numbers up to 1000 a) number notation and place values b) addition and subtraction c) multiplication and division within the 2, 3, 4, 5 and 10 times tables	1 Measurement of a) length : metre, centimetre b) weight : kilogram, gram c) time : hour, minute d) area : in non-standard units e) volume : litre 2 Concept of capacity 3 Addition and subtraction of money 4 Addition and subtraction a) length b) weight c) volume	1 Picture graphs with scales	1 Equal parts of a whole 2 Idea of simple fractions	1 Shapes and patterns 2 Lines, curves and surfaces

CONTENT CHART – SECONDARY ONE (SPECIAL/EXPRESS COURSE)

ARITHMETIC	MENSURATION	ALGEBRA	GEOMETRY
1 Whole numbers • the four operations • ordering • factors and multiples 2 Fractions and decimals • concept and notation • ordering • the four operations 3 Approximation and estimation • rounding off • estimation 4 Use of a calculator 5 Squares, square roots, cubes and cube roots 6 Number sequences 7 Measure and money 8 Ratio, proportion and rate 9 Percentage 10 Simple financial transactions 11 Real numbers • integers • rational and irrational numbers	1 Perimeter and area 2 Volume and surface area	1 Algebraic expressions and formulae 2 Algebraic manipulation 3 Simple linear equations	1 Simple plane figures and simple solid figures 2 Angle properties • angles formed with common vertex • angles formed with parallel lines • angle properties of triangle • angle properties of square, rectangle, parallelogram and rhombus • angle properties of polygon 3 Similarity and congruence 4 Geometrical constructions • construction of simple geometrical figures • scale drawing 5 Symmetry • line symmetry • rotational symmetry

WHOLE NUMBERS

LEVEL : PRIMARY 1

TOPICS	OUTCOMES	NOTES
Numbers to 100	Pupils should be able to	
1 Number notation and place values (tens, ones)	a) count to 100	a) • include counting in tens • include other activities such as completing sequences of consecutive numbers Examples: 15, 16, 17, ____ , 24, 23, 22, ____ ,
	b) read and write numbers up to 100	b) • include writing in words • include the concept of place values
2 Cardinal and ordinal numbers	c) give a number to indicate the number of objects in a given set	c) • include the cardinal concept that the last count gives the number of objects in the set • For small sets up to 5 objects help pupils to visualize the number instead of counting one by one
	d) represent a given number by a set of objects	
	e) Use ordinal numbers such as first, second, up to tenth	e) • include symbols, e.g. 1st, 2nd, 3rd, etc.
3 Comparing and ordering	f) compare two or more sets in terms of the difference in number	f) • include the concept of one-to-one correspondence • include finding 'How many more/less?' • Provide opportunities for pupils to use 'more than', 'less than' and 'fewer than'

Sec 1E geometry

TOPICS	OUTCOMES	NOTES
3 Similarity and congruence	• recognize similar/congruent figures	• Provide activities such as making identical, enlarged and reduced copies of a given figure to help pupils conceptualise similar and congruent figures • Lead pupils to see that i) a figure and its enlarged/reduced copy are similar ii) similar figures of the same size are congruent • Provide investigative activities for pupils to discover properties such as the following: i) the corresponding sides of two similar figures are proportional ii) the corresponding angles of two similar figures are equal iii) the corresponding sides/angles of congruent figures are equal
	• find unknown sides/angles of similar/congruent figures	• Exclude tests for similarity/congruency between two triangles
4 Geometrical constructions		
4.1 Construction of simple geometrical figures	• measure a line segment and an angle	• Emphasis is on correct use of ruler, set squares, protractor and compasses
	• draw line segments, angles, parallel lines and perpendicular lines	• Include i) distance from a given point to a given line ii) distance between two parallel lines

Teaching Approaches

- Schools should implement this syllabus in ways that best cater to the abilities, needs and interests of their pupils….
- The following elements should be an integral part of mathematics teaching and learning:
  - Practical and investigative work
  - The communication aspect of mathematics
  - Problem solving should be an essential part of mathematics activity
  - Mental calculation should be an essential component in mathematics attainment.

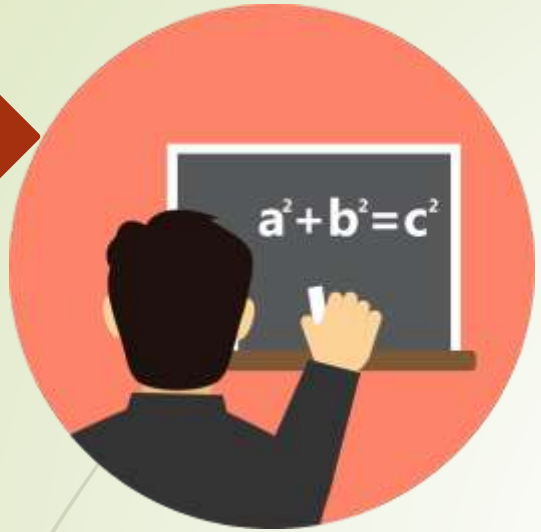
Assessment

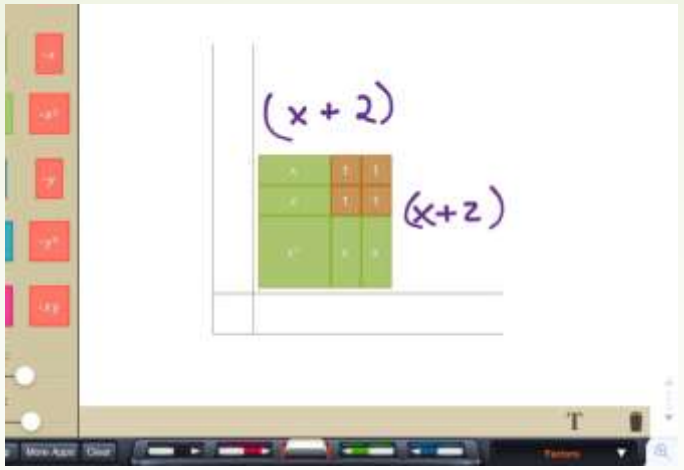
- Purpose of assessment
- Types of assessment
  - Formative
  - Summative
- Assessment methods
  - Observation
  - Oral communication
  - Practical and project work
  - Written assignments and tests

