

Instructional Tasks in Mathematics Classes in Singapore.

June 2, 2011

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National Institute of Education
Singapore

2011 Mathematics Teachers Conference

- 6th Annual Conference
- Congratulations Professor Berinderjeet Kaur!



Today's Presentation: Research Questions

- What are instructional tasks?
- Why are tasks important?
- What kind of instructional tasks do teachers ask students to engage in in Singapore?
- What is the intellectual quality of the instructional tasks in Secondary 3 Mathematics that teachers set for their students?
- What might be a possible framework to guide teachers in the design of high quality instructional tasks?

Outline

Part One. Instructional Tasks

Part Two. Improving Task Design and Implementation

Part Three. Three Conclusion

Core 2 Design

- Panel 2
 - Surveys (S, T) x 2 Primary 5 and Secondary 3 English and Mathematics (n=64 schools)
 - Assessment x 2
- Panel 3
 - Classroom observation (1 unit of work in each class in each subject in each year) = 620 lessons in 42 schools (M=360 lessons approx)
 - Coded in Excel and Studio Code
- Panel 5
 - Teacher assessment and assignment tasks
 - Student artifacts

PART ONE.

INSTRUCTIONAL TASKS

Why Tasks?

“It is what teachers get students to do in the class that emerged as the strongest component of the accomplished teachers repertoire, rather than what the teacher, specifically does.” In effect, in learning, what teachers do matters, *but what students do matters even more.*

John Hattie, *Visible Learning*, 2009, p.22.

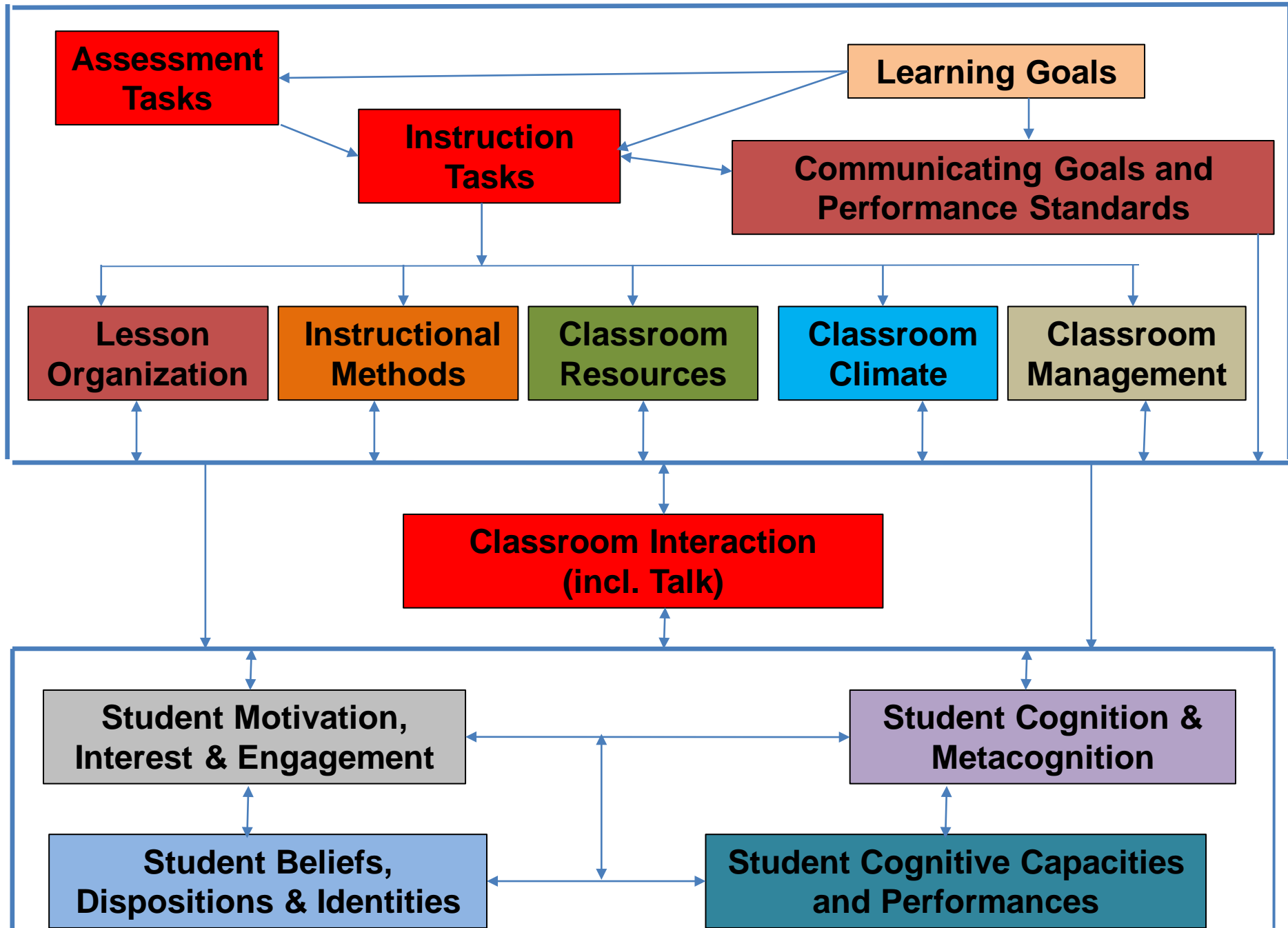
“Task analysis is the single most important component process in the instructional design process, whether that process is used to produce direct instruction, performance support, or constructivist learning environments.”

David Jonassen, Martin Tessmer, and Wallace Hannum, *Task Analysis Methods for Instructional Design*. London: Routledge, 1999, p. 1

Why Tasks?

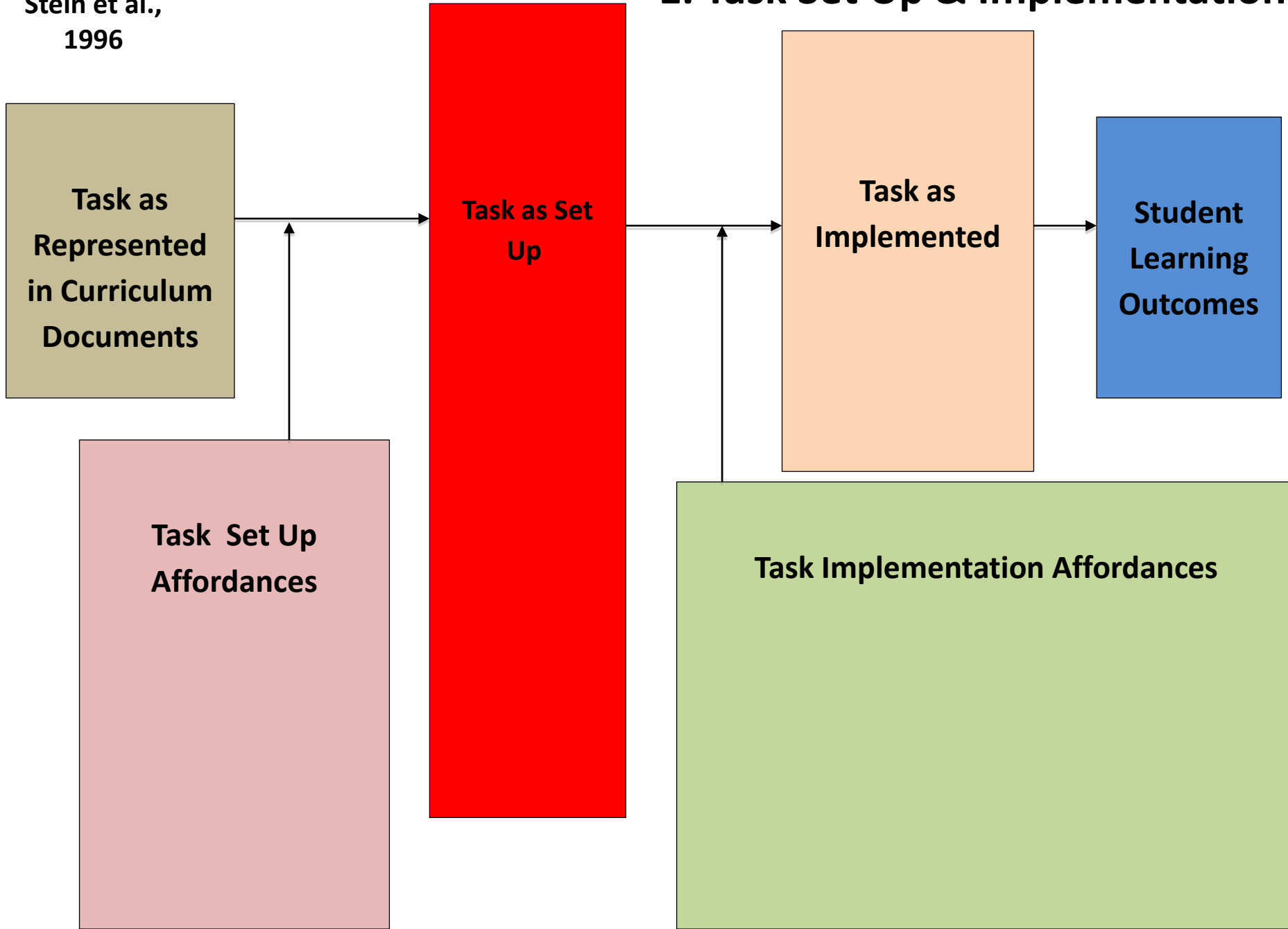
1. Principal determinant of the **intellectual quality of knowledge** work in the classroom & student learning
2. In principle, the **key organizing principle** of instructional practices
 - Instructional methods
 - lesson organization/participation structure
 - Classroom learning environment
 - Resources & technology
 - classroom talk

