1. Change the Givens

Many mathematical problems involve several pieces of information and steps of computation. We can change what is given in the problems. When all the givens are on the same side, the problem is usually relatively easy. However, when the givens appear on different sides of an equation, the problem becomes more difficult.

**Example 1.** Find 15% of 200.

**Example 2.** An arc subtends an angle $30^\circ$ at the centre of a circle of radius 5 cm. Calculate the arc length.

**Example 3.** Given that $h = 3$ and $k = -2$, find the value of $3h + 4k$.

**Example 4.** Find the next number: 1, 3, 6, 10, _, ….

**Example 5.** Consider shape 1 in Figure 1, which is divided into four equal parts. Remove one quarter of shape 1 to obtain shape 2. What fraction of shape 2 must be added on to get back to shape 1?

![Figure 1](image)

2. Use More Open-ended Tasks

A closed problem is one that has only one correct answer and possibly also one most efficient method of solution that the pupils are trained to master. On the other hand, an open-ended problem has several or infinitely many solutions and may be solved using many different methods. These open-ended problems can be used to probe deeper understanding of concepts and to promote creative thinking. Their use also helps to change the common perception that any mathematics problem always has only one right answer or one correct method of solution.

**Example 1.** Round 34.56 to the nearest one decimal place.

**Example 2.** Solve $2x - 3 = 7$.

**Example 3.** A tiger weighs 220 kg. What is the weight of three similar tigers?

**Example 4.** Calculate the hypotenuse of the triangle in Figure 2.

![Figure 2](image)
Example 5. Calculate the area enclosed between the two circles in Figure 3.

Figure 3.

3. Include Interesting or Meaningful Contexts

Problem solving in school is for the sake of solving problems in life. Other things being equal, problems where the situation is real are better than problems where it is described in words and problems which might really occur in a sane and reasonable life are better than bogus problems and mere puzzles.

Real life problems are set in some contexts. The contexts may be everyday situations (meaningful), current affairs, historical events (heritage), and cultural practices. Including real contexts in problems will widen pupils’ perspectives of their local environments and the wider world and provide links to other subjects. For problems embedded in these contexts, the method may be hidden, so the pupils have to find the correct method before they can solve the problem.

Example 1. Two plant pots are geometrically similar. The height of the smaller pot is 5 cm. The height of the larger pot is 15 cm. Find the ratio of the volume of the smaller pot to that of the larger pot.

Example 2. Calculate the area of the annulus between two concentric circles of radii 4 cm and 7 cm.

Example 3. Every time a heart beats, it ejects about 56 g of blood. Given that the heart beats about 72 times per minute, calculate the amount of blood ejected from the heart into the aorta.

Example 4. The number of apples is twice the number of oranges. Write this in algebra.

4. Use Creative Imagination

Not all mathematics problems need to include real contexts. They may involve fantasy and imagination to create some fun and make mathematics more enchanting. It is, indeed, very rare that teachers help pupils develop creative imagination in mathematics lessons. Because of this shortcoming, school mathematics is predominantly left hemispheric thinking, leaving little opportunity to develop right hemispheric visualisation, even in geometry work that is supposed to develop our intuition about objects in space.

Imagine a large white equilateral triangle. At each corner imagine a small black equilateral triangle, as in Figure 5(a). Close your eyes and slowly enlarge each small triangle. Describe the shape of the interior of the white triangle. Repeat with a square, as in Figure 5(b).

Figure 5. (a) Triangle with black corners. (b) Square with black corners.
5. **Use Different Formats**

The same question may be given in a combination of different modes, for example, in symbols, diagrams, numbers, or words. The presentation can have significant effects on pupil achievement. A study has found that among Singapore pupils, a problem presented with a diagram and a “simple” explanation was easier than the same problem stated in words only. To help the weaker pupils to master problem solving, it is helpful to start with problems presented in a diagram + explanation form and slowly move to a verbal format. In fact, public examinations are now giving more problems in a combination of diagrams and words. In some cases, the pupils have to obtain the relevant information from the diagrams.

**Example.** Villages $B$ and $C$ are each 5 kilometres from village $A$, and $\angle BAC = 150^\circ$. The village $C$ is due south of a point $X$ and the villages $A$ and $B$ are both due east of $X$.

6. **Other Ways**

There are other ways to design test items that we have not dealt with here. These may involve using pupils’ errors, asking for specific cases of a general rule, generalising from specific cases, and applying the SOLO* taxonomy to develop items that have increasing levels of complexity. Whichever ways are used, the test items must cover a range of lower to high-order skills so that pupils of different abilities and interests can demonstrate what they understand and can do. This requires teachers to acquire new mathematical knowledge as well as knowledge of other fields that make use of mathematics, such as the sciences, technology, social sciences, and psychology.

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*SOLO, which stands for Structured Observation of Learning Outcome, provides a systematic way of describing how a learner's performance grows in complexity when mastering many academic tasks. It can thus be used to define curriculum objectives, which describe where a student should be operating, and for evaluating learning outcomes so that we can know at what levels individual students actually are operating.