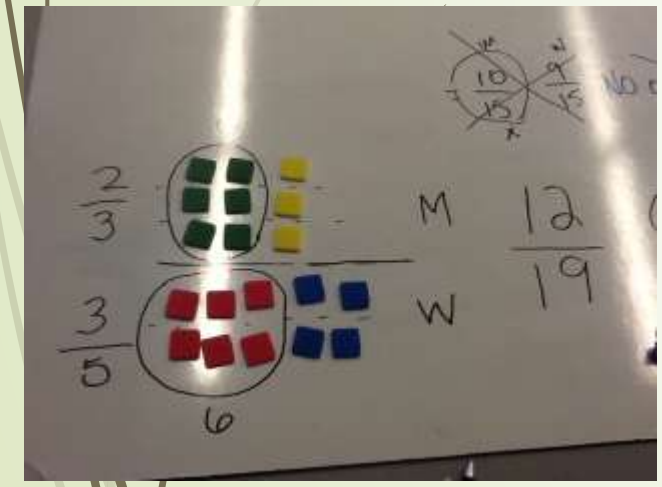
A progressive school mathematics curriculum

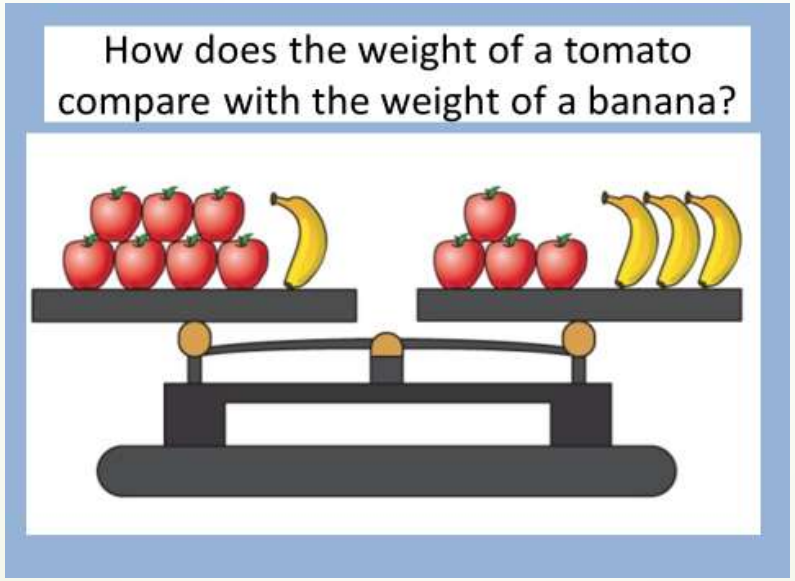
- Periodic revisions:
  - 2000
  - 2006
  - 2012
  - 2018 - -> 2020

Enactment of school mathematics curriculum


$a^2+b^2=c^2$

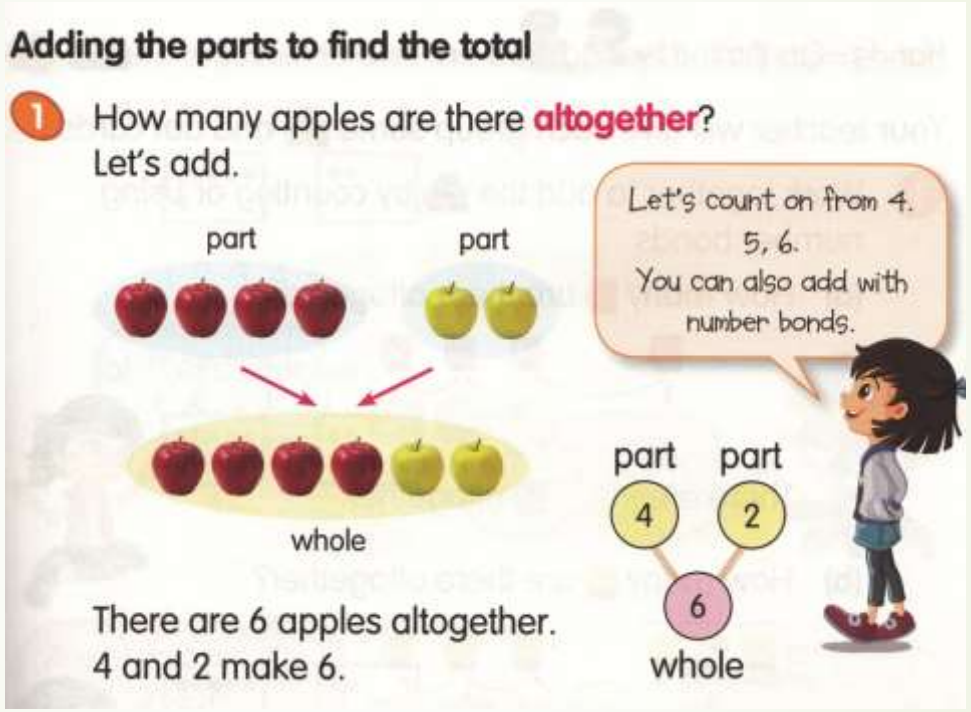

(x + 2)
(x+2)



