

# VISUAL AND SPATIAL REASONING: THE *CHANGING* 'FORM' OF MATHEMATICS REPRESENTATION AND COMMUNICATION

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# INTRODUCTION

- How we represent mathematics and mathematical ideas shapes students' thinking
- Visual and graphic representations are increasingly influential in students sense making and everyday lives

# CONTEXT

- Although mathematics curricula has changed little in the past ten years the way in which mathematical ideas are represented and communicated have shifted dramatically.
- Different forms of sense making are required as (even young) children become increasingly exposed to visual forms of communication.
- Whether playing computer games, navigating web pages, or interpreting the rich design tasks students require a range of spatial reasoning skills to interpret information.

# REPRESENTATION

- **Encoding** generally occurs when students construct their own representations in order to solve a task. e.g., drawing diagrams or visualising
- **Decoding** techniques are used to make sense of information within a given task, when the information has been represented visually for others to solve

# ENCODING TASKS

1. Some sparrows are sitting in two trees, with each tree having the same number of sparrows. Two sparrows then fly from the first tree to the second tree. How many sparrows does the second tree then have more than the first tree?
2. A saw in a sawmill saws long logs, each 16m long, into short logs, each 2m long. If each cut takes 2 minutes, how long will it take for the saw to produce eight short logs from one long log?

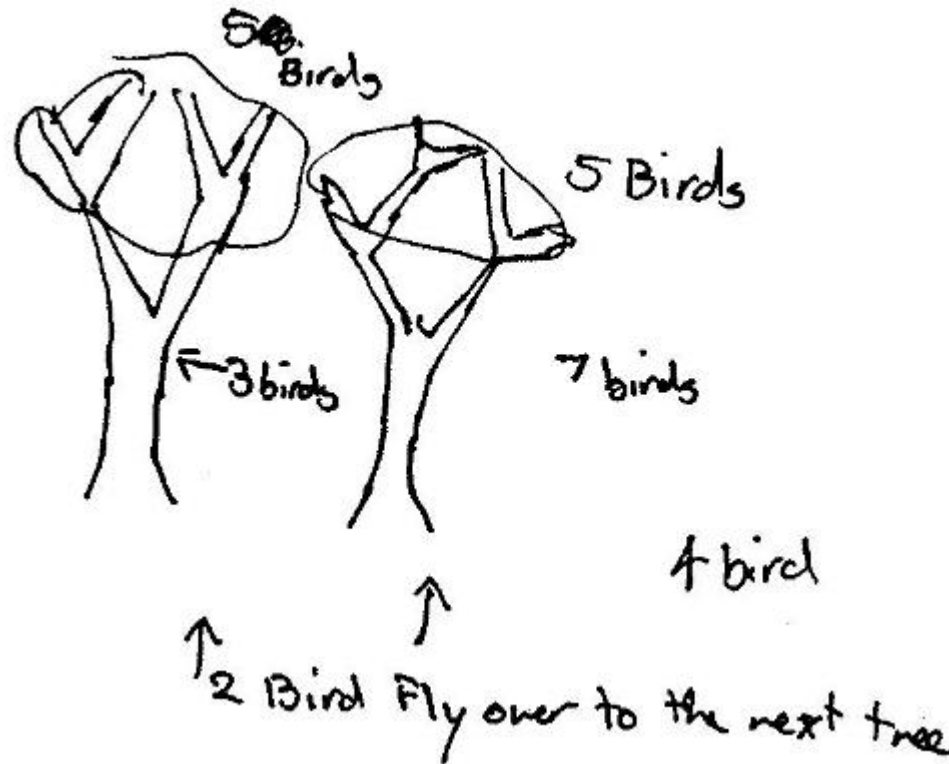
# ENCODING TASKS

## Discussion

- What strategies did you use to solve the tasks?
- Would the availability of “working out space” have influenced your approach?

# STUDENT WORK SAMPLES

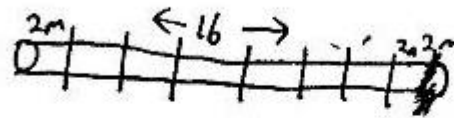
## Task 1.