An Anatomy of Teaching and Learning



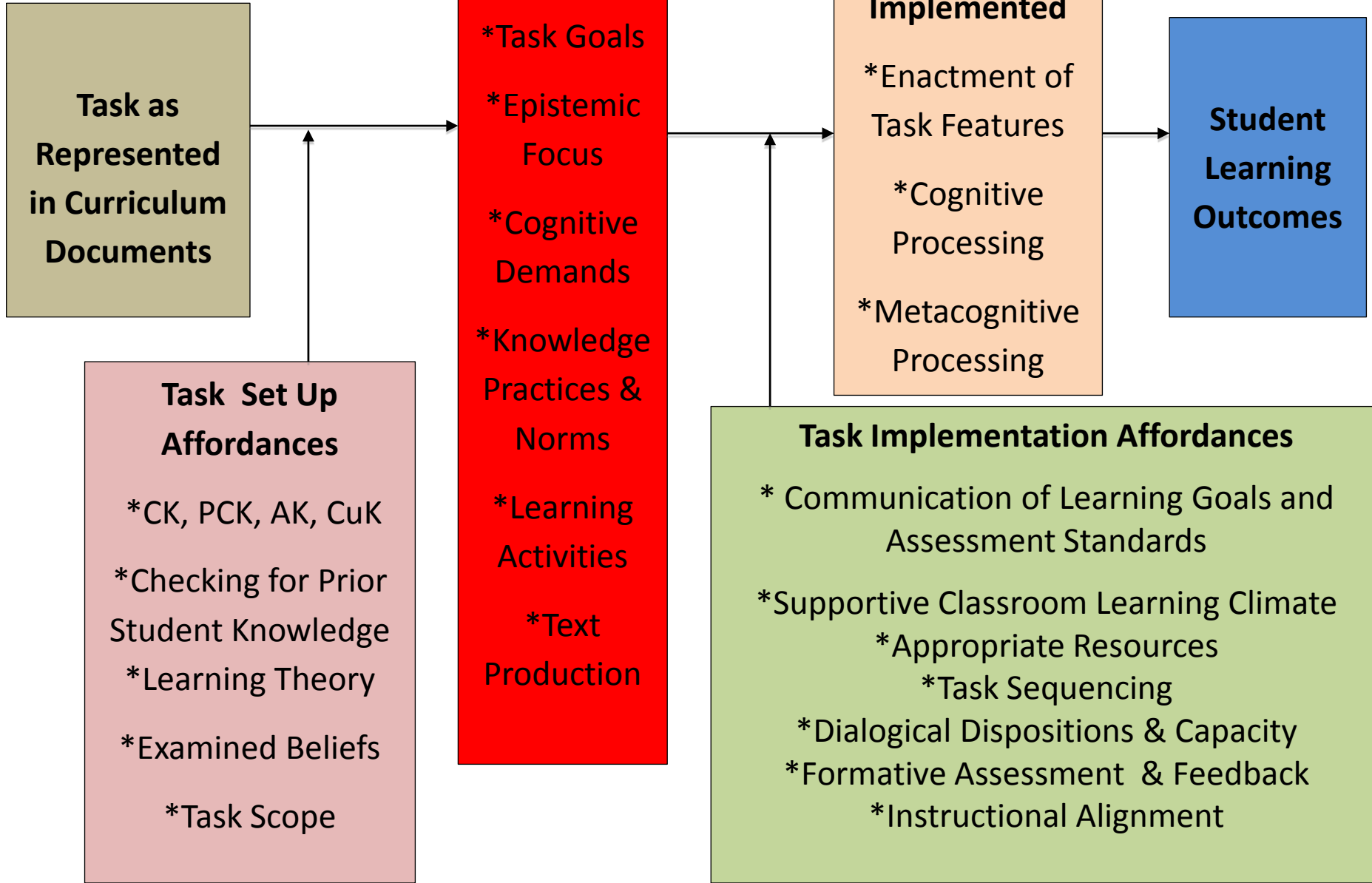
Adapted from
Stein et al.,
1996

1. Task Set Up & Implementation



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Stein et al.,
1996

Task Set Up & Implementation



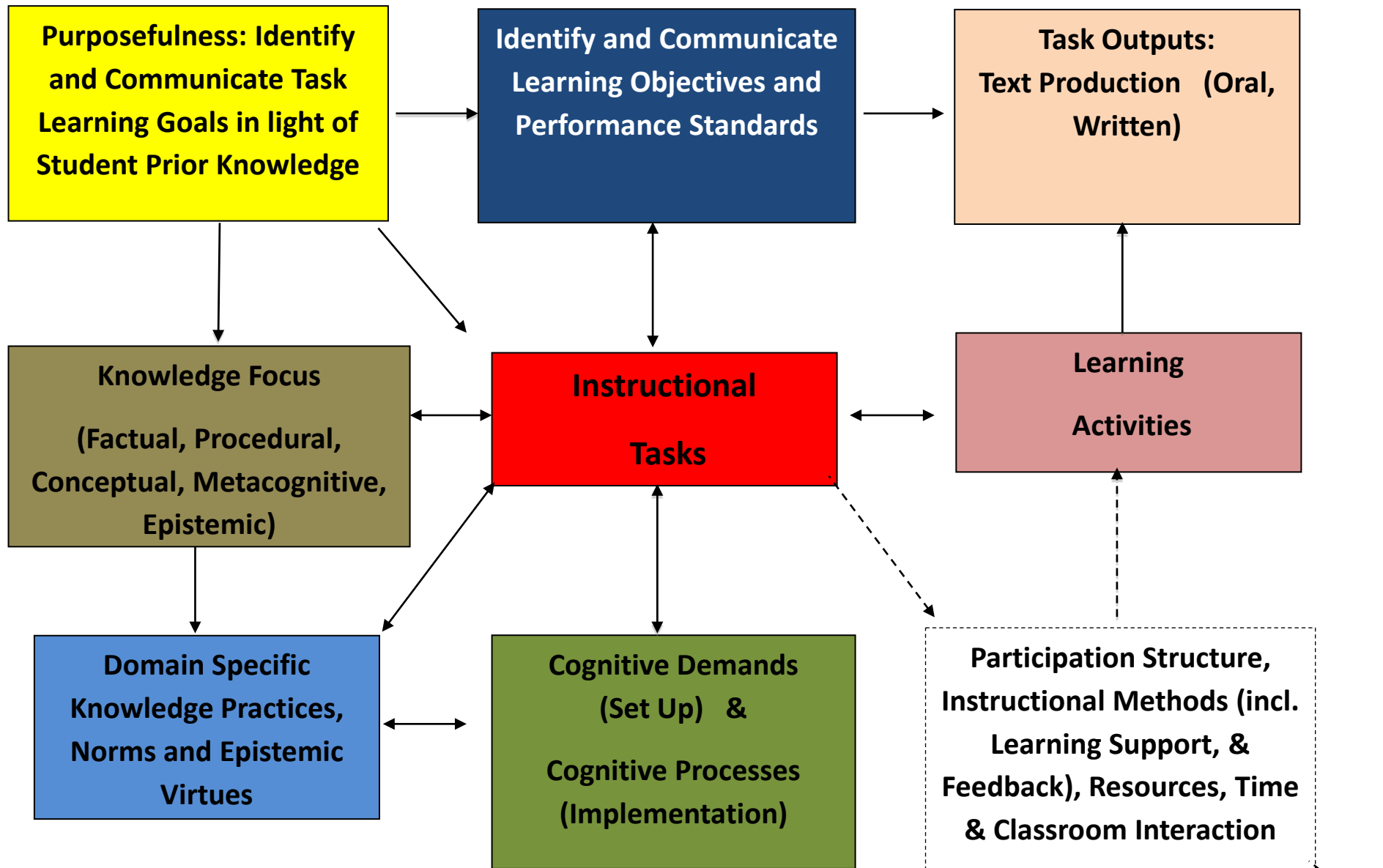
So – What are Tasks?

Instructional Tasks? A Normative Definition

Valuable instructional tasks

- are discrete, purposeful, goal-directed **cognitive activities** focused on some form of **worthwhile knowledge work** that has educational and/or institutional value
- are subject to clear **standards** of performance
- impose specific kinds of normative (including epistemic), communicative and behavioral **demands** on students
- provide the **organizing principle** for the selection of activities, resources and learning
- give **meaning and purpose** to the activities, resources and learning chosen to enact the tasks

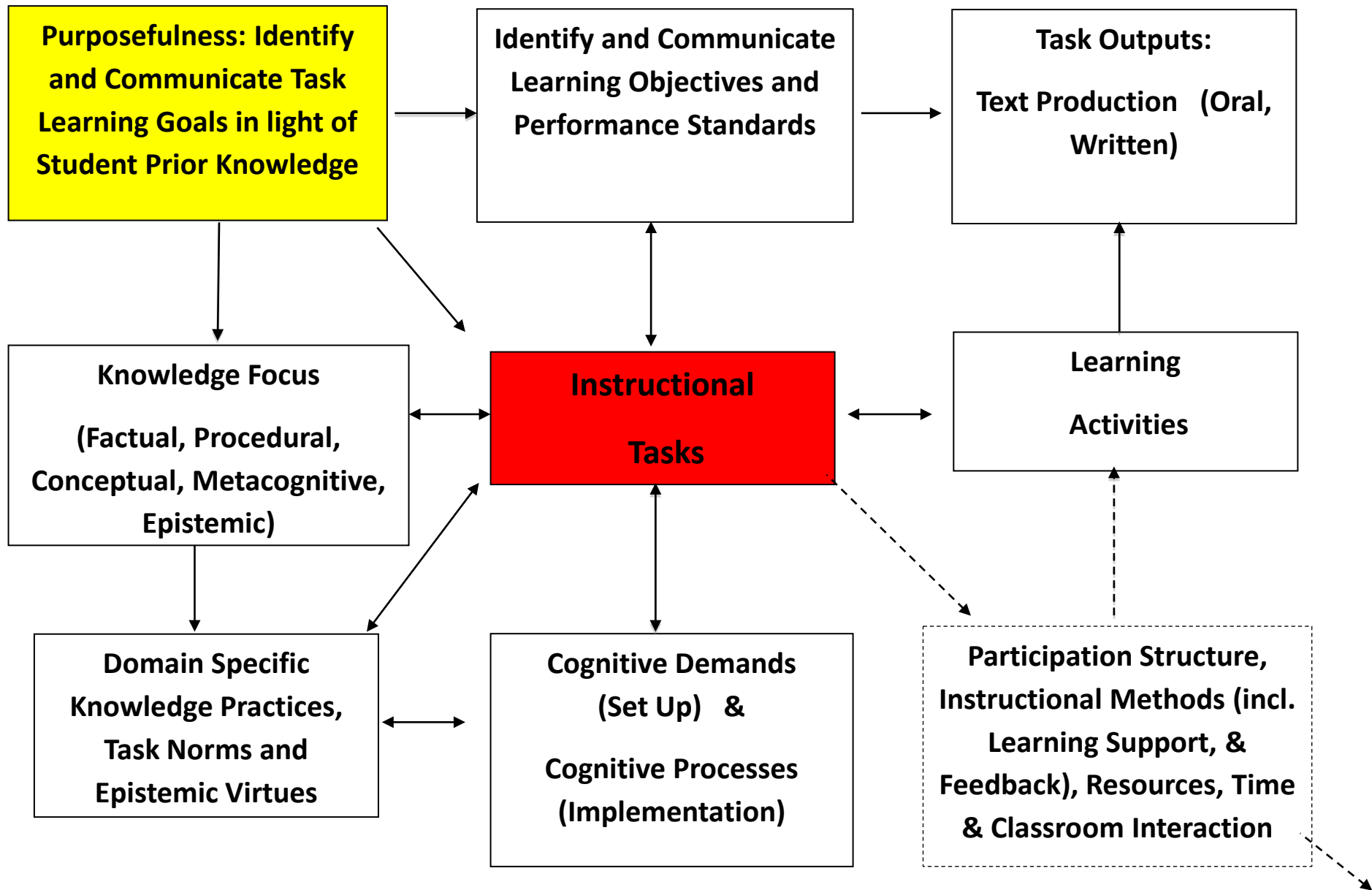
Conceptual Model of Instructional Tasks



1.1.

Purposefulness: Identify and
Communicate Lesson Goals

Conceptual Model of Instructional Tasks

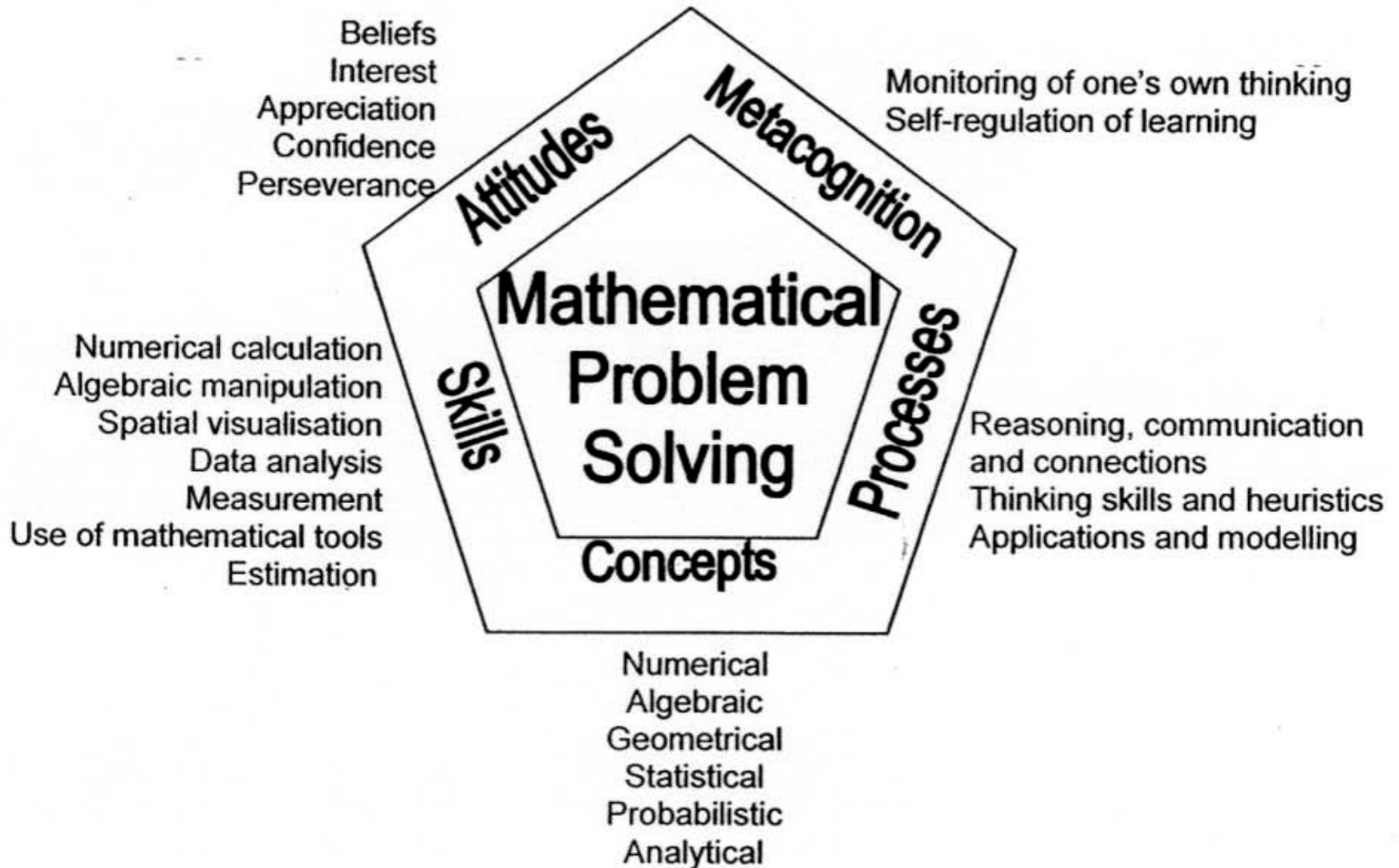


The Importance of Goals

“The key to making your students’ learning experiences worthwhile [and engaging] is to focus on your planning on major instructional goals, phrased in terms of desired outcomes – the knowledge, skills, attitudes that you want to develop in your students. ***Goals, not content coverage or learning processes, provide the rationale for curriculum and instruction.*** All of the elements of ... instructional program – content sources, discussion questions, activities, assignments, and assessment methods – should be included ... as means to accomplish important instructional goals.”

Jere Brophy, *Motivating Students To Learn*, 2nd ed., 2004, p.33.

MOE Framework of School Mathematics



Framing Mathematics: MOE

Mathematical processes refer to the knowledge skills (or process skills) involved in the process of acquiring and applying mathematical knowledge. This includes reasoning, communication and connections, thinking skills and heuristics, and application and modeling.

- **Mathematical reasoning** refers to the ability to analyze mathematical situations and construct logical arguments. It is a habit of mind that can be developed through the application of mathematics in different situations and contexts.
- **Communication** refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their own understanding of mathematics and sharpen their mathematical thinking.
- **Connections** refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and everyday life. This helps students make sense of what they learn in mathematics.

Mathematical reasoning, communication and connections should pervade all levels of mathematics learning, from primary levels to the advanced-levels.

Learning Goals: Core 2 Research Focus

1. Content specific knowledge

2. Knowledge Focus*

- Factual
- Conceptual: Meaning-making
- Procedural
- Metacognitive
- Epistemic

3. Skills

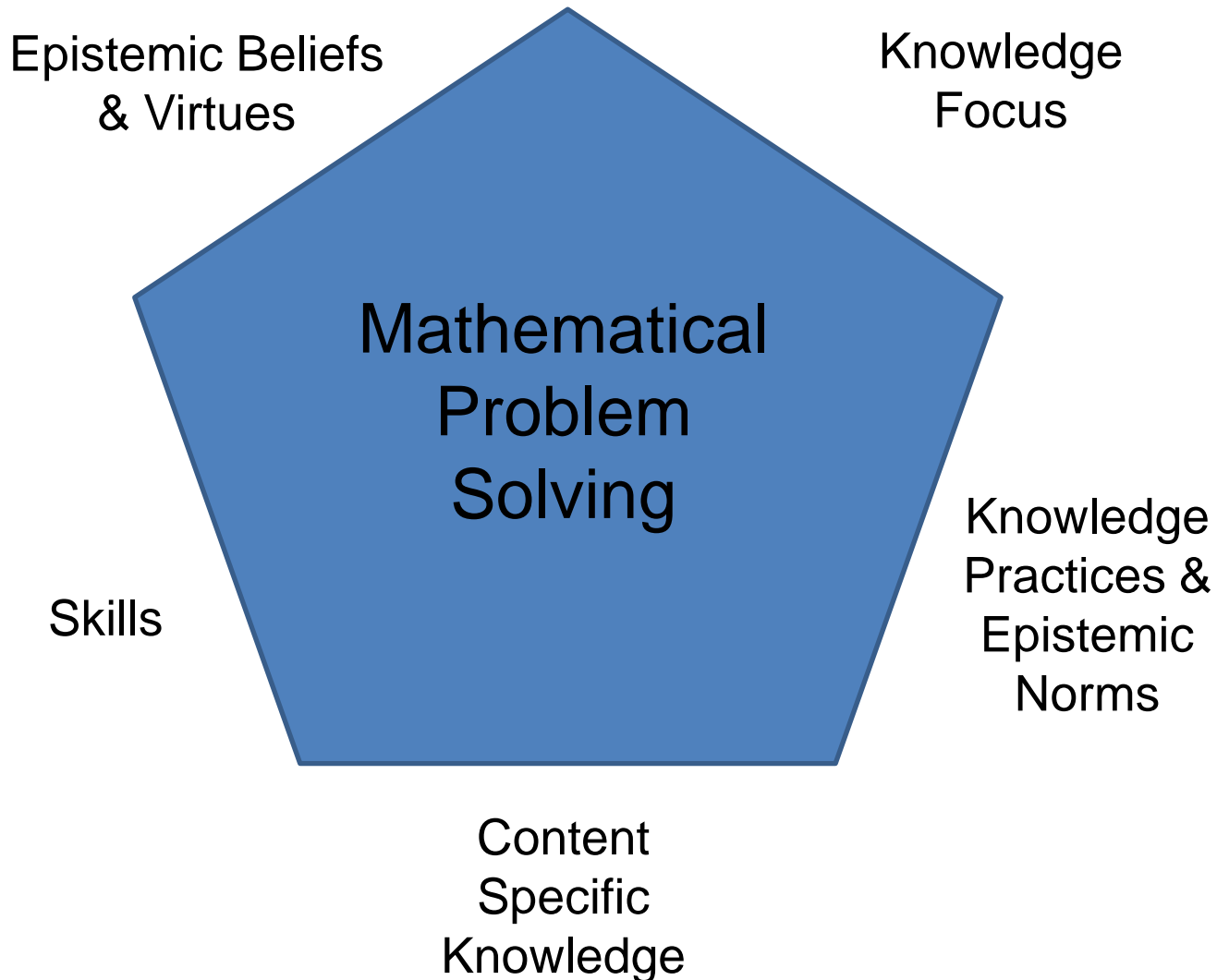
- Mathematics-specific Skills
- Metacognitive Skills
- Strategic Skills
- Process Skills