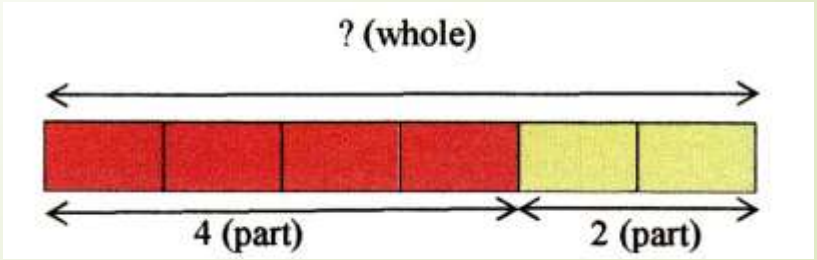

How does the weight of a tomato compare with the weight of a banana?

Resources

Concrete-Pictorial-Abstract



From 1983 onwards…
Pedagogical Approach based on Bruner's theory of learning (construction of knowledge by learners)

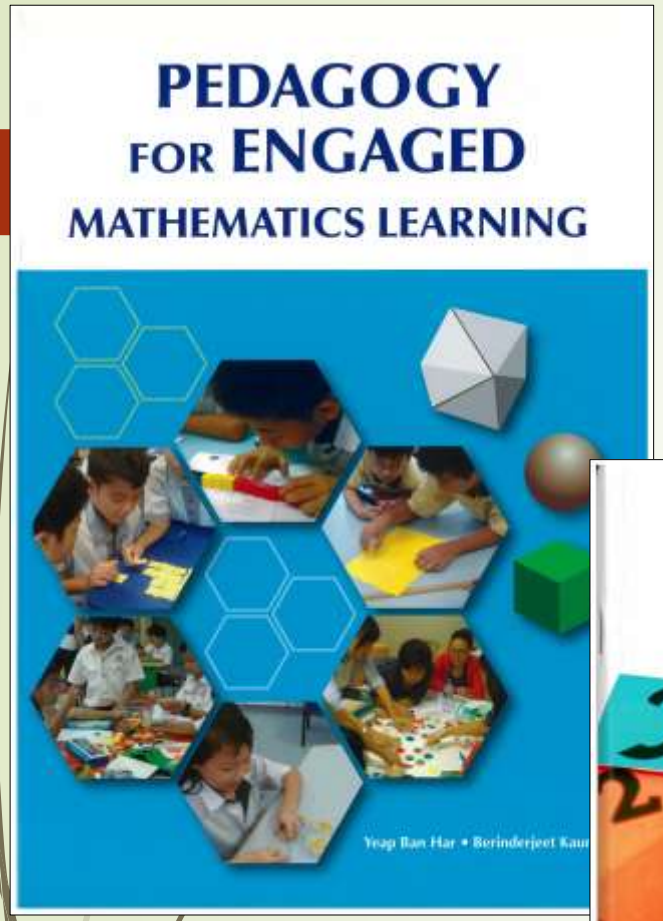


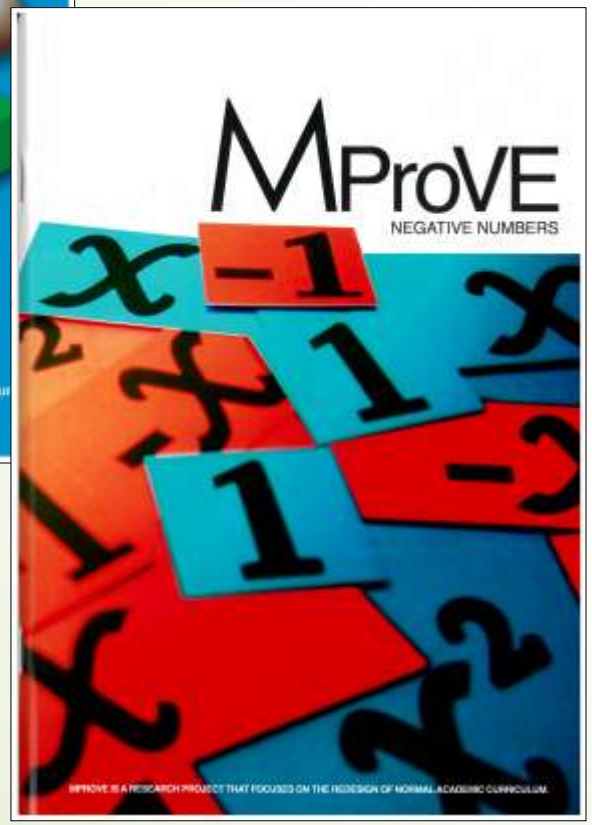
“The quality of an Education System cannot exceed the quality of its teachers”