# STUDENT WORK SAMPLES

## Task 2.

17. ~~16m~~<sup>14</sup>, 2m,



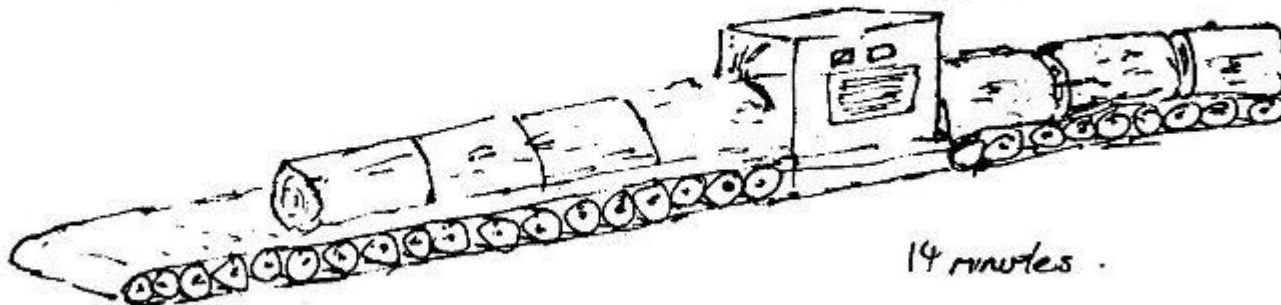
I've learnt that to double  
check it and be more,  
carefull and for each  
~~piece~~ log you need  
to have one less cut.

# STUDENT WORK SAMPLES

## Task 2.

17) 16 minutes. The first sentence was not really needed so I paid attention to the second sentence; then the answer was pretty straight forward;

$$\begin{array}{r} 2 \\ 8 \times \\ \hline 16 \text{ mins.} \end{array}$$



14 minutes.

I learnt by drawing a diagram that there was only 7 cars not eight thus subtracting 2 minutes.

# THE CHANGING NATURE OF ENCODING

- What are non graphic tasks measuring?
  - Problem solving skills as opposed to content knowledge
  - Content knowledge as opposed to problem solving skills
  - Or both?



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**RIPPLE**

# CONTENT KNOWLEDGE?

A group of students paid \$4 each to see a show.  
Altogether the group paid \$124.

How many students were in the group?

30

31

34

51



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UNIVERSITY

Professor Tom Lowrie Mathematics Teachers Conference:  
Communication, Reasoning and Connections. Singapore 2011

# PROBLEM SOLVING SKILLS?

- Siti started saving some money on Monday. On each day from Tuesday to Friday, she saved 20¢ more than the amount saved the day before. She saved a total of \$6 from Monday to Friday. How much money did she save on Monday?
- One afternoon, 6 friends rented 4 computers from 2.00 p.m. to 4.30 p.m. and took turns to play games on them. At any time, 4 of them played on the computers while the other 2 friends watched. If each of them had the same amount of playing time, how many minutes did each person play on the computer?

Singapore PSLE questions

# ENCODING EXAMPLE FROM NAPLAN

- Encoding as a support system.
- Aspects of realistic problem solving



Question 6

Lin is packing 34 cakes into boxes.  
Each full box holds 5 cakes.

Write your answer  
in the box.

What is the smallest number of boxes Lin needs  
to pack all the cakes?

# ENCODING EXAMPLE FROM NAPLAN

Question 11

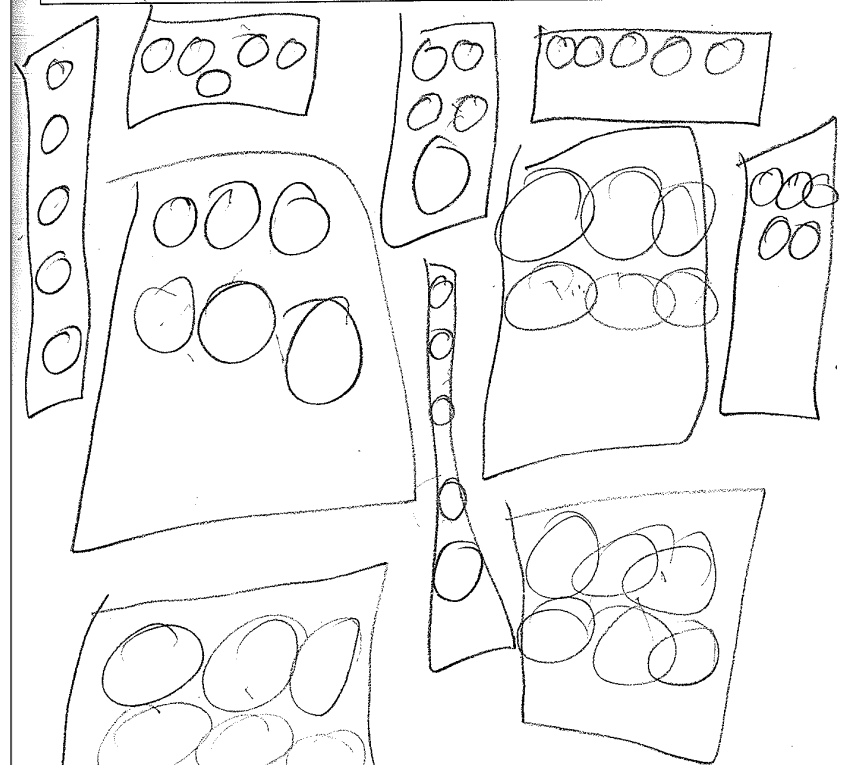
A school has 10 boxes of footballs.  
Four boxes have 6 footballs in each box.  
The other boxes have 5 footballs in each box.  
How many footballs are there **altogether**?