4. Epistemic Beliefs and Virtues*

5. Domain-Specific Knowledge Practices*

- Knowledge Communication: Syntax
- Knowledge Representation
- Knowledge Generation
- Knowledge Deliberation
- Knowledge Validation/Justification
- Knowledge Communication: Presentation

Core 2 Pentagon



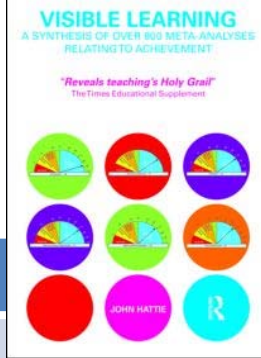
Likert Scale Values

1. Very Poor/ Never /Strongly Disagree
2. Poor / Only very occasionally / Disagree
3. OK / Sometimes / Neither Agree or Disagree
- 4. Good / Often / Agree**
5. Very good / All the time / Strongly Agree

Visible Learning: Communicating Learning Goals

	Alpha	Secondary 3 Mathematics		Secondary 3 English		<i>d</i>
		Mean (1-5)	SD	Mean (1-5)	SD	
Communicating Learning Goals	.88, .91	3.55	0.84	3.46	.84	.107
The teacher tells us the learning objectives of the lesson.		3.60	0.98	3.50	.96	.103
The teacher explains to us the learning objectives of the lesson in detail.		3.53	0.98	3.46	.94	.073
The teacher links the objectives of the lesson to previous lessons.		3.47	0.98	3.42	.96	.052
The teacher helps me understand what I am supposed to learn at the beginning of the lesson.		3.61	0.97	3.48	.96	.135

Visible Teaching and Learning



Visible Teaching	Visible Learning
Learning goals	Student prior knowledge
Performance standards	Monitoring of student learning
Exemplars of successful performance	S-T feedback: Formative Evaluation
T-S feedback	Metacognitive wisdom/self regulation
Public reasoning: “understanding talk”	Public reasoning: “understanding talk”

The remarkable feature of the evidence is that the biggest effects on student learning occur when teachers become learners of their own teaching, and when students become their own teachers. ... That is, it is visible teaching and learning by students that makes the difference.”

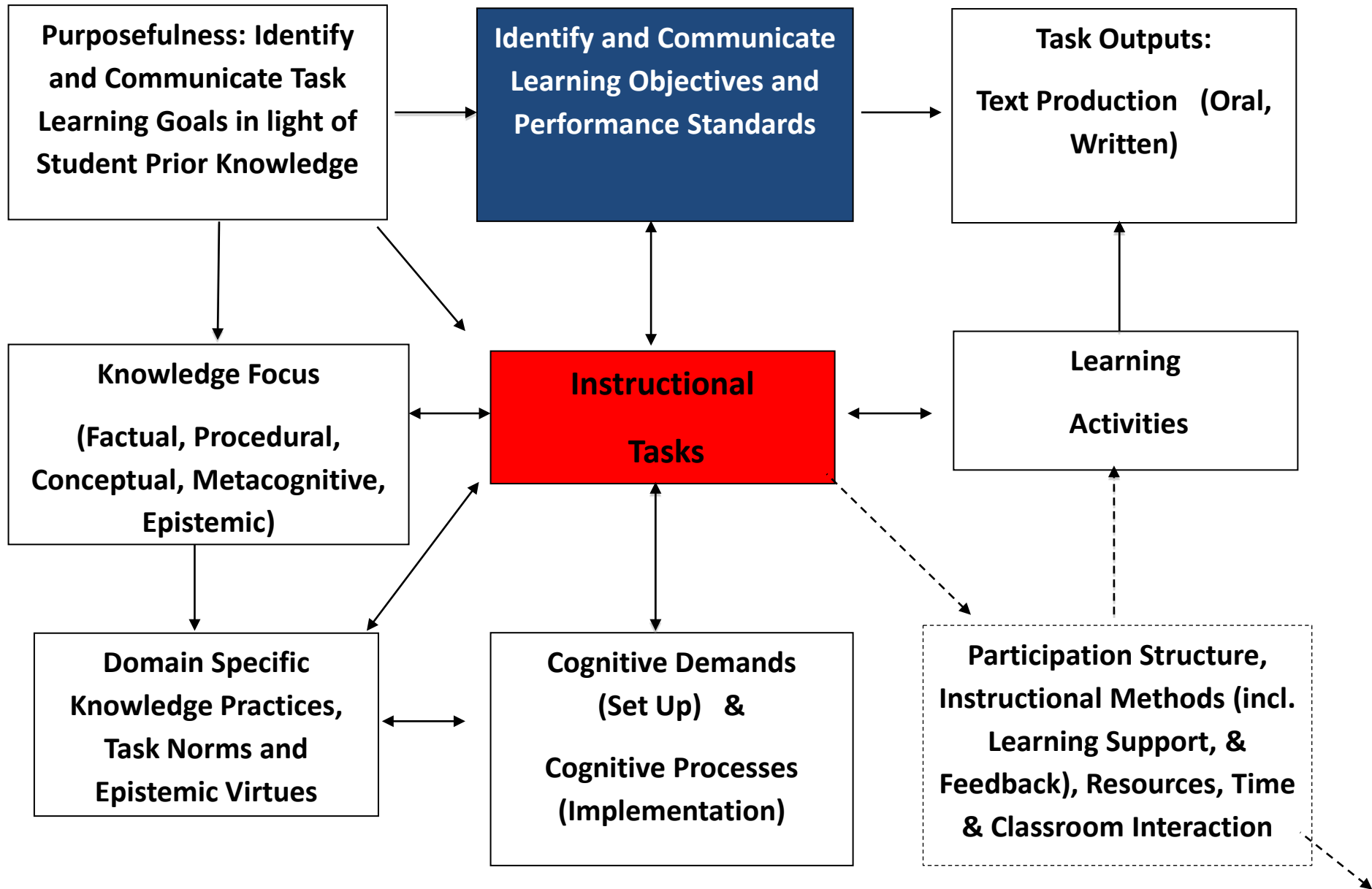


John Hattie, *Visible Learning*. London: Routledge, 2009, p.22.

1.2.

Identify and Communicate
Learning Objectives and
Performance Standards

Conceptual Model of Instructional Tasks

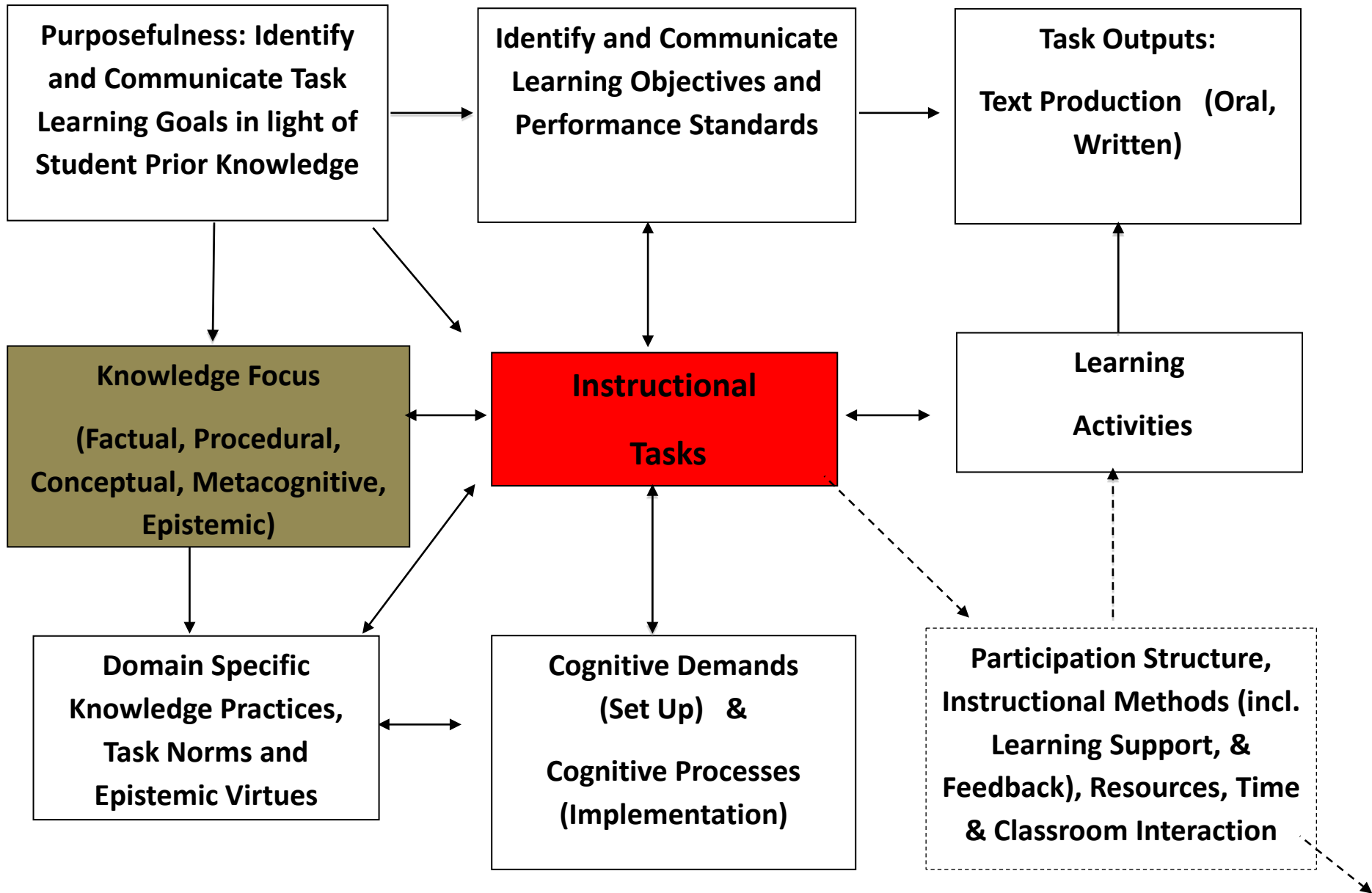


Visible Learning: Communicating Assessment Standards

	Secondary 3 Mathematics		Secondary 3 English		<i>d</i>
	Mean (1-5)	SD	Mean (1-5)	SD	
Communicating Assessment Criteria	3.64	.83	3.61	.81	.037
The teacher explains the standard of good work	3.64	.95	3.67	.93	.032
The teacher explains the standard of good performance in our tests and exams.	3.67	.98	3.63	.93	.042
The teacher explains clearly what a good solution to a problem is.	3.72	.97	3.53	.93	.200
The teacher shows us examples of good work.	3.53	1.02	3.62	.95	.091

1.3. Knowledge Focus

Conceptual Model of Instructional Tasks

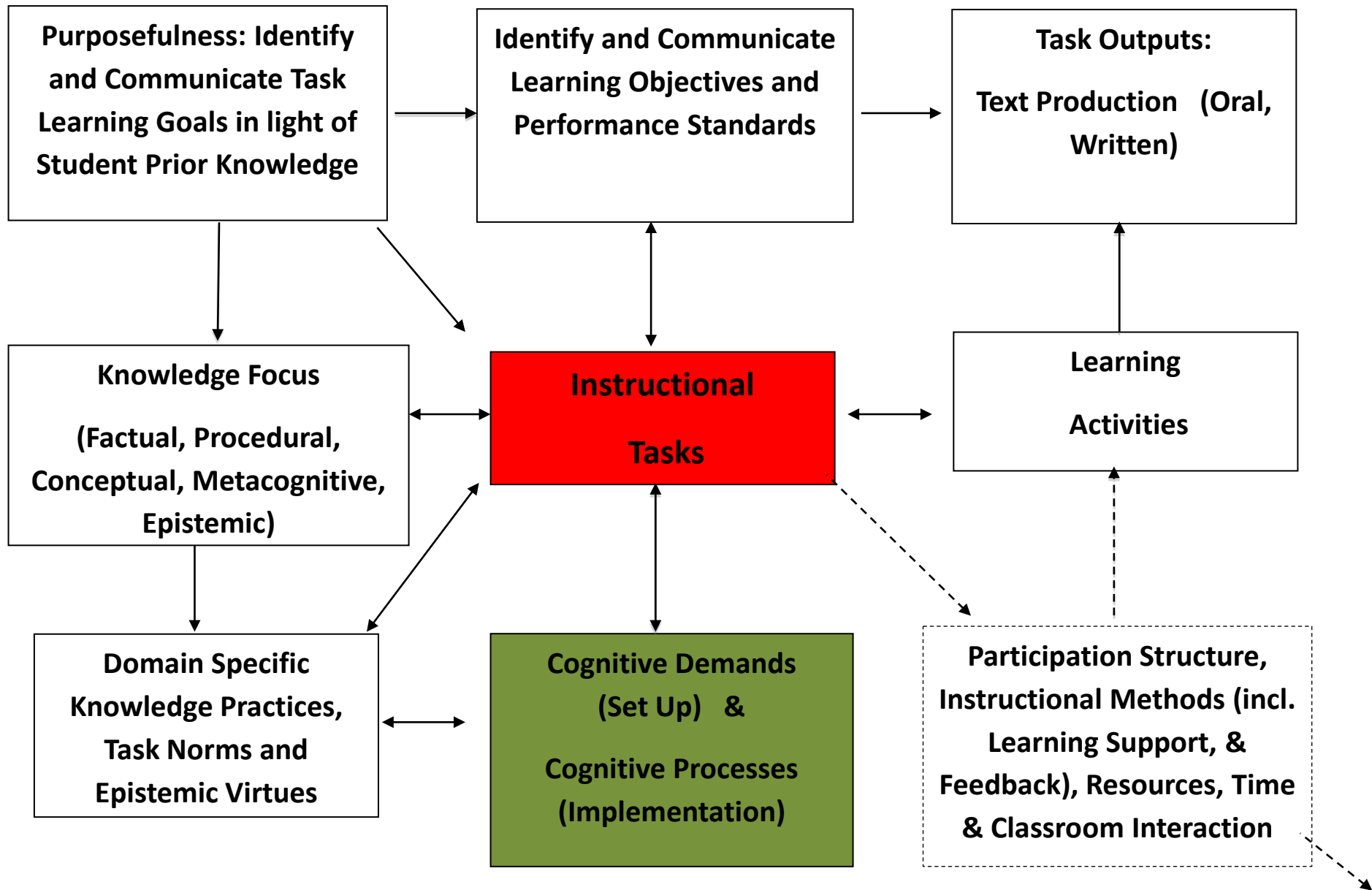


Summary: Mathematics Knowledge, Secondary 3 M&E (2010)