- Mourshed & Barber (2007) McKinsey Report -

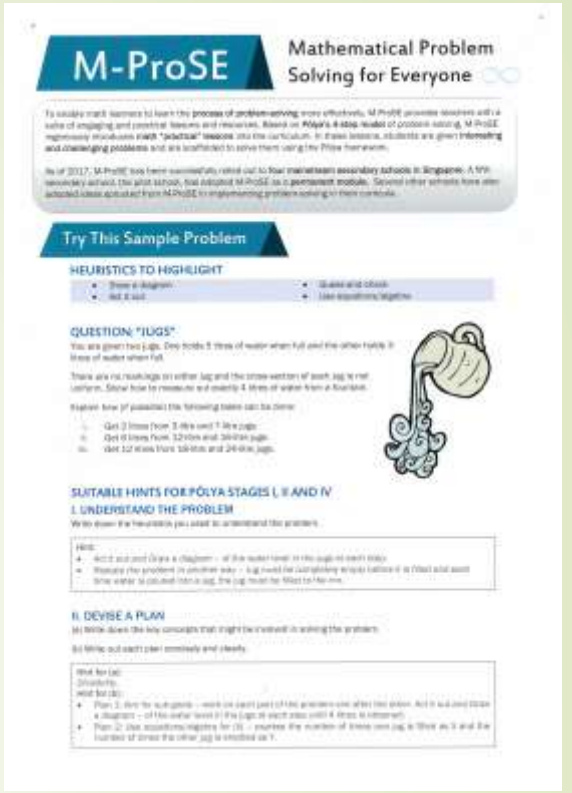
- World class pre-service education for mathematics teachers at the National Institute of Education (NIE)
- Life-long professional development of mathematics teachers
  - NIE, AST (Academy of Singapore Teachers),
  - AME (Association of Mathematics Educators),
  - School-based professional learning communities, etc…