Write your answer  
in the box.

54

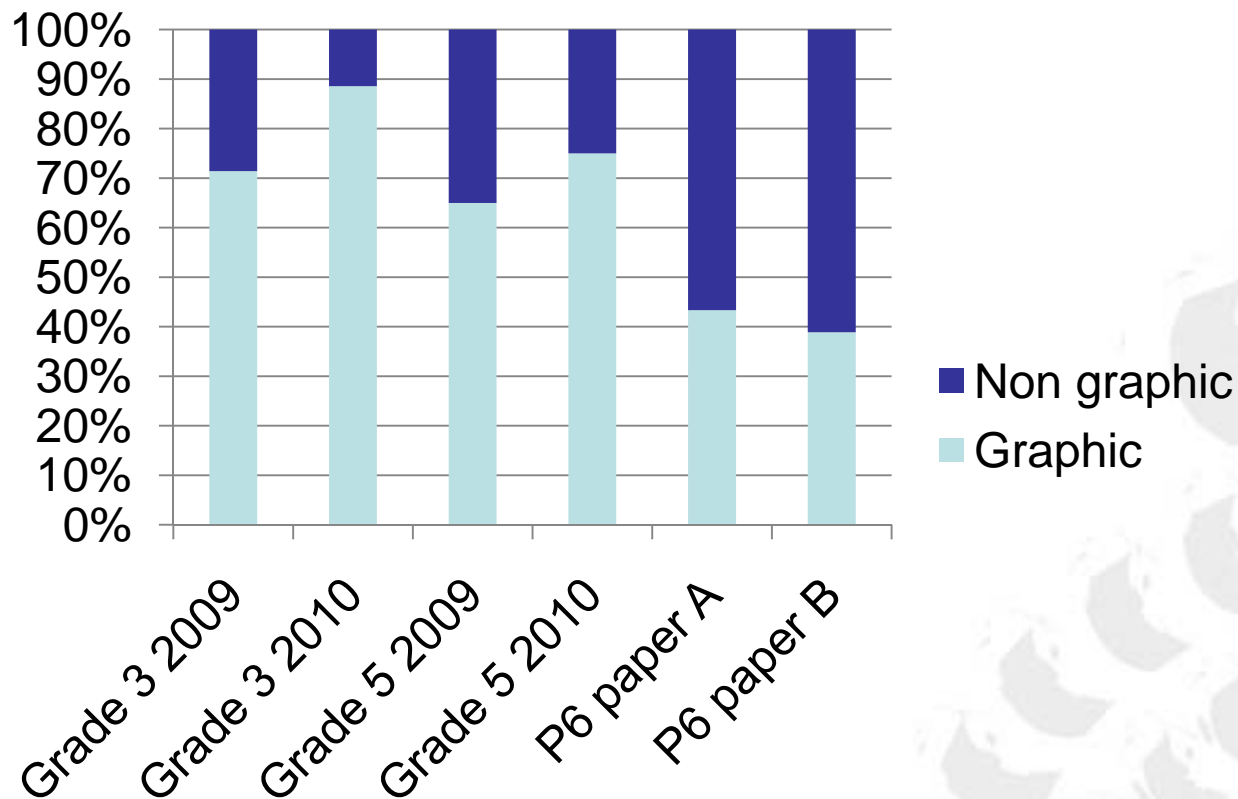
A school has 10 boxes of footballs.  
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box.

How many footballs are there  
**altogether**?