	Alpha	EV%	Mean	SD
Mathematics*				
Metacognitive Knowledge	.729	65.0	3.59	.797
Procedural Knowledge	.909	68.7	3.43	.756
Conceptual Knowledge	.831	59.7	3.39	.716
Epistemic Knowledge	.844	48.3	3.36	.730

1.4. Tasks: Cognitive Demand

Conceptual Model of Instructional Tasks



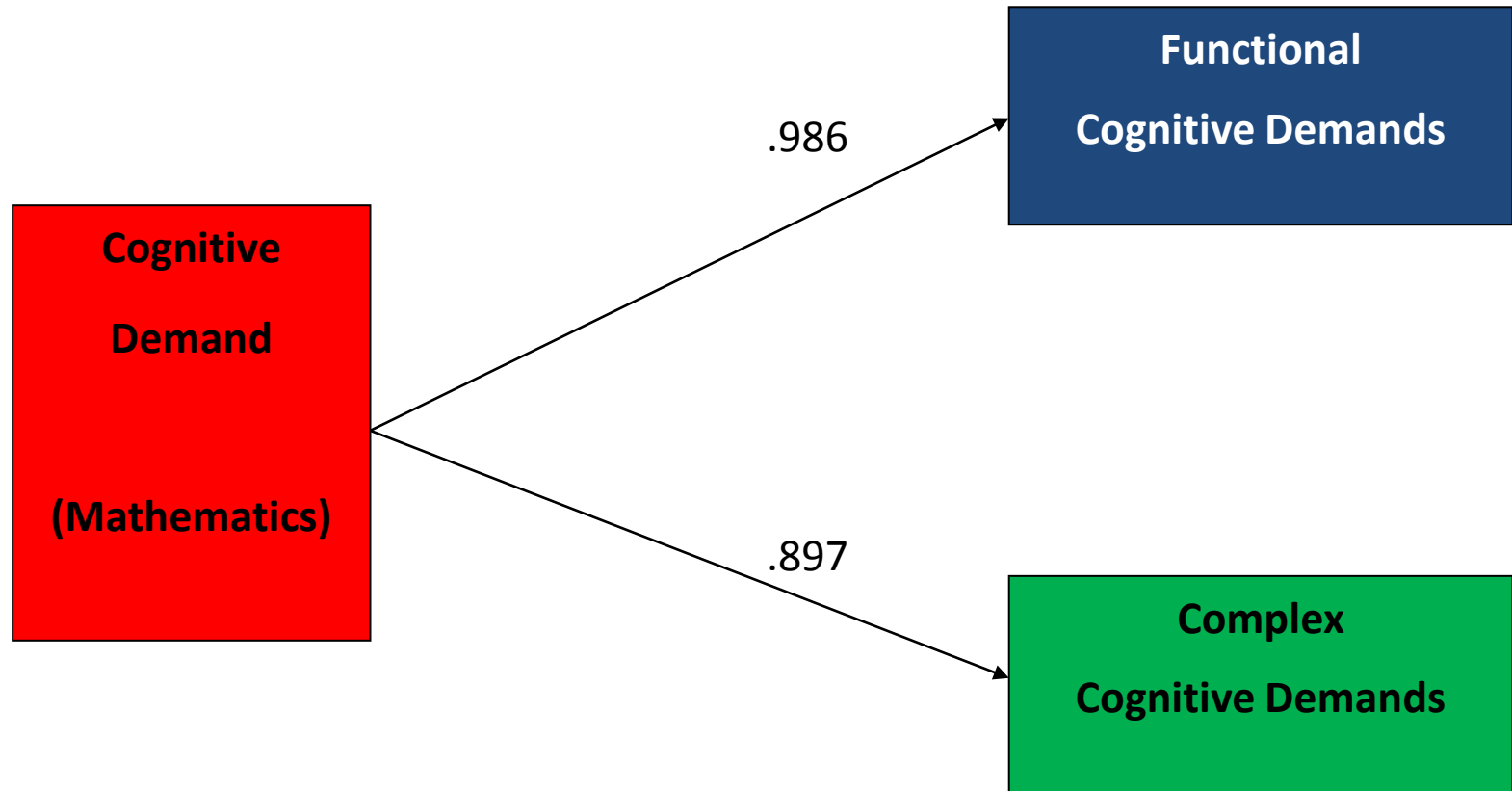
Instructional Tasks: Cognitive Demand

	Mean (1-5)	SD
To <i>remember</i> [memorize] formulae or rules	4.03	.642
To <i>practise</i> what you have learnt	3.92	.908
To <i>remember</i> or recall information you have learnt in a previous lesson	3.66	.920
To <i>review</i> what you have learnt	3.55	.900
To <i>check</i> the correctness of a solution to a problem	3.54	.909
To <i>apply</i> what you have learnt to a new problem or situation	3.53	.890
To <i>make the meaning</i> clear	3.50	.930
To <i>understand</i> a word problem, graph or table	3.47	.890
To <i>explain</i> something	3.46	.928
To <i>analyze</i> information	3.41	.910
To <i>discuss</i> a problem with one or more students	3.39	.957
To <i>compare</i> solutions to a problem	3.37	.891
To <i>summarize</i> information you have learnt or gathered	3.35	.938
To <i>work out</i> a new solution to a problem	3.35	.895

Cognitive Demand Instructional Tasks - 2

	Mean	SD
To <i>give reasons</i> for why a guess or a solution that someone has made in class is correct	3.34	.937
To <i>give an example</i> of a mathematical idea (e.g., Give an example of a four-sided figure)	3.33	.940
To <i>explain</i> the difference between two ideas (e.g., Area and Volume)	3.32	.940
To <i>classify</i> problems you have learnt	3.32	.939
To <i>make a connection</i> between what you have learnt and something else (e.g., finding area of a composite shape using regular shapes)	3.31	.937
To <i>investigate</i> a problem	3.28	.925
To <i>represent</i> or state a problem in a different way (e.g., drawing models , graphs or tables)	3.25	.912
To <i>represent</i> something differently	3.18	.903
To <i>write the solution</i> to a problem and explain it to your classmates	3.15	1.006
To <i>find out new information</i> from the textbook, the library, the internet or some other source	2.97	.996

Confirmatory Factor Analysis: Cognitive Demand of Mathematical Tasks



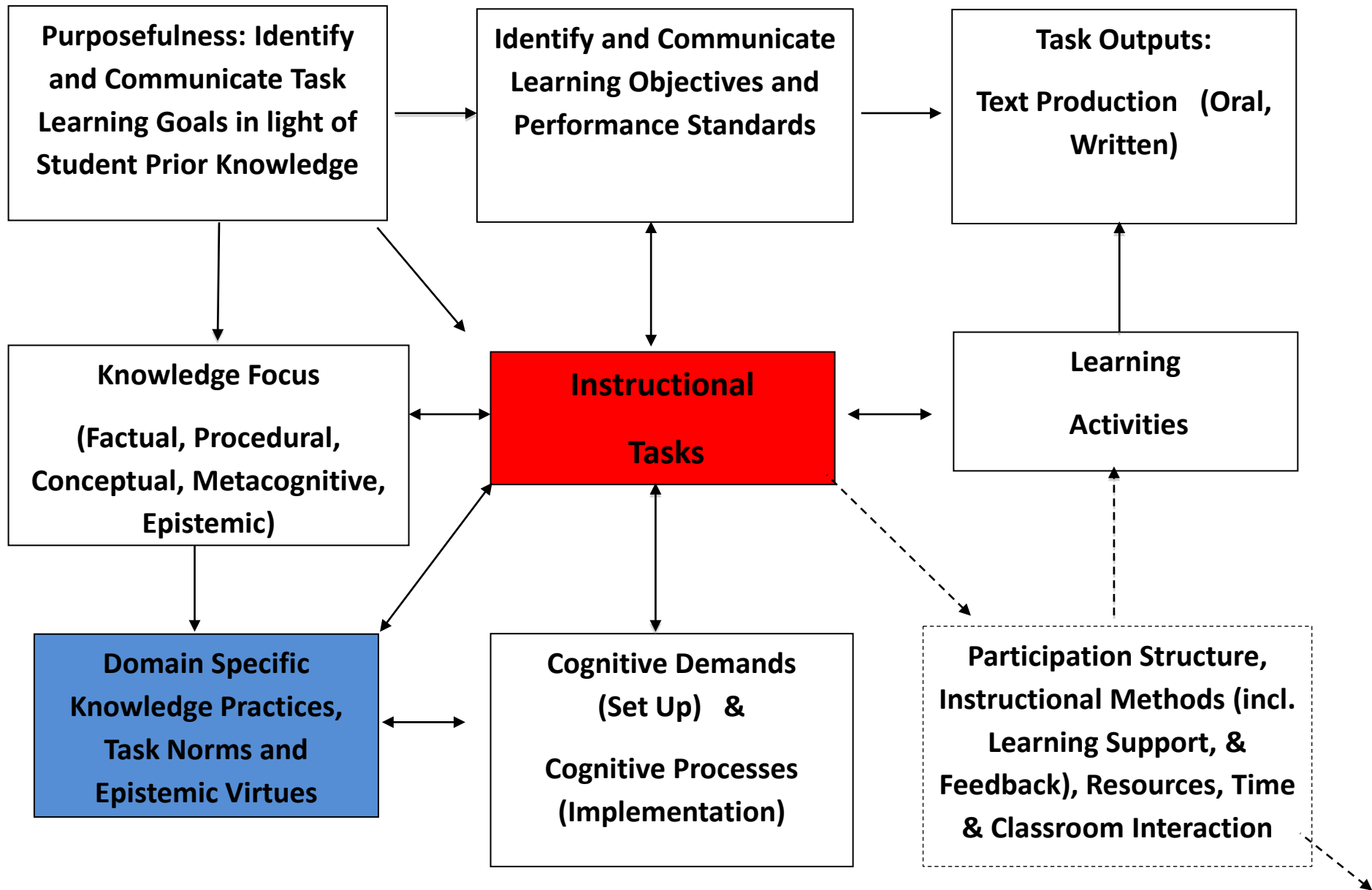
Goodness-of-fit statistics:	
Chi-Square / df / p-value	274.837 / 118 / .000
CFI / TLI	.982 / .980
RMSEA (90% C.I.)	.034 (.029-.040)
SRMR	.022

Descriptive Statistics: Cognitive Demands, Secondary 3 Mathematics and English

	Mean (1-5)	SD	Corr (r)
Mathematics			
Functional Cognition	3.81	.643	.691
Complex Cognition	3.38	.672	
English			
Functional Cognition	3.43	.748	.340
Complex Cognition	3.34	.657	

1.5. Knowledge Practices

Conceptual Model of Instructional Tasks



Domain Specific Knowledge Practices...

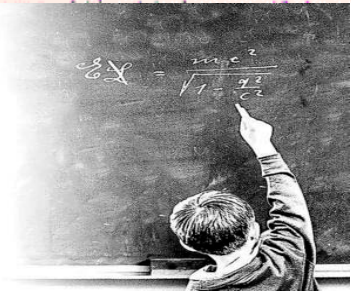
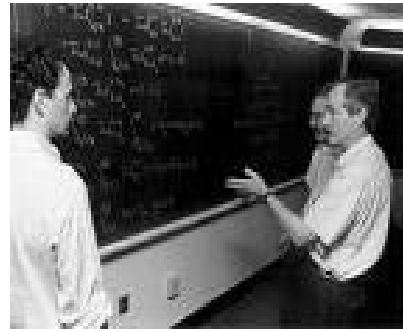
Knowledge Practices

PHILOSOPHIA
MATHEMATICA

SERIES III

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Professor Deborah Ball

“Mathematical practices involve more than what is normally thought of as mathematical knowledge. This area focuses on the mathematical know-how, beyond content knowledge, that constitutes expertise in learning and using mathematics. *The term “practices” refers to the specific things that successful mathematics learners and users do. Justifying claims, using symbolic notation efficiently, defining terms precisely, and making generalizations are examples of mathematical practices.*”

D. Ball, *Mathematical Proficiency for All Students: Towards a Strategic Research and Development Program in Mathematics Education*. Santa Monica: Rand, 2003: p. xviii

Core 2 Taxonomy of Mathematics Knowledge Tasks .