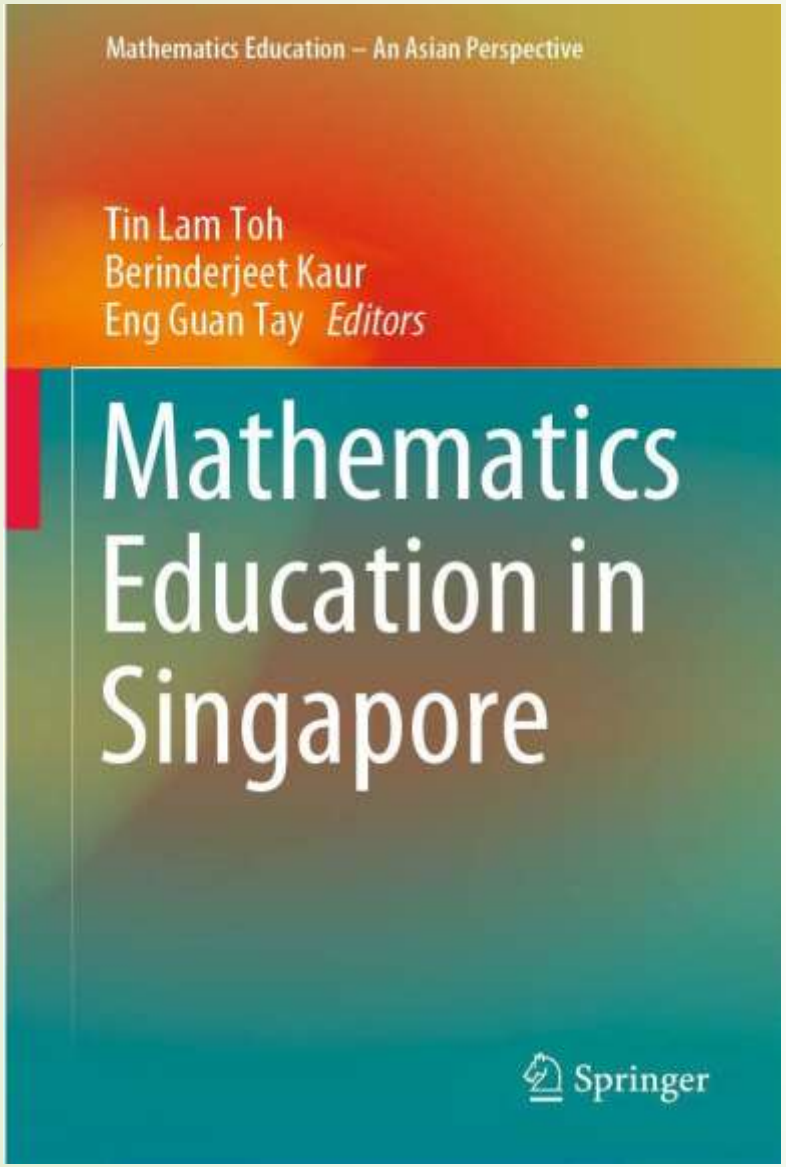
Evidence-Based Pedagogy

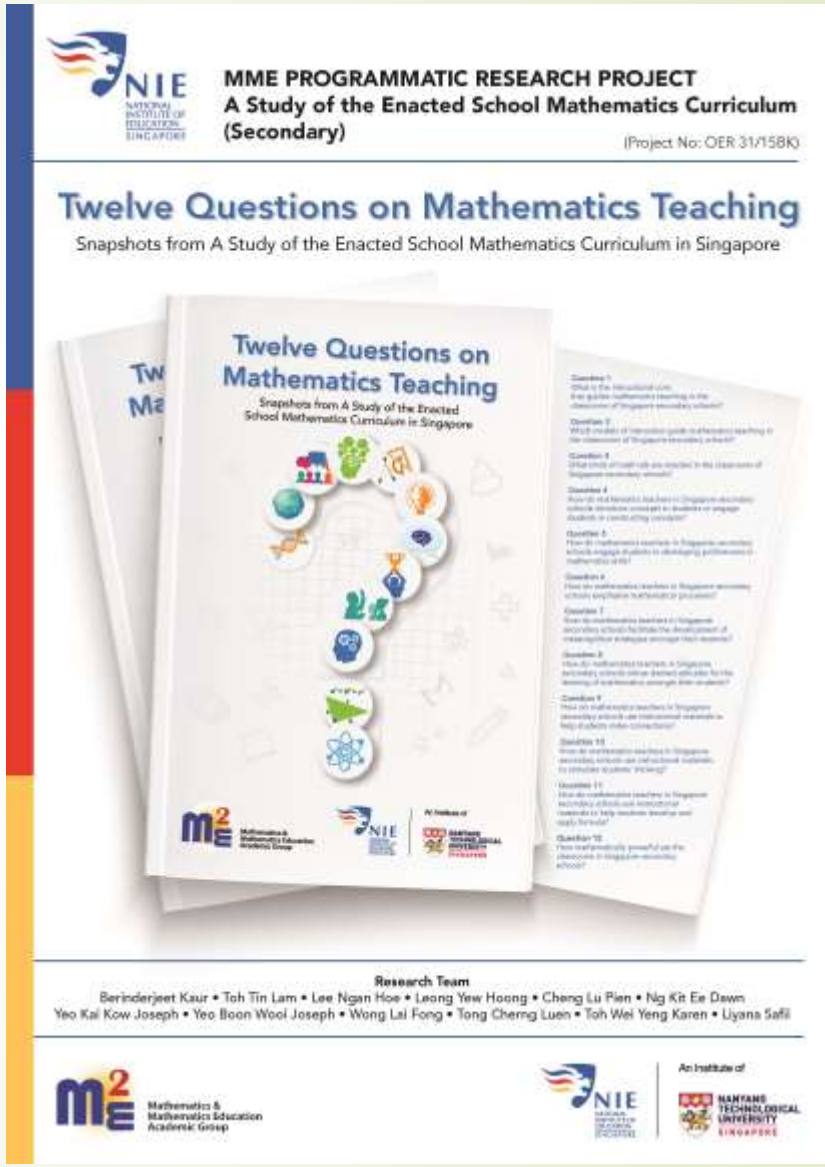

PEDAGOGY
FOR ENGAGED
MATHEMATICS LEARNING
Yeap Ban Har • Berinderjeet Kaur


MProVE
NEGATIVE NUMBERS
-1

MAGICAL
VOL 4: DATA ANALYSIS
SAM
SARAH


M-ProSE
Mathematical Problem Solving for Everyone
Try This Sample Problem
HEURISTICS TO HIGHLIGHT
QUESTION: "JUGS"
SUITABLE HINTS FOR PÓLYA STAGES I, II AND IV
I. UNDERSTAND THE PROBLEM
II. DEVISE A PLAN


Mathematics Education – An Asian Perspective
Tin Lam Toh
Berinderjeet Kaur
Eng Guan Tay Editors
Mathematics Education in Singapore
Springer