# THE INFLUENCE OF DECODING IN ASSESSMENT

Proportion of Graphics and Non graphics items from the Australian NAPLAN (Grade 3 & 5) booklets and the specimen papers from the 2006-2010 Singapore PSLE (Grade 6) booklet.



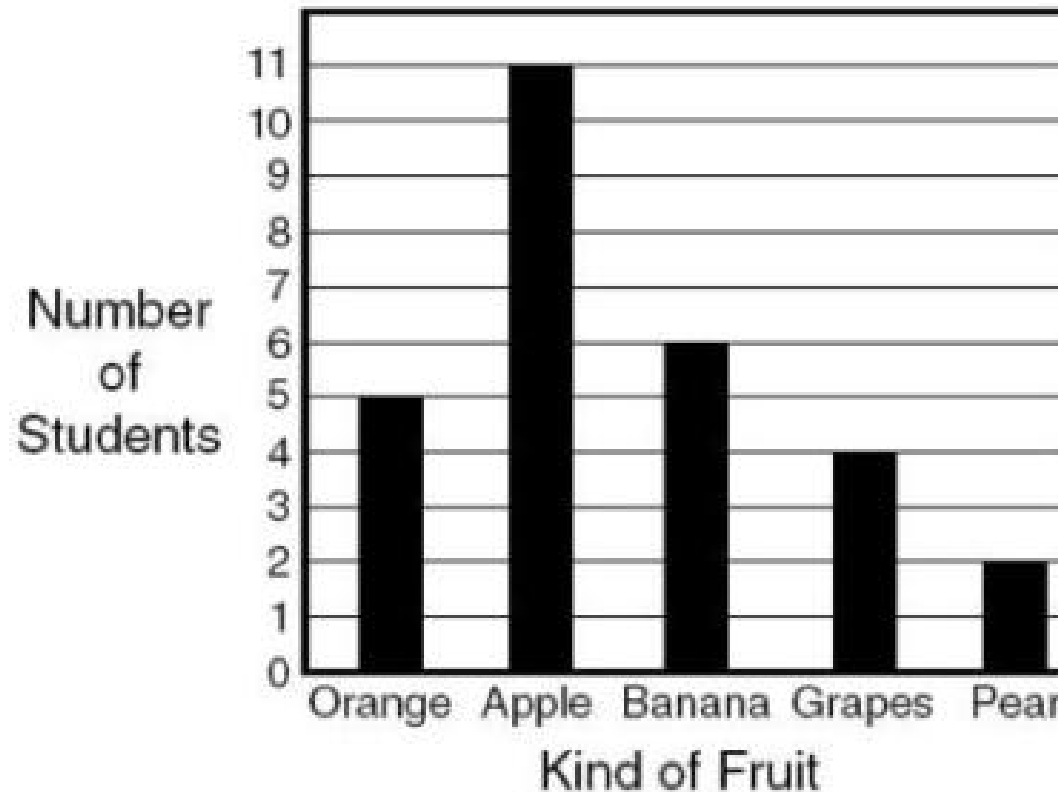
# DECODING

- There are three levels of decoding with graphics
  - \* Elementary—extracting information from the data (reading the data)
  - \* Intermediate—finding relationships between the data (reading between data)
  - \* Advanced—moving beyond the data, predicting and generating

Source: Friel, Curcio & Bright, 2001

# ELEMENTARY EXAMPLE

Lunch Fruits

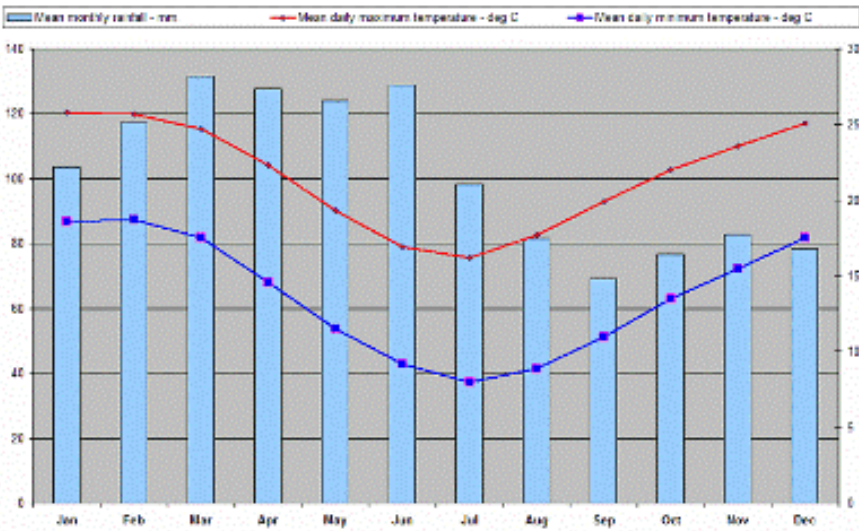


How many students eat Grapes?