- ***Investigating*** knowledge claims
- ***Generating*** knowledge claims (inquiry, interpretation, reading)
- ***Representing*** knowledge claims
- ***Communicating*** knowledge claims (including expression)
- ***Deliberating*** knowledge claims
- ***Justifying/validating*** knowledge claims
- ***Applying*** knowledge claims to new problems or contexts

Knowledge Tasks: Mathematics

Secondary 3 Mathematics	Mean (1-5)	SD
Generic Disciplinary Tasks	3.35	.665
Knowledge Activation and Consolidation Tasks	3.62	.716
Analytical Tasks	3.30	.782
Application Tasks	3.13	.650
Specific Disciplinary Tasks	3.11	.611
Knowledge Communication Tasks (Syntax)	3.50	.725
Knowledge Validation Tasks	3.15	.852
Knowledge Deliberation Tasks	3.13	.845
Knowledge Representation Tasks	3.11	.762
Knowledge Generation Tasks	3.09	.760
Knowledge Communication Tasks (Presentation)	2.67	.890

Disciplinary Practices: Knowledge Communication: Syntax

3. Knowledge Communication (Syntax) ($a=.770$, $EV=68.5\%$)	3.50	.725
To use the correct symbols at the proper places (e.g., equal signs, letters to represent the unknown)	3.82	.974
To write correct mathematical sentences	3.58	.983
To present a solution to a problem in a logical step-by-step manner	3.41	1.054
The teacher asks us to use mathematical vocabulary in our class.	2.98	.982

Disciplinary Practices: Knowledge Representation

2. Knowledge Representation Tasks (a=.913; EV=65.8%)	3.11	.762
To change from one way of representing an idea to another so that you understand the idea better	3.20	.950
To represent an idea in different ways (e.g., using graphs, tables or algebraic symbols)	3.17	.968
To use different representations (for e.g., symbols, tables, diagrams, graphs) to highlight differences or similarities between mathematical ideas	3.17	.980
To compare different types of representations that can be used in solving a problem	3.17	.953
To use a mathematical representation of an idea you have learned in a new context or situation	3.15	.937
To represent a real world problem as a mathematical problem	3.02	1.029
To create your own way of representing (e.g., make your own diagram or table) a mathematical idea	2.93	.982

Knowledge Representation

Types of Representation

Concrete

Word – Spoken

Word – Written

Numerical – Spoken

Numerical – Written

Pictorial

Schematic

Graphical

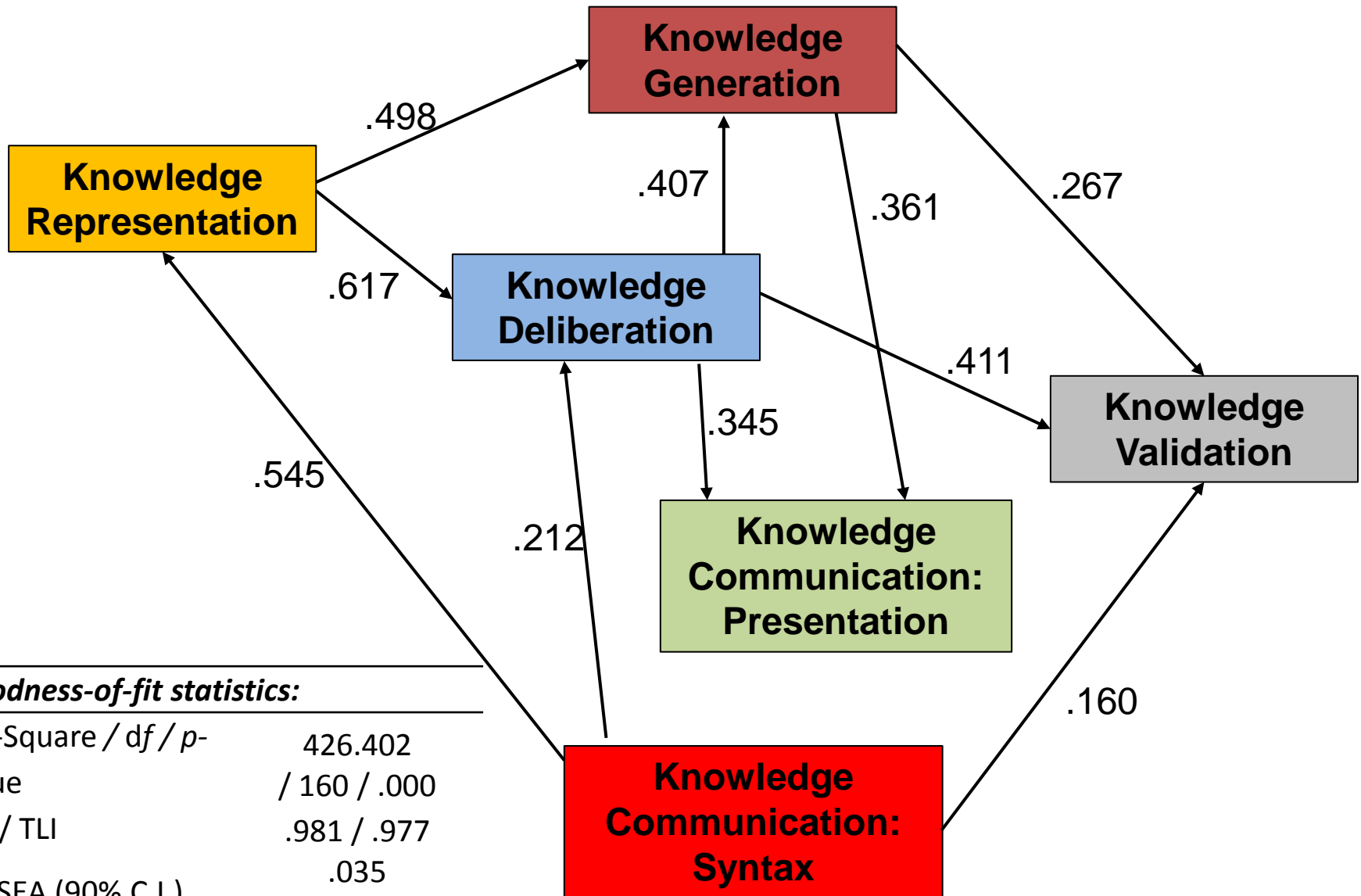
Symbolic – Written

Symbolic – Spoken

Story – Spoken

Story – Written

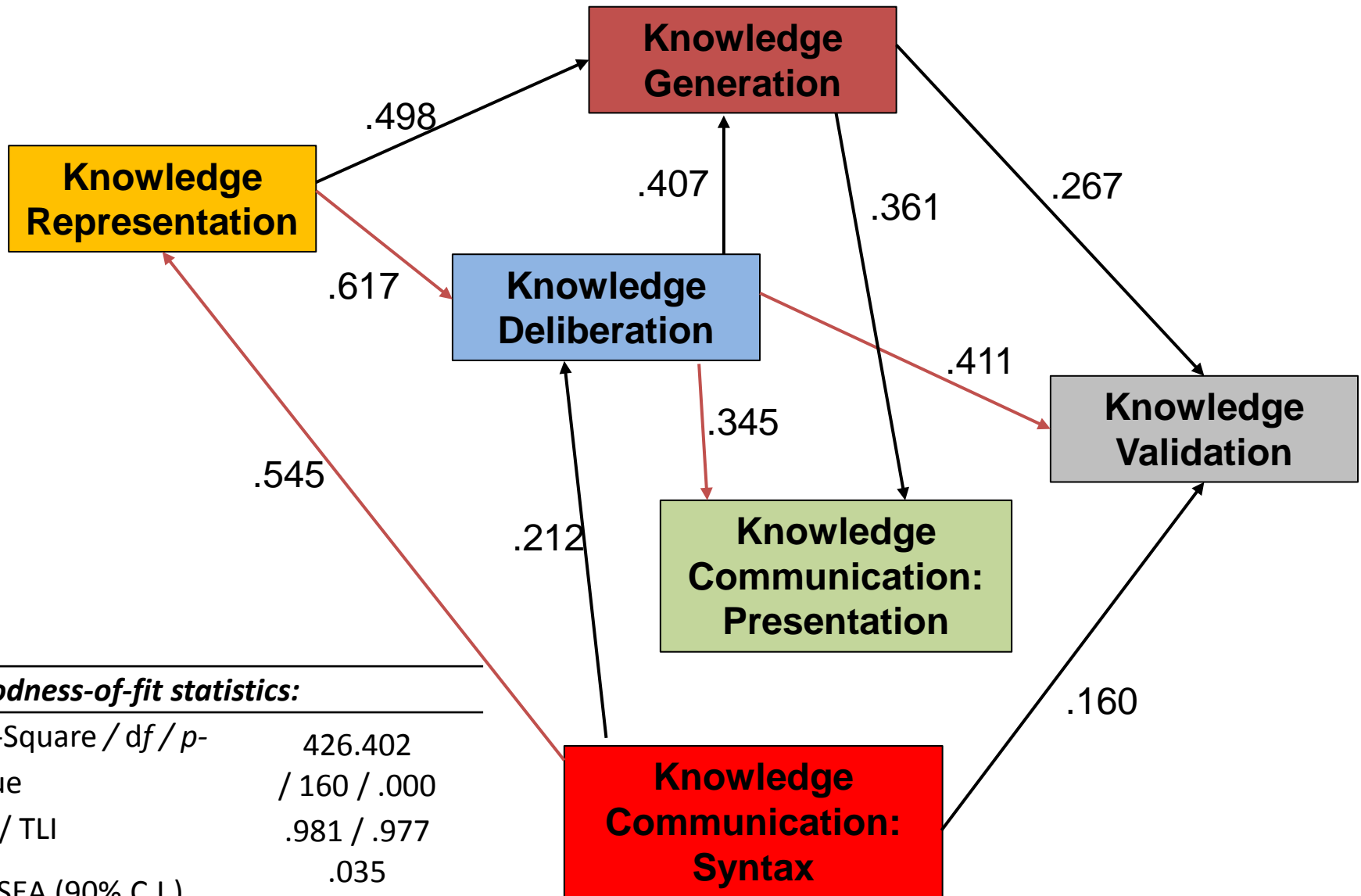
Path Model of Disciplinary Knowledge Practices: Secondary 3 Mathematics



Goodness-of-fit statistics:

Chi-Square / df / p-value	426.402 / 160 / .000
CFI / TLI	.981 / .977
RMSEA (90% C.I.)	.035 (.031-.039)
SRMR	.025

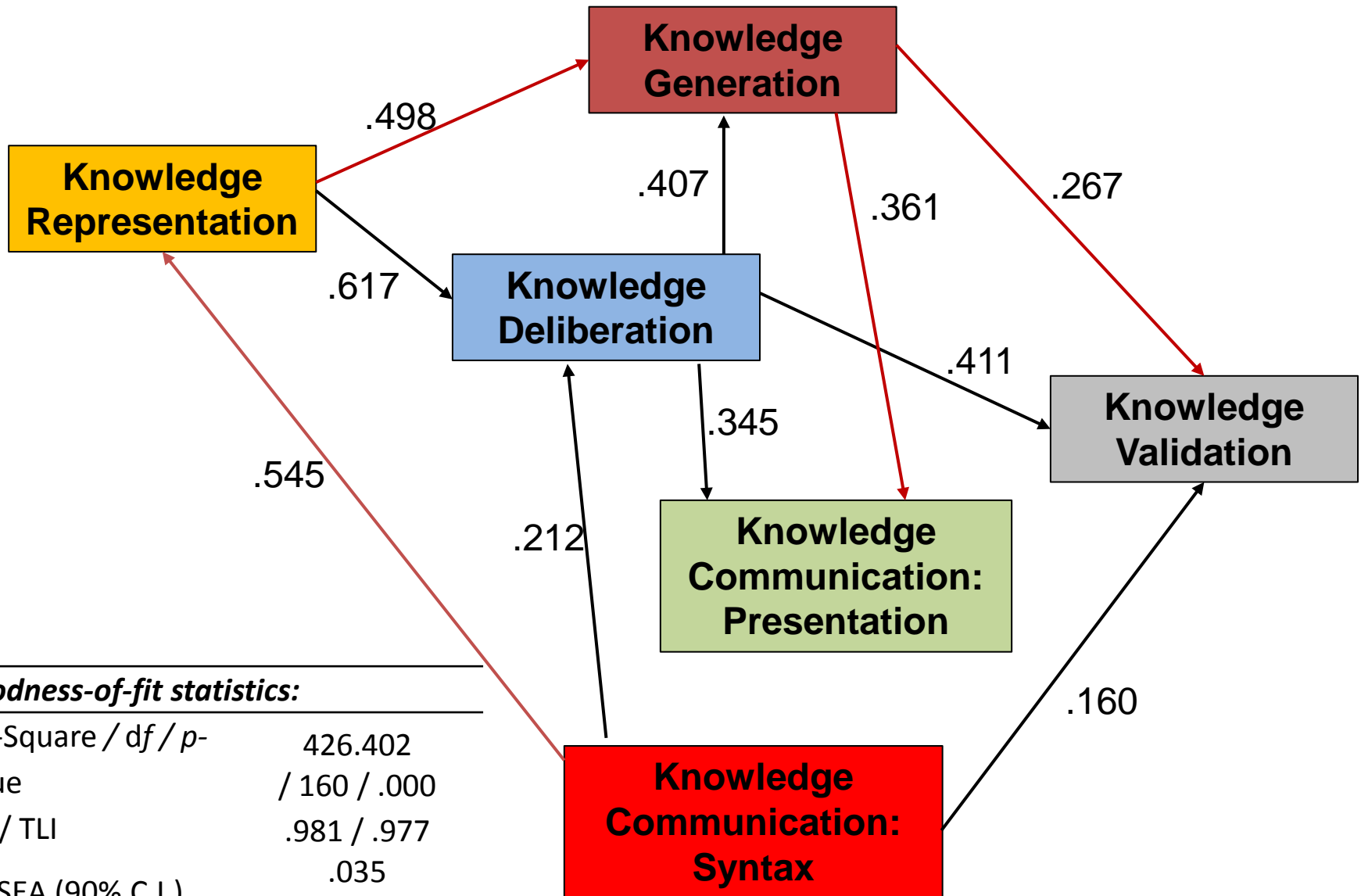
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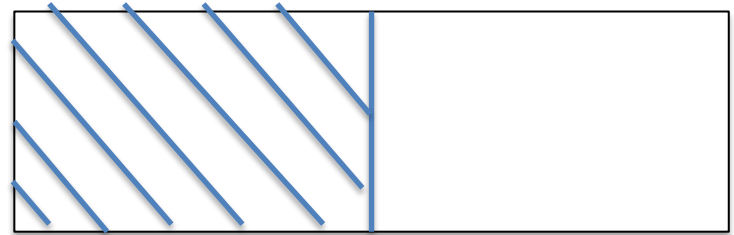


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Knowledge Practice in Action

$$\frac{1}{2} \div 3 = \frac{1}{2} \times \frac{1}{3}$$



T: How do you represent the half?

S: You have to cut

T: I break into two. How do I show my half here?

S: Shade one part

Knowledge Practice in Action

The image shows a chalkboard with handwritten mathematical work. On the left, there is a small circle with a vertical line through it, and the equation $\frac{1}{2} = 3$. In the center, a diagram shows a large rectangle divided into three vertical sections. The top section is labeled with a bracket and the number 2. Below it, a smaller rectangle is labeled with a bracket and the fraction $\frac{1}{3}$. To the right of the diagram, the text reads: "1 whole = $\frac{2}{2}$ ", "Total (apple) \rightarrow 6 units", "Each person \rightarrow 1 unit", and "Fraction \rightarrow $\frac{1}{6}$ ".