NIE
MME PROGRAMMATIC RESEARCH PROJECT
A Study of the Enacted School Mathematics Curriculum (Secondary)
(Project No: OER 31/15BK)
Twelve Questions on Mathematics Teaching
Snapshots from A Study of the Enacted School Mathematics Curriculum in Singapore
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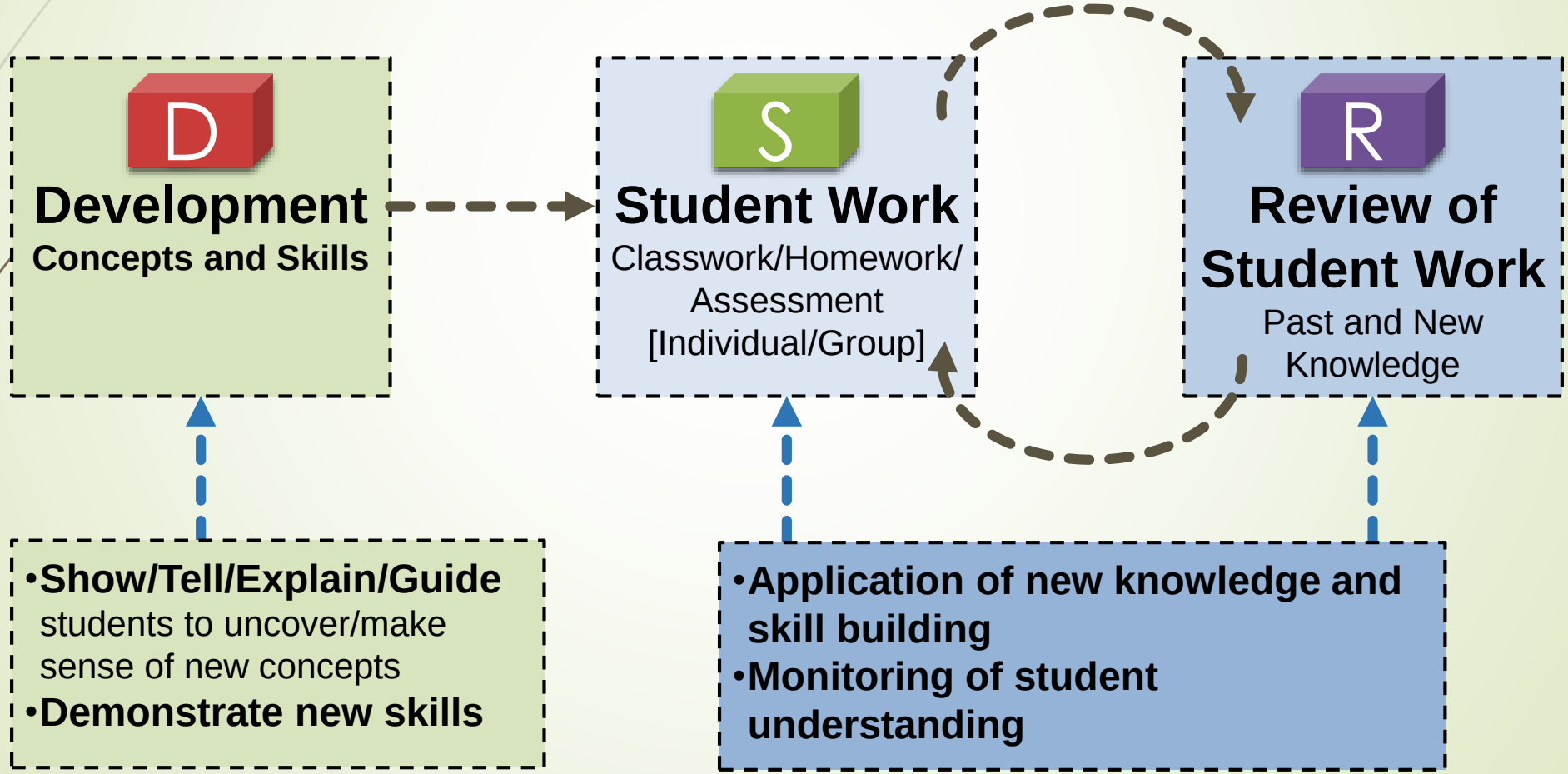
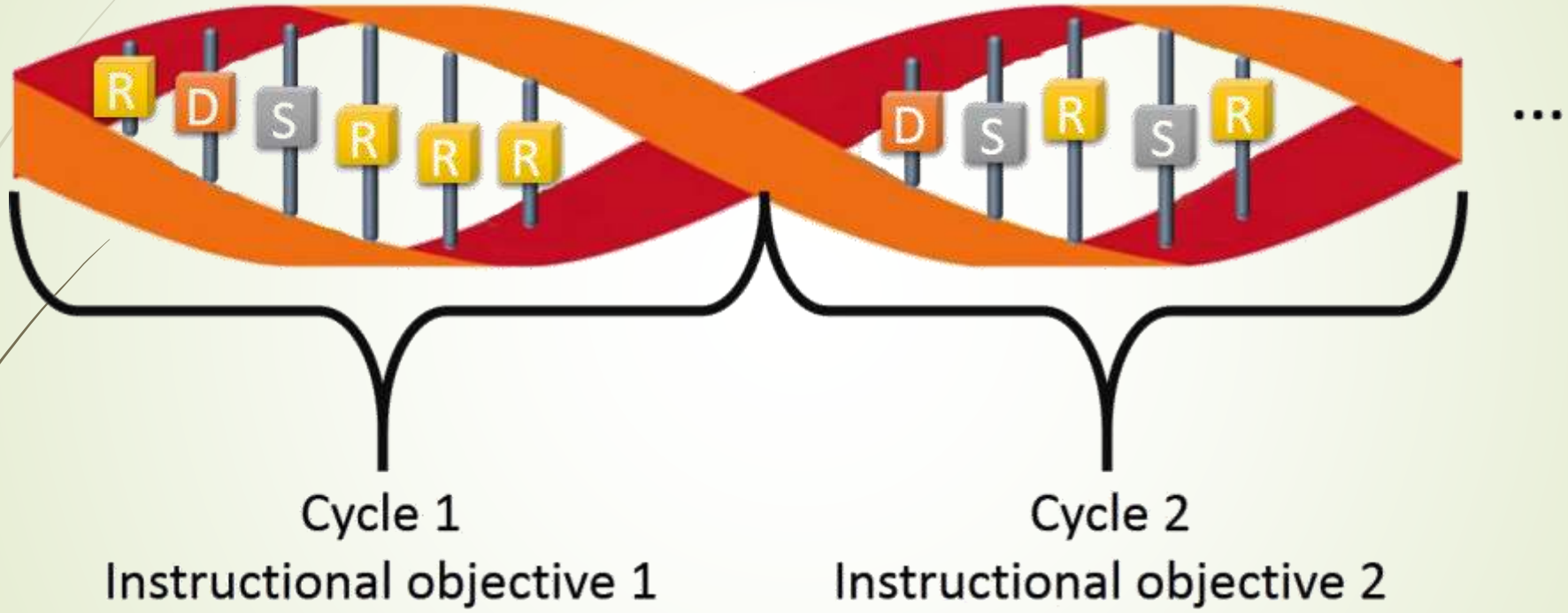
Snapshots from A Study of the Enacted School Mathematics Curriculum in Singapore
Research Team
Berinderjeet Kaur • Toh Tin Lam • Lee Ngan Hoe • Leong Yew Hoong • Cheng Lu Pien • Ng Kit Ee Dawn
Yeo Kai Kow Joseph • Yeo Boon Wool Joseph • Wong Lai Fong • Tong Cherng Luen • Toh Wei Yeng Karen • Liyana Safii
Mathematics & Mathematics Education Academic Group
An Institute of
NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