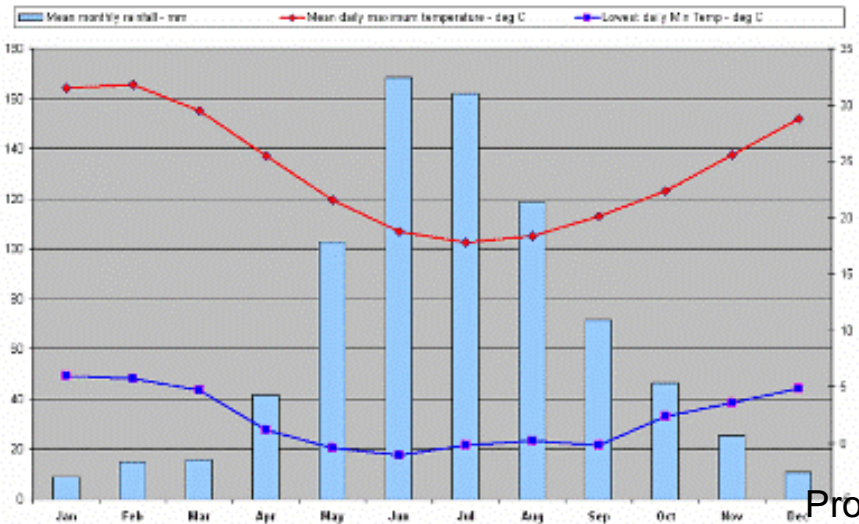


# INTERMEDIATE EXAMPLE

What are the differences between climate variables in Sydney and Perth?