$\frac{1}{2} = 3$

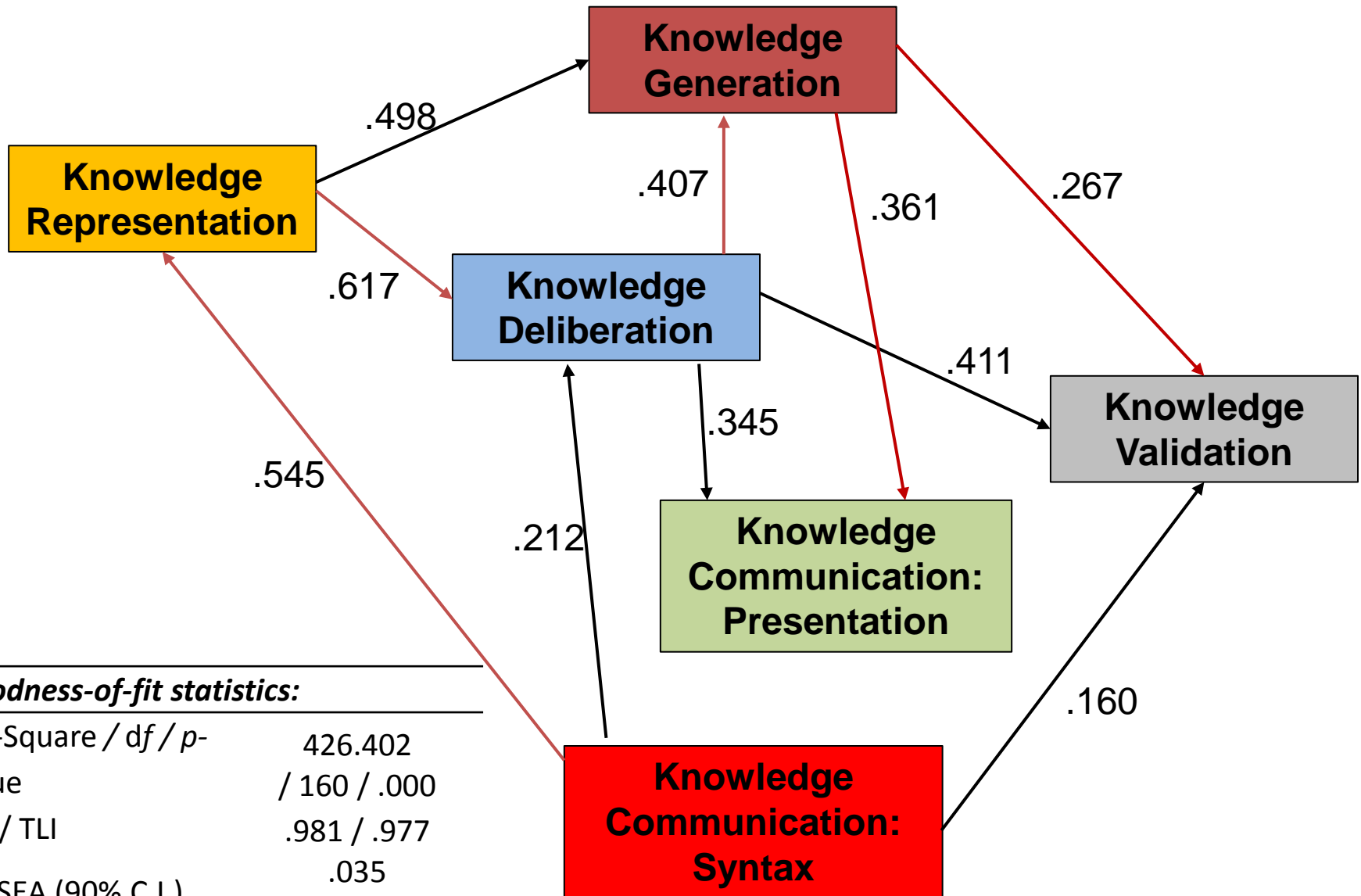
1 whole = $\frac{2}{2}$

Total (apple) \rightarrow 6 units

Each person \rightarrow 1 unit

Fraction \rightarrow $\frac{1}{6}$

Path Model of Disciplinary Knowledge Practices: Secondary 3 Mathematics



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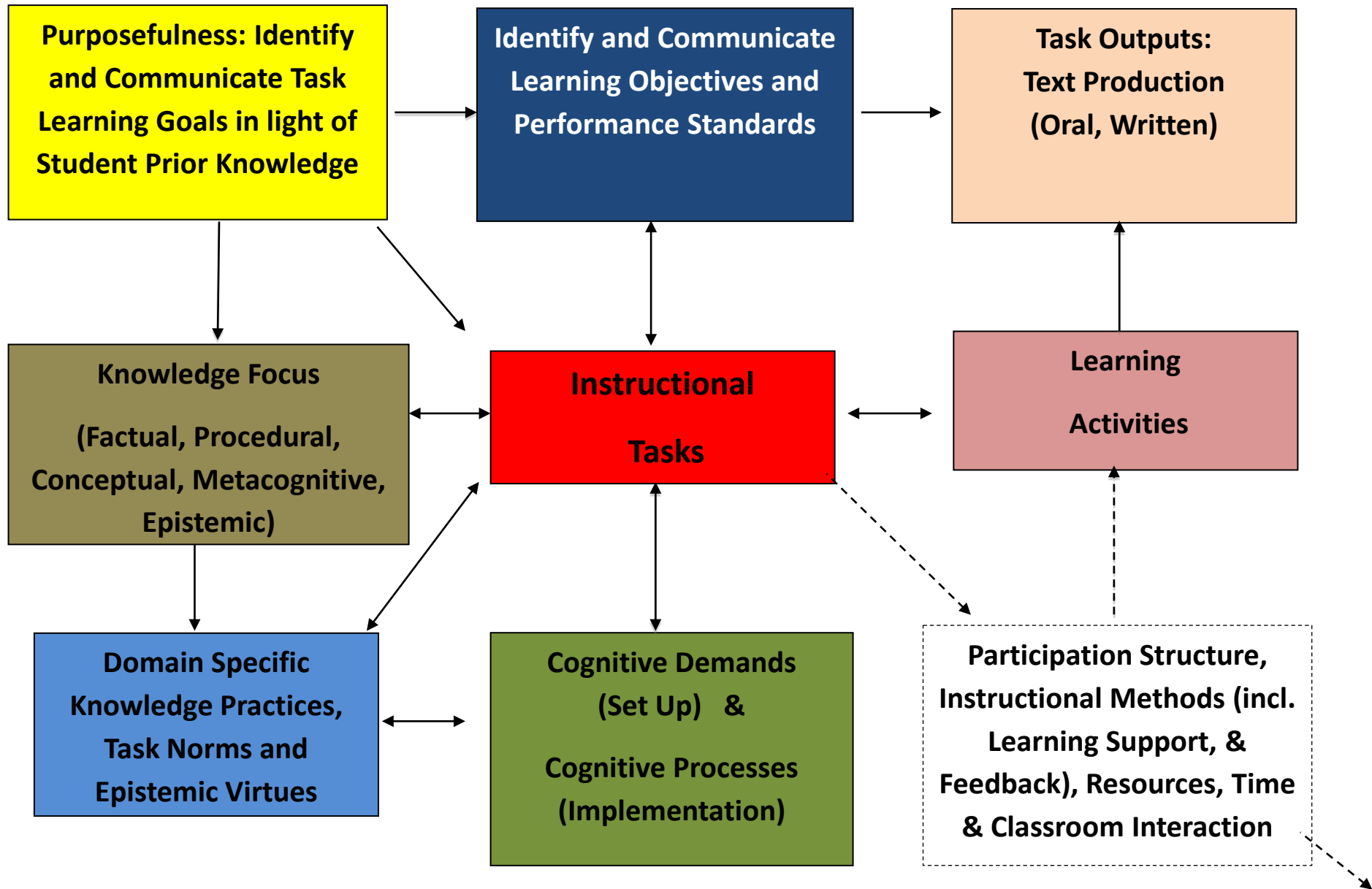
In short:

- Mathematics knowledge construction via disciplinary knowledge practices
- Foundation in knowledge of syntax but knowledge of representation key gateway to complex cognition and understanding
- Representation of the problem facilitates complex cognition and understanding
- Knowledge representation generates possible ways to solve a problem OR it also allows students to deliberate on the problem , for example, by guessing or making hypotheses, then checking to see if it makes sense or is possible
- Finally, pathway finishes with some form of public presentation and/or validation

Note that this set of knowledge practices is highly consistent with Polya's heuristics: understand the problem, devise a plan, implement the plan and examine what was done/learned/stronger propositions

Taxonomy of Tasks: Examples

Conceptual Model of Instructional Tasks



Glossary: Taxonomy of Learning Tasks

- Non-mathematical tasks
- Recall / remembering tasks
- Repetition (Drilling & Memorization) tasks
- Review and Revision Tasks
- Routine Procedural tasks
- Understanding tasks
 - Knowledge Manipulation
 - Procedural tasks with connections (including Application)
- Doing Mathematics

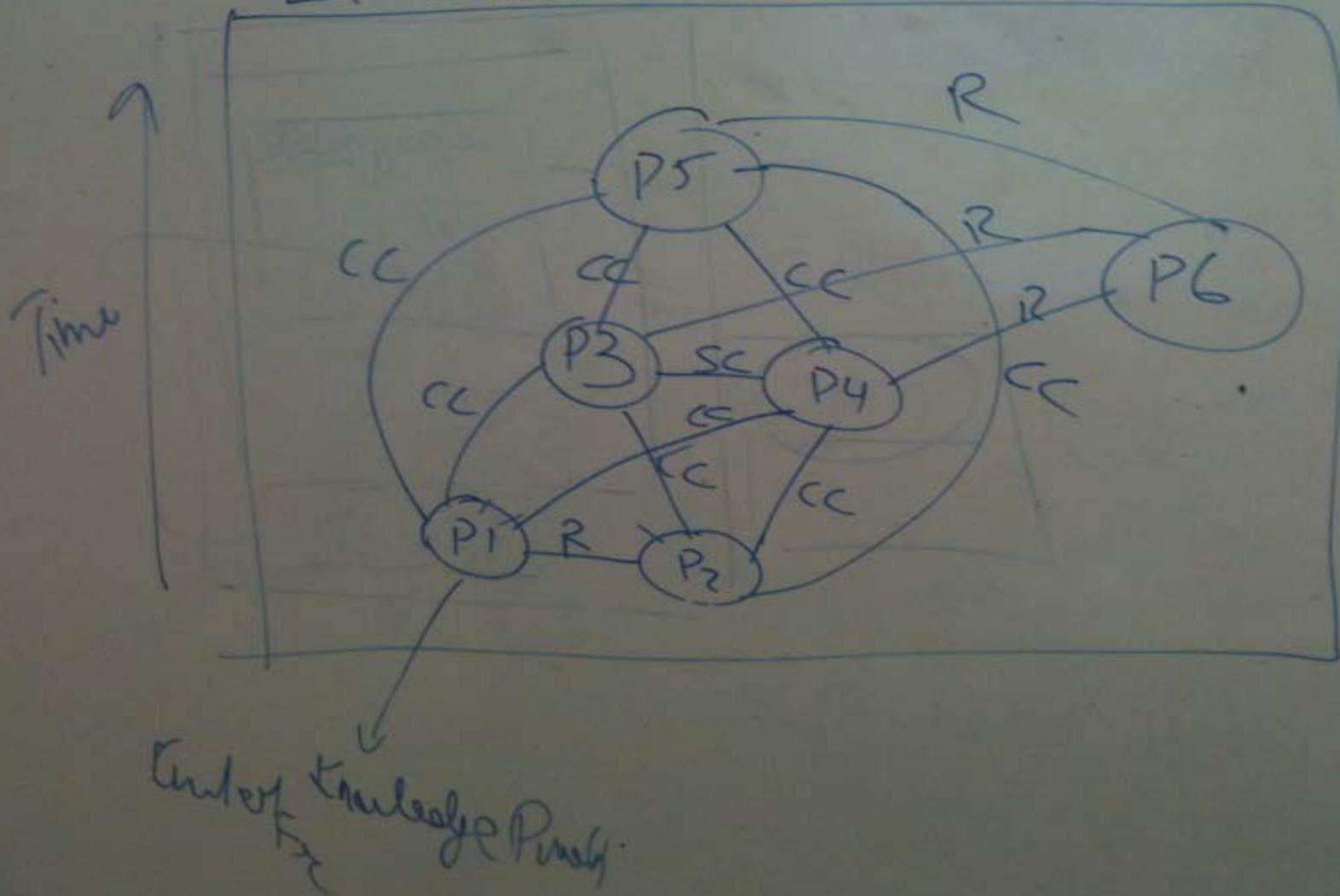
Task Example

Problem	Goals	Knowledge Focus	Cognitive Demand	Learning Task	Disciplinary Knowledge Practice
What is 3×5 ?	Automaticity	Procedural Knowledge	Remember (memorize)	Memorization	Syntax
Mr Jones' bedroom is 7m long and 6m wide. What is the area of bedroom?	Apply learned procedure (calculation of area, given dimensions of area)	Procedural Knowledge	Recall information/ practice	Procedure without connection	Syntax, Representation

Task Example

Problem	Goals	Knowledge Focus	Cognitive Demand	Learning Task	Disciplinary Knowledge Practice
<p>Miss Lim wants to build a rectangular hen pen for a science fair in school. She has 24m of fencing material.</p> <p>If she wants the hens to have as much room as possible, how long would each of the sides of the pen be?</p> <p>How long would each of the sides of the pen be if she had only 16m of fencing?</p> <p>How would you go about determining the pen with the most room for any amount of fencing?</p> <p>Organize your work so that someone else who reads it will understand it and be convinced by your solution and arguments.</p>	<p>Calculate area and perimeter</p> <p>Apply problem solving strategies to explore relationship between area or perimeter of rectangles</p> <p>Explain the work done (mathematical ideas and solutions) in a clear manner</p>	<p>Procedural Knowledge</p> <p>Conceptual Knowledge</p> <p>Epistemic Knowledge</p>	<p>Understand, Investigate, Represent a problem</p> <p>Analyze information</p> <p>Write out solution and explain the solution</p> <p>Generalize</p> <p>Apply what has been learnt to a new problem</p> <p>Make connections between two mathematical ideas/concepts</p>	<p>Procedures with connections</p> <p>Doing Mathematics</p>	<p>Syntax</p> <p>Representation</p> <p>Generation</p> <p>Deliberation</p> <p>Presentation</p> <p>Validation/Justification</p>