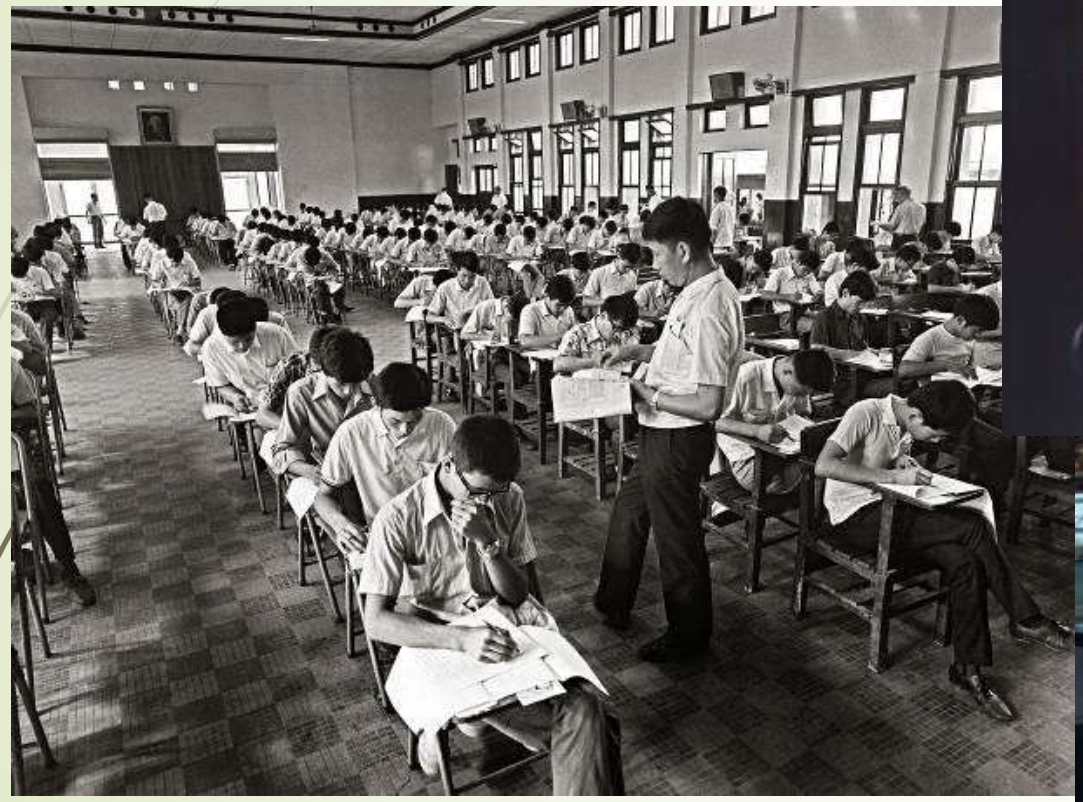
Evidence-based pedagogy
DNA of mathematics lessons (Kaur, 2019).



Towards mastery learning …



Student Outcomes



The Past … (Ministry of Education, 1981).

In 1975, a study carried by the MOE revealed that 25% of students after 6 years of primary school failed to meet the minimum numeracy level set by the ministry.

In 1981, diagnostic tests of basic skills of mathematics were administered to 17 000 Primary 1 to Primary 4 students.

- The findings were dismal.
- More than half of Primary 3 and 4 students did poorly on items that tested division.
- 87% of Primary 2–4 students could solve word problems when key words like ‘altogether’ and ‘left’ were provided but only 46% of them could solve word problems without key words

Progressive assessment of the problem solving curriculum

PSLE Questions

The total mass of 7 apples is 1 kg 50 g.

What is the average mass of the apples?

The average mass of 3 boys, Ali, Boon and Chai, is 44 kg.

Ali is heavier than Boon by 6 kg.

The total mass of Ali and Boon is the same as the mass of Chai.

What is the mass of Ali?

1. 30 kg 2. 36 kg 3. 63 kg 4. 69 kg

GCE 'O' Level Questions

The table shows the number of cars owned by each of 25 families….

Find i) the median number of cars

ii) the modal number of cars
iii) the mean number of cars

The three numbers ***x***, ***y*** and 25 have a mean of 25. the five numbers ***x***, ***y***, 25, ***p*** and ***q*** have a mean of 41.

Find i) the value of ***x*** + ***y***
ii) the mean of ***p*** and ***q***

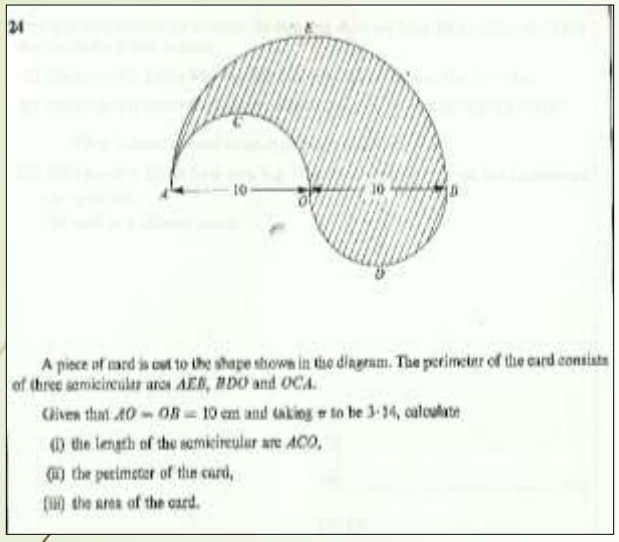
Alec has written down five numbers.

The mean of these numbers is 7, the median is 5 and the mode is 4.

The largest number is three times the smallest number.

Find the five numbers.

Nov 1986 P1/24



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A piece of card is cut to the shape shown in the diagram. The perimeter of the card consists of three semicircular arcs AEB, BDO and OCA.

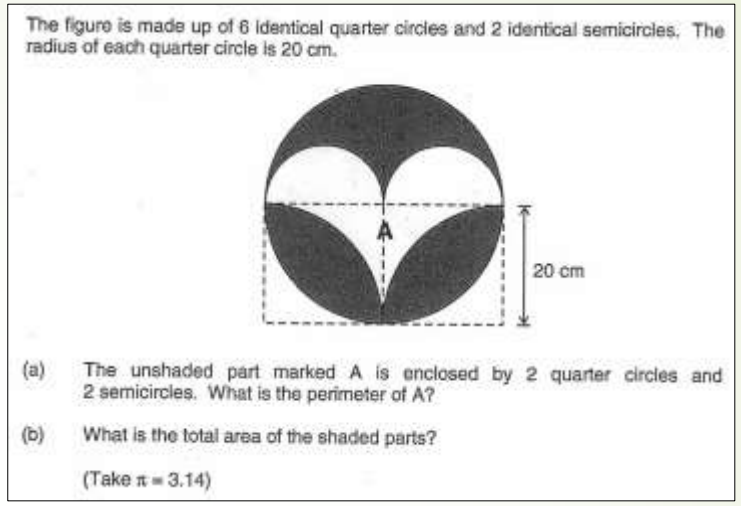
Given that $AO = OB = 10$ cm and taking π to be 3·14, calculate

(i) the length of the semicircular arc ACO,

(ii) the perimeter of the card,

(iii) the area of the card.

PSLE 2001 onwards…



The figure is made up of 6 identical quarter circles and 2 identical semicircles. The radius of each quarter circle is 20 cm.

A

20 cm

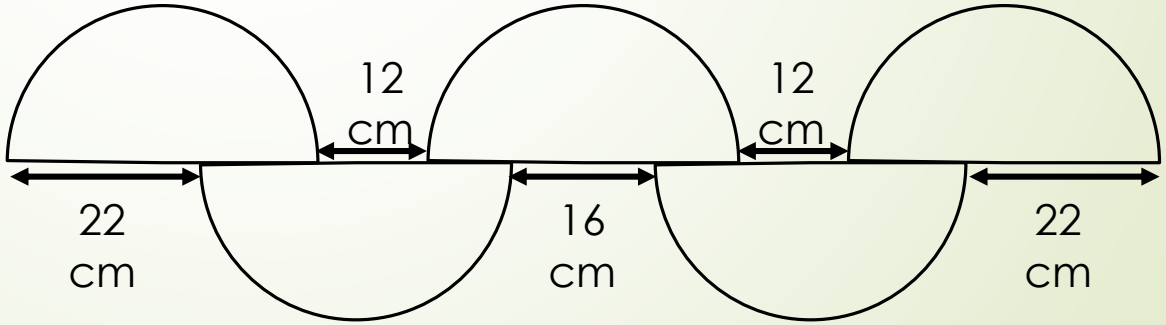
(a) The unshaded part marked A is enclosed by 2 quarter circles and 2 semicircles. What is the perimeter of A?

(b) What is the total area of the shaded parts?

(Take $\pi = 3.14$)

Looking at 2019 PSLE math question …

The figure consists of 5 identical semicircles arranged as shown. Find the diameter of one of the semicircles.



Achievement in mathematics of our students at the national examinations.

For the last 3 years …..

- Almost 90% of students passed O Level Maths
- More than 94% of students passed N(Academic) Level Maths
- More than 80% of students passed N(Technical) Level Maths
- More than 85% of students obtained A+ to C in PSLE Standard Maths

Source: https://data.gov.sg

November 29, 2016 Straits Times

Singapore students top global achievement test in mathematics and science

SINGAPORE - Singapore students are the world's best in mathematics and science, according to a global benchmarking study released on Tuesday (Nov 29).

Primary 4 pupils and Secondary 2 students here topped both subjects in the Trends in International Mathematics and Science Study (TIMSS), a widely recognized achievement test by policymakers and educators worldwide.

Around 12,600 students here took part in the latest test which was conducted in Singapore in October 2014. Students across all schools - 179 primary schools and 167 secondary schools - as well as streams were included in the sample.

December 6, 2016 Straits Times

Singapore students top in maths, science and reading in PISA international benchmarking test

SINGAPORE - One week after scoring top marks in a mathematics and science study, Singapore students aced an even more prestigious international benchmarking test, dubbed the "World Cup for Education".

The Republic's 15-year-olds were ranked No 1 for mathematics, science and reading in the Programme for International Student Assessment (PISA), a study done every three years, and run by the Organization for Economic Cooperation and Development (OECD) to measure how well students use their knowledge and skills to solve real-world problems. In the last PISA test in 2012, Singapore students were ranked second in mathematics and third in science and reading.

Mathematics Education in Singapore Schools

The Past, Present and Future

OECD Education and Skills Today

OECD Education Today October 11, 2019

Why it's important to think like a mathematician

Andreas Schleicher,

Director, OECD Directorate for Education & Skills

*The **mechanics of math** are becoming **less important for humans** , because **computers** are so much quicker and more accurate in carrying them out. If you don't know how to calculate a logarithm, don't worry, you won't need to.*

*But a **deep understanding of mathematical ideas and principles**, and our **capacity to think like mathematicians**, are becoming more important. Because without that capacity, we will be unable to navigate the data, numbers, graphs or diagrams around us.*

*We can always find an expert to code an app, but if we **lack** the **basic elements of computational thinking**, we won't understand the nature of the algorithms that now penetrate every facet of our lives.*

Doer ------> Thinker -----> Creator

Thank You

Comments, Questions & Clarifications

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