Problem: After gathering a pile of coconuts one day, three sailors on a desert island agreed to divide them evenly after a night's rest. During the night one sailor got up, divided the coconuts into three equal piles with a remainder of