Knowledge Building

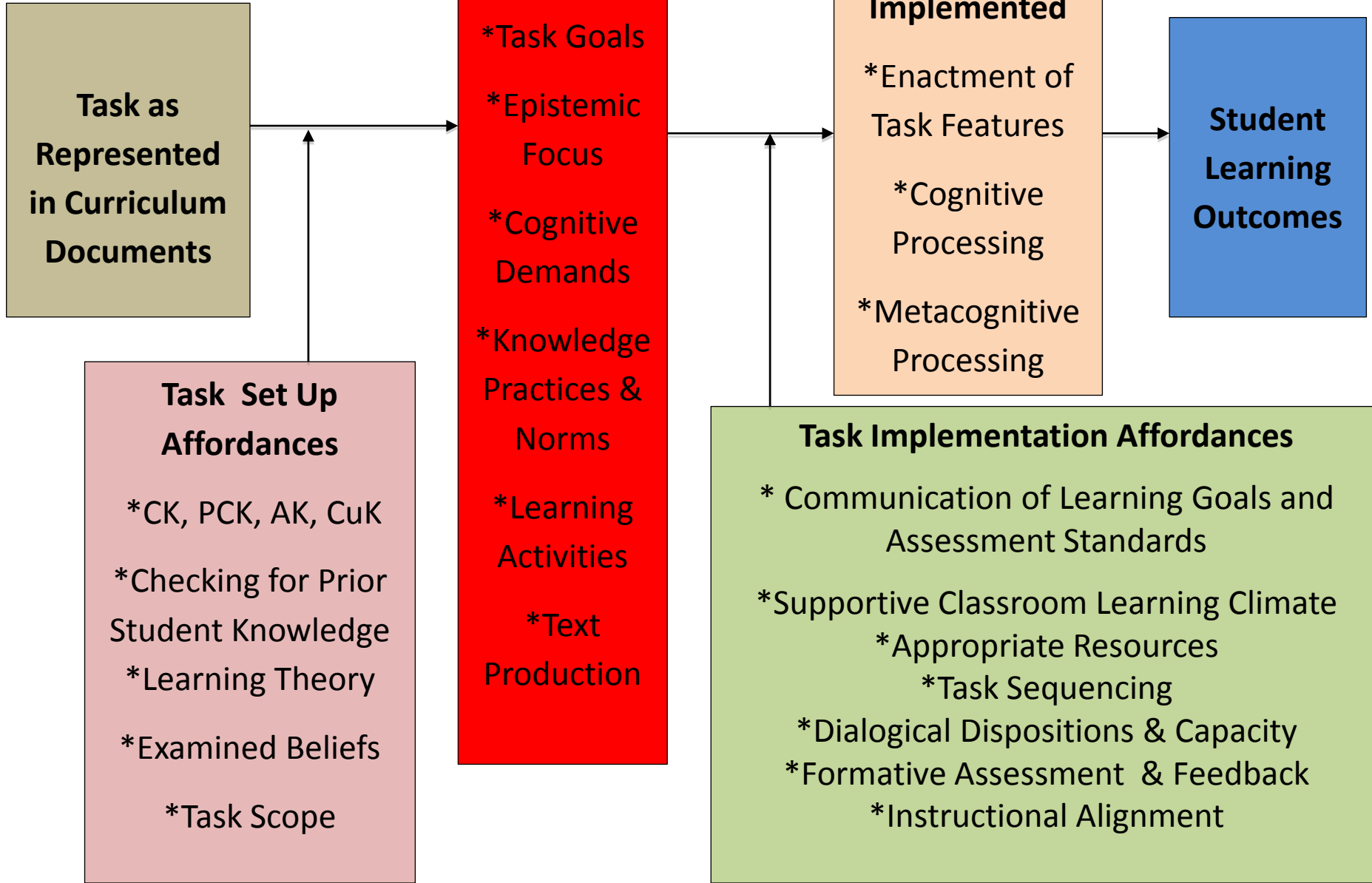


PART TWO

IMPROVING MATHEMATICS TEACHING AND LEARNING

Adapted from
Stein et al.,
1996

Task Set Up & Implementation



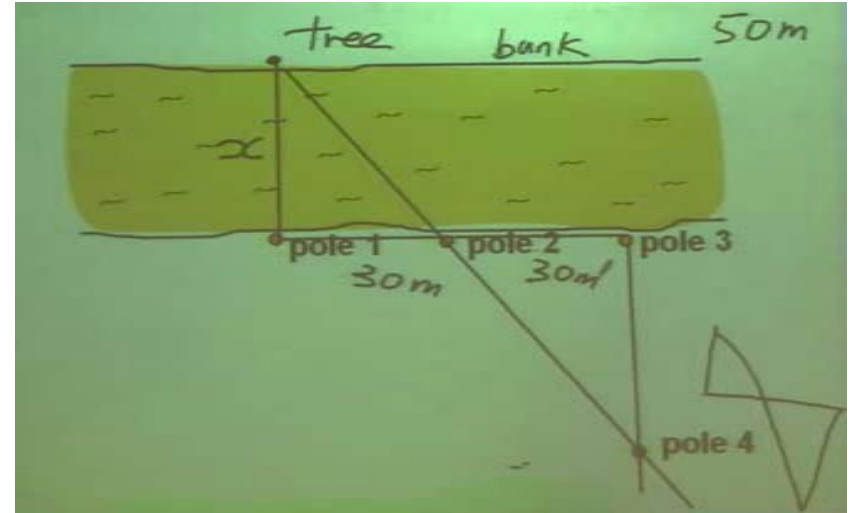
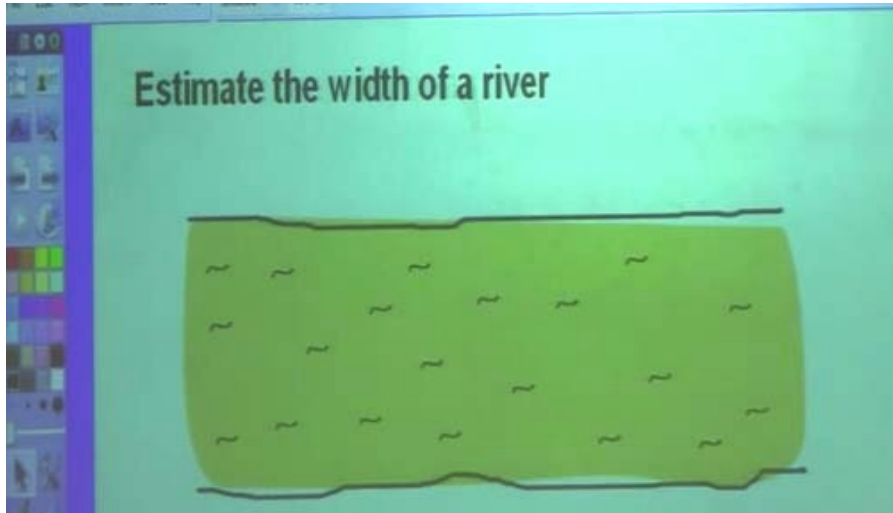
Improving Quality of Task Design: Set Up

- Developed understanding of **content knowledge and PCK**
- Clear, challenging, transparent **learning goals and specific objectives and performance standards**
- Clear understanding of **prior student knowledge**
- Clearly articulated **learning theory** and cognitive principles to inform instructional tasks
- Clear domain-specific **knowledge (epistemic) focus**
- Developmentally appropriate domain-specific **knowledge practices** high in disciplinarity
- Appropriate mix of routine procedural and **complex knowledge production tasks** informed by authentic real world problems and disciplinary standards
- Strong focus on **meaning making/making connections** and **knowledge representation**
- Clear expectations of desired **text production / outputs**

Factors Promoting Low Fidelity of Task Implementation

- Lack of clarity about **learning goals**
- Inadequate **communication** of learning goals and assessment standards
- **Inappropriateness of task**, given prior student understanding (outside ZPD): misalignment of task cognitive demands and prior knowledge
- **Classroom management** problems
- Too little – or too much -- **time**
- Lack of **accountability** for work done: need for summative assessment
- **Watering down** of task complexity to low ambiguity / low risk tasks
- Inappropriate **sequencing** of learning activities, problems, tasks
- Focus shifts to **correct answer** rather than working through problem
- **Excessive scaffolding**: reduces ambiguity and challenge
- **Too little monitoring** of student work and detailed feedback/formative assessment
- Low **mastery norms** / low **metacognitive** monitoring
- **Misalignment** of instructional practices (methods, lesson organization, talk) with task demands
- Inappropriate or inadequate **resources** and technology applications

Example



- Time taken moving from problem posing to giving the first clue (use similar triangles): less than a minute
- Time taken in giving the biggest clue (drawing of the similar triangles at the bottom right hand corner of the second diagram): 3 minutes
- Goals of the task not articulated, therefore the challenging goal is not achieved (unproductive success?) Goal of teaching how to solve such problems achieved.

Factors Promoting High Fidelity of Task Implementation

- Clarity about **learning goals**
- **Communication** of learning goals and assessment standards
- **Appropriateness of task**, given prior student understanding (outside ZPD): misalignment of task cognitive demands and prior knowledge
- **Well managed classroom**
- Neither too little – or too much -- **time**
- **Accountability** for work done: need for summative assessment
- **Maintaining** task complexity/keeps focus on working through problem
- Appropriate **sequencing** of learning activities, problems, tasks
- Productive **scaffolding**: neither too little or too much
- **Constant monitoring** of student work and detailed feedback/formative assessment
- **Mastery norms**
- **Metacognitive** monitoring and reflection
- **Alignment** of instructional practices (methods, lesson organization, talk) and task demands
- Student **self monitoring**
- Appropriate and adequate **resources** and technology applications

An Example of High Fidelity Task Implementation

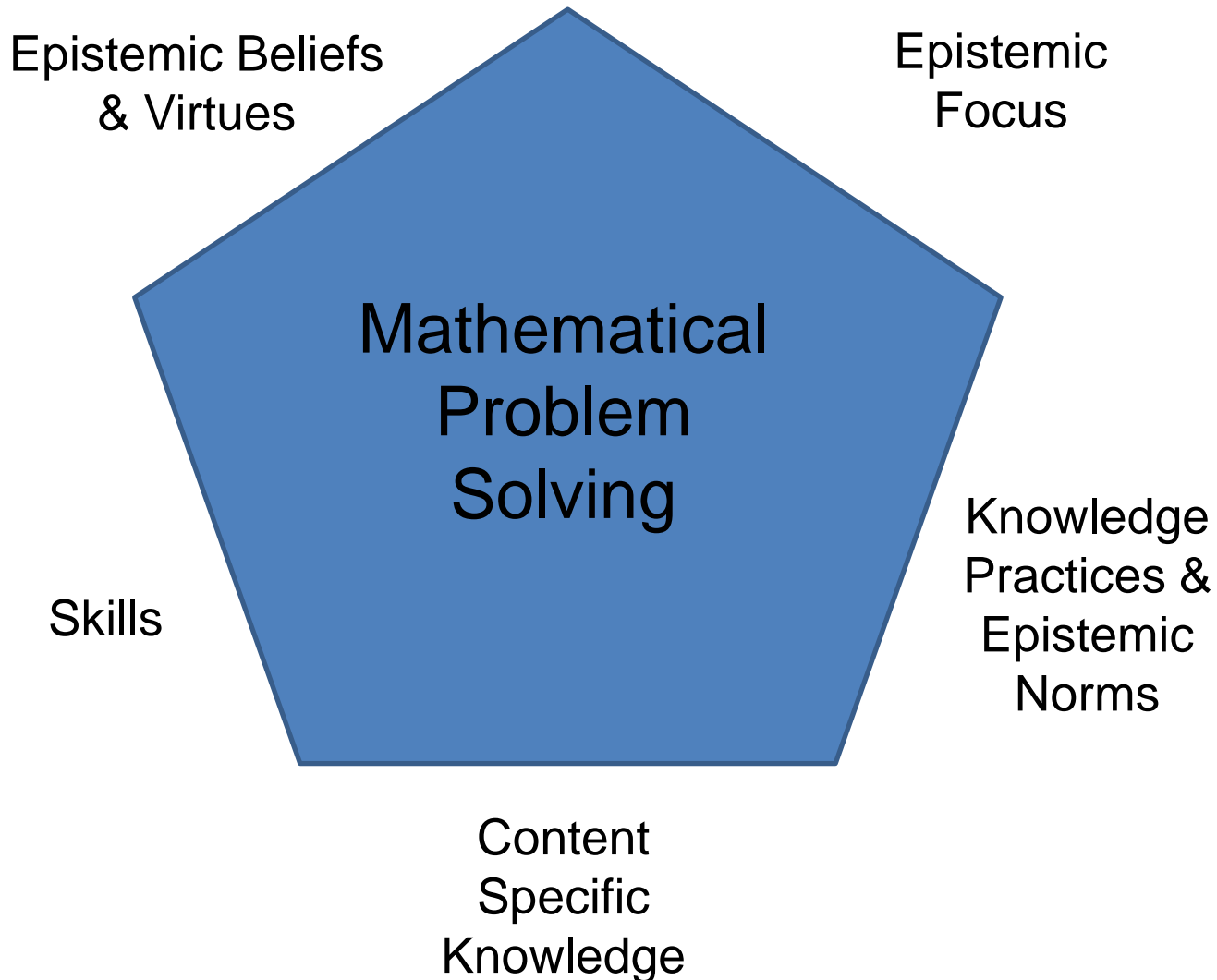
Fencing task again

- Both teacher and students were aware of the (challenging) goals of the tasks as set up, and maintained high fidelity to the cognitive demands of the task setup at the implementation stage
- While teacher gives hints, T does not show students exactly how to go about solving the problem. After 1 period of student discussion, none of the groups have completed the task but several have begun systematically laying out different pen configurations for the same amount of fencing.
- Several groups are on their way to discover that a square would enclose the greatest amount of area for any given amount of fencing.