Sydney

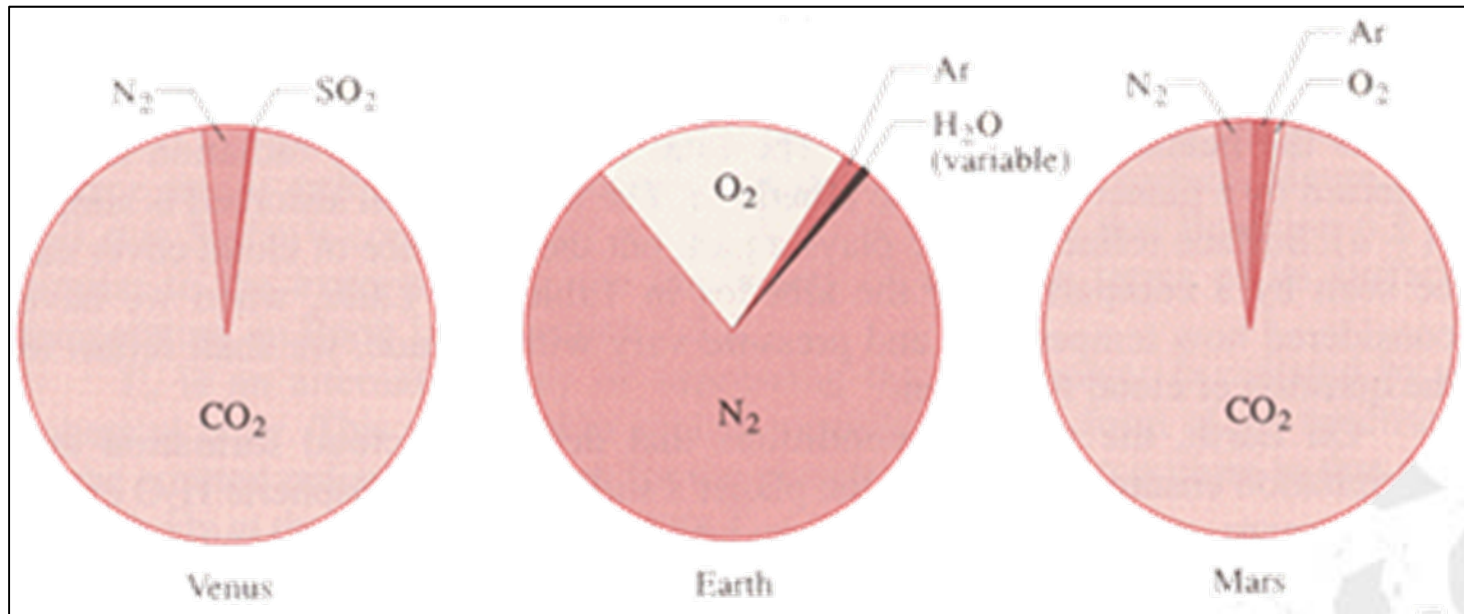


Perth

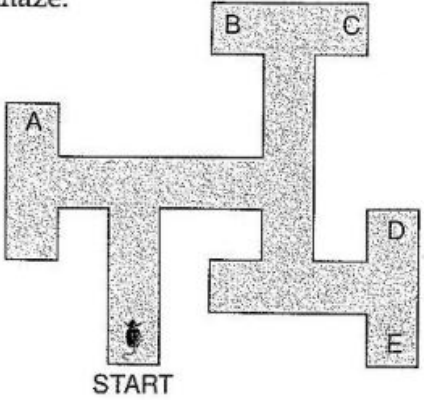
- █ = Mean yearly rainfall
- = Mean daily maximum temperature
- = Mean daily minimum temperature



# ADVANCED EXAMPLE



Joseph put a mouse at the start of this maze.

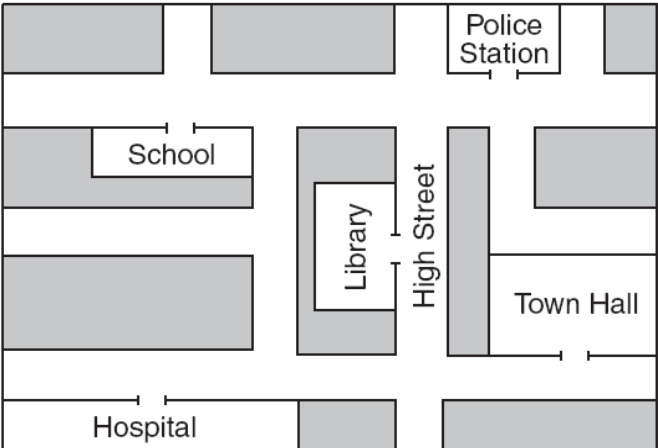


It ran through the maze.  
It turned right, then right,  
then left, then left.

Where did the mouse finish?

A                       D  
 B                       E  
 C

NSW DET, (1994). NSW Year 3  
Basic Skills Test



Jenny leaves the Library and turns right into High Street.  
She then turns left at the next corner and walks straight ahead.

Which building does she pass?

School                       Hospital                       Police Station                       Town Hall

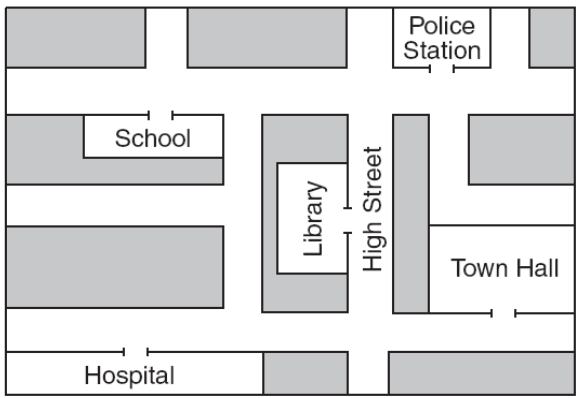
Australian Curriculum, Assessment and Reporting  
Authority. (2009). National Assessment Program Literacy  
and Numeracy: Numeracy Year 3 2009 (p. 14). Sydney:  
Australian Curriculum and Assessment Reporting Authority.

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# COMPARING LIKE TASKS

- On similar items *15 years* apart...
  - ❑ More steps required in the '94 task
  - ❑ Perhaps higher graphics demands in '09 task
  - ❑ '09 task certainly more contextually rich and 'realistic' in nature

# SPATIAL REASONING ON A MAP TASK

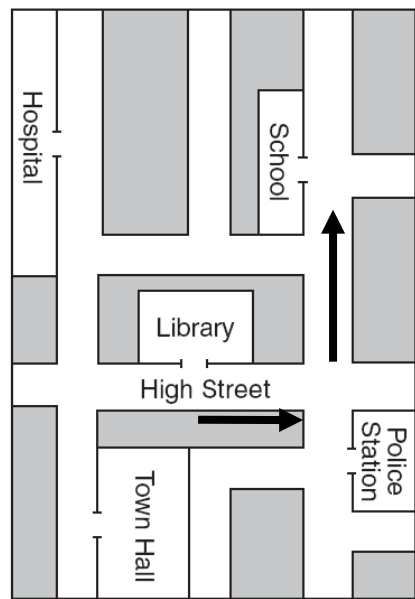


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Above: The Map task as it is presented in the NAPLAN.

Right: Gemma turned the Map 90° clockwise

# SPATIAL ORIENTATION



Gemma solving the Map task

- A case of 'inside' space
- Inappropriate position disrupts movement

# ENCODING-DECODING REPRESENTATIONS

A box contains 6 red marbles, 10 blue marbles and 4 yellow marbles.

Which colour marble is **impossible** to take from the box?

red

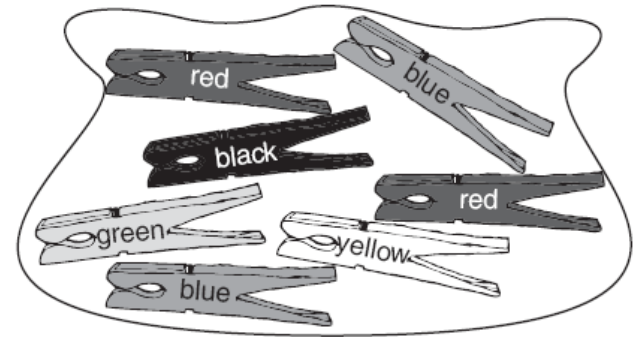
blue

white

yellow

Australian Curriculum, Assessment and Reporting Authority. (2009). National Assessment Program Literacy and Numeracy: Numeracy Year 3 2009 (p. 7). Sydney: Australian Curriculum and Assessment Reporting Authority.

Jess takes 2 pegs out of this bag at the same time.



Which of these is **impossible**?

- a blue peg and a black peg
- a red peg and a red peg
- a green peg and a green peg
- a yellow peg and a black peg

Australian Curriculum, Assessment and Reporting Authority. (2010). National Assessment Program Literacy and Numeracy: Numeracy Year 3 2010 (p. 10). Sydney: Australian Curriculum and Assessment Reporting Authority.

Above: A Probability task without a graphic

Right: A Probability task with a graphic that is essential in order to answer the task

# DECODING THE GRAPHIC



Barry solving the graphic probability task

- Realistic context disrupting sense making

# DECODING THE GRAPHIC

- Realistic context  
disrupting sense  
making

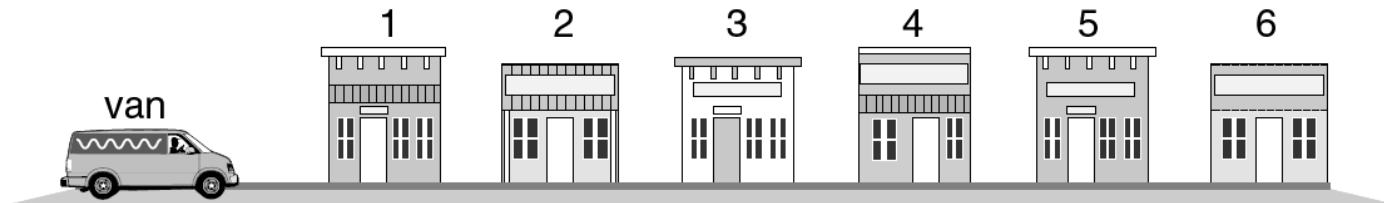


Stacy solving the graphic probability task

# CONTEXT GRAPHIC

What purpose is this graphic serving?

There are 60 boxes in a van.  
There are 6 shops.  
Seven boxes are delivered to each shop.



How many boxes are left in the van after shop 6?

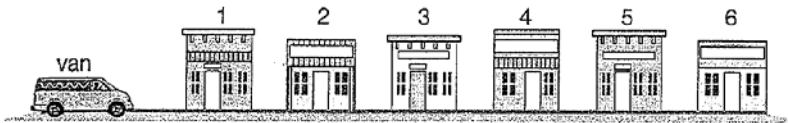
Australian Curriculum, Assessment and Reporting Authority. (2010). National Assessment Program Literacy and Numeracy: Numeracy Year 3 2010 (p. 15). Sydney: Australian Curriculum and Assessment Reporting Authority.

# ENCODING

## (WITHOUT DECODING!)

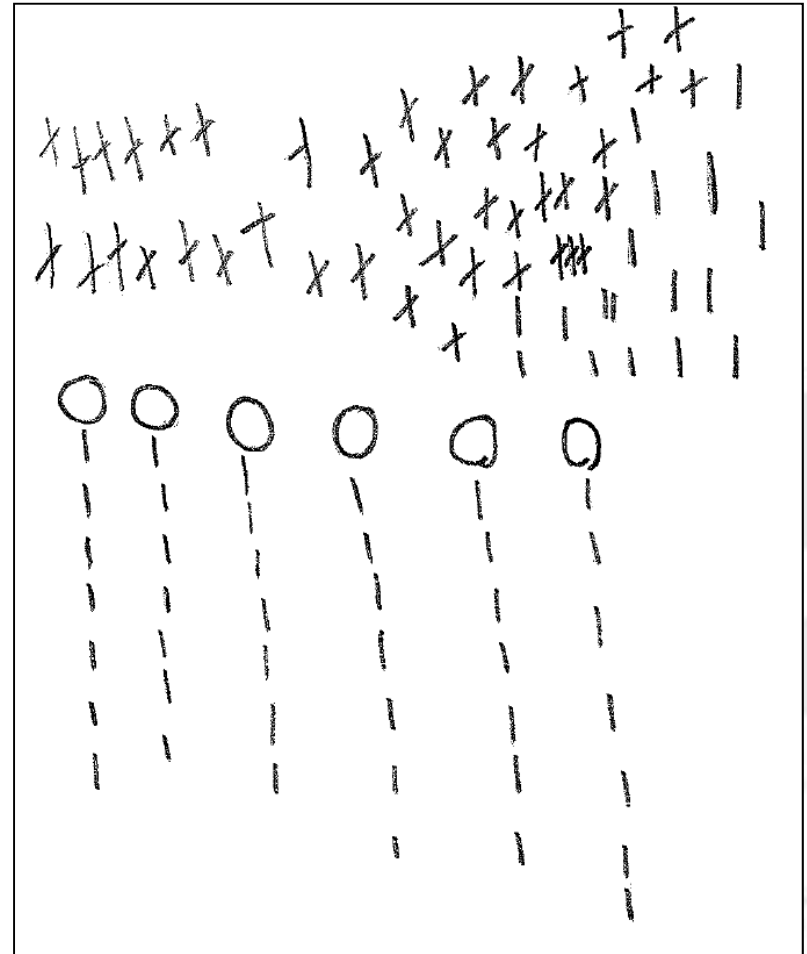
### Question 12

There are 60 boxes in a van.  
There are 6 shops.  
Seven boxes are delivered to each shop.



How many boxes are left in the van after shop 6?

16



# DECODING THE CONTEXTUAL GRAPHIC



- Using the graphic as a tool

# CONCLUSIONS

- Further research needs to be undertaken on the nature (composition and structure) and intent (what are we measuring?) of graphics tasks in assessment.
- Given the increasing reliance of graphics in society, it is not surprising that graphic representations hold a prominent place in *current* forms of assessment. And since assessment tends to influence and even drive practice, the way in which mathematics ideas and conventions are represented impact greatly on teaching practices and student learning.

# IMPLICATIONS

- A number of practical implications emerge from the study.
- Students are required to decode external representation with more regularity than the process of evoking internal representations through encoding. Although both require high levels of spatial reasoning, most representations are now constructed *for* the student rather than *by* the student.
- Students need to acquire different spatial-reasoning skills which allow them to consider all the elements of a task, including specific features of a graphic and the surrounding text, when solving mathematics tasks.

# IMPLICATIONS

- Using traditional word-based problem solving, provides opportunities for students' to utilise encoding techniques to make sense of mathematics ideas. If these encoding skills are not encouraged and promoted elsewhere, students' general reasoning skills will be restricted since such techniques are necessary when students encounter novel or complex problems.
- Conversely, the introduction of mathematics tasks rich in graphics requires a different skill base. Explicit attention needs to be given to specific types of graphics since they have different structure and conventions. Teaching map-based graphics, for example, requires different approaches and techniques than graph-based graphics. Indeed bar graphs and line graphs require specific and independent attention.

# DISCUSSION