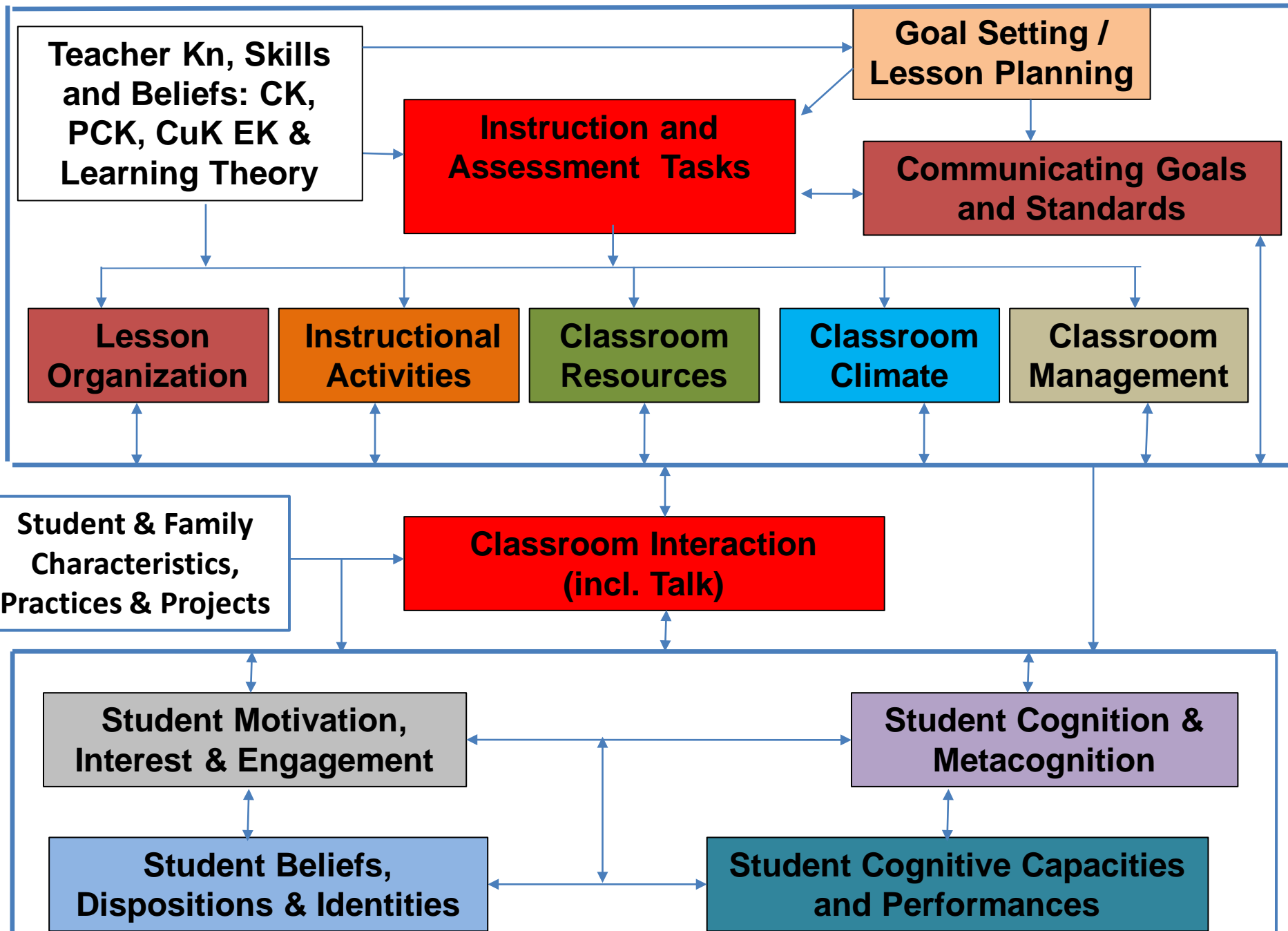
PART FIVE

CONCLUSION

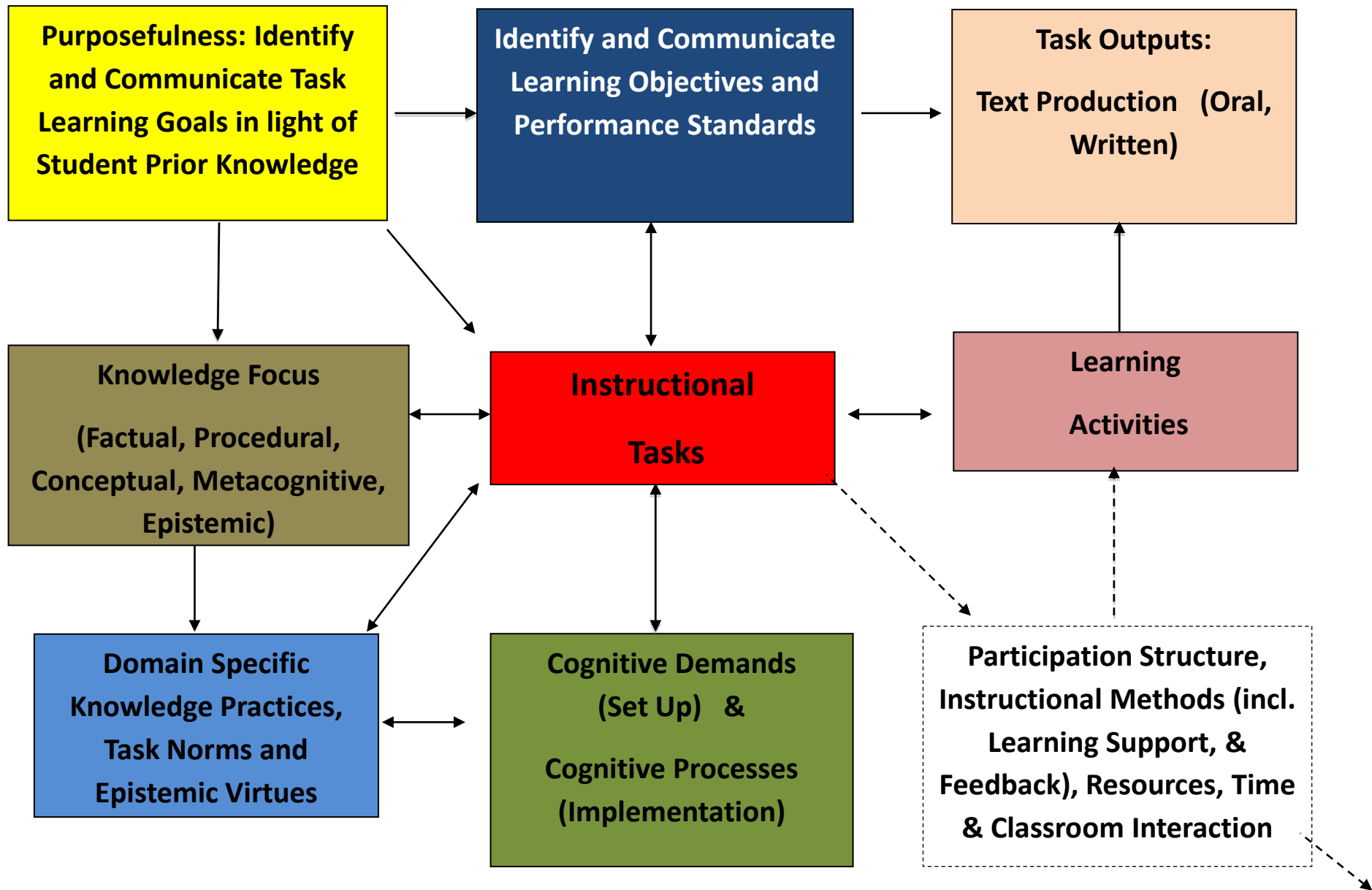
Core 2 Pentagon



Framing Instruction: An Anatomy of Teaching and Learning

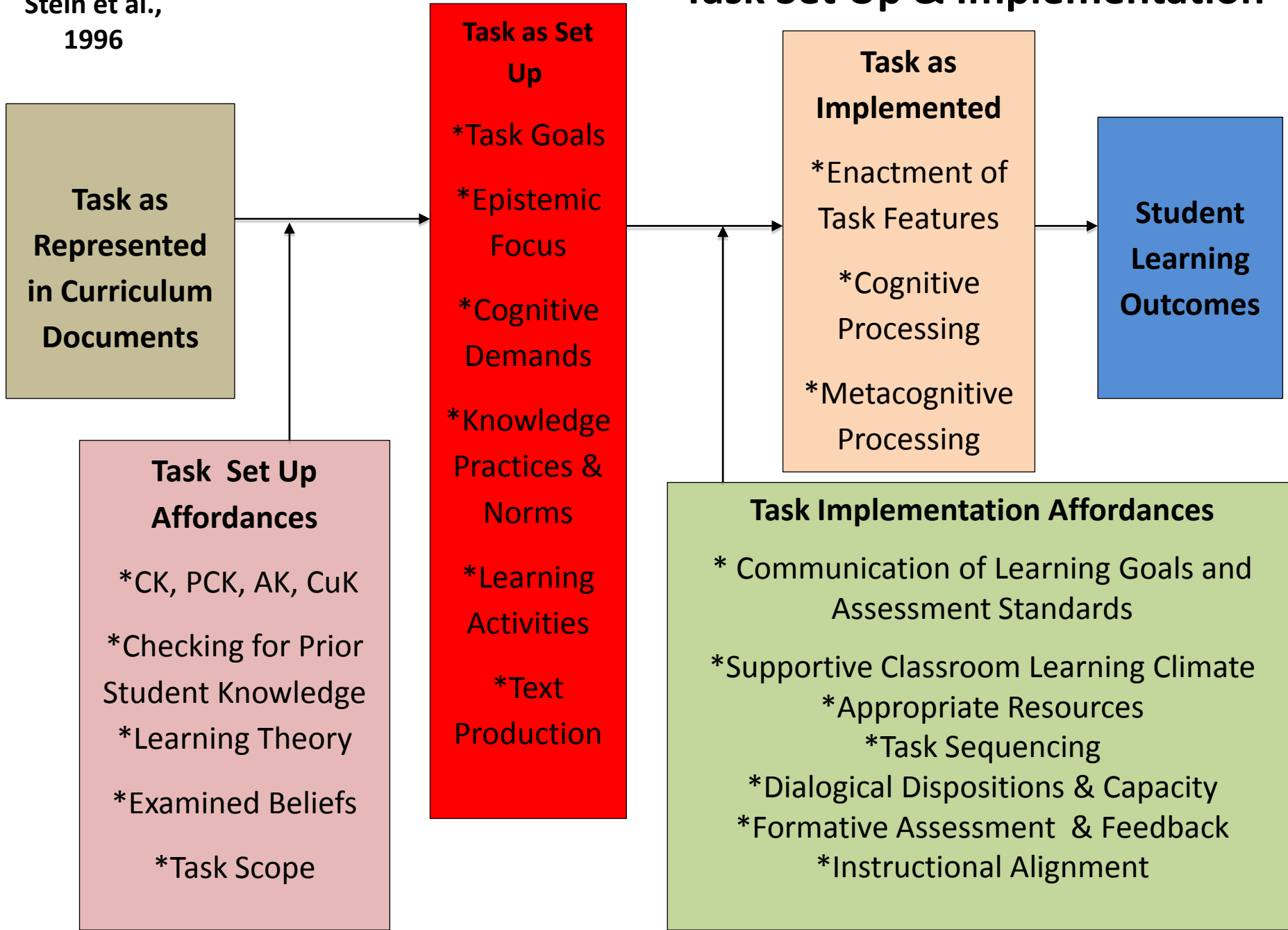


Conceptual Model of Instructional Tasks

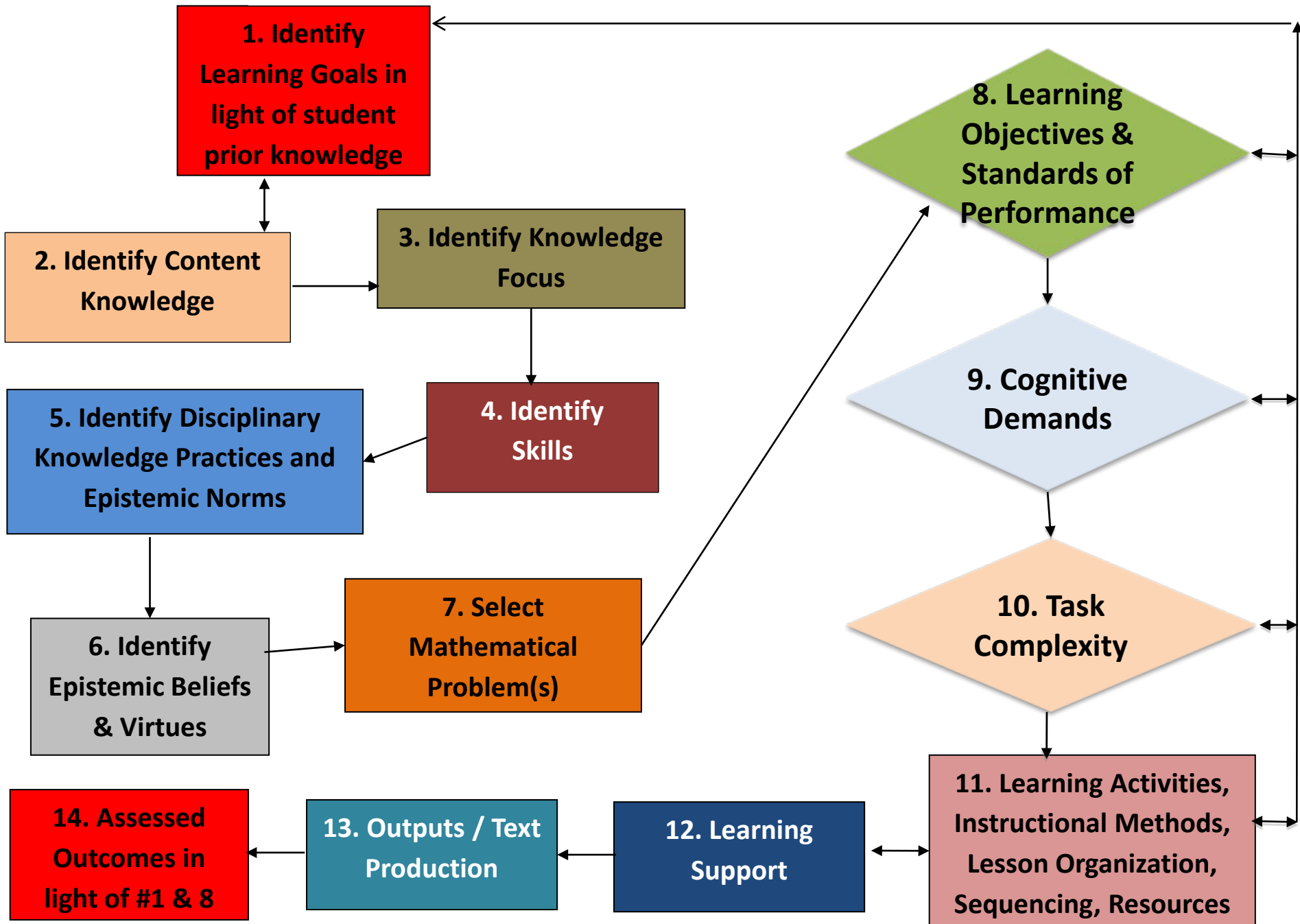


Adapted from
Stein et al.,
1996

Task Set Up & Implementation



A Framework to Guide Task Design





Thank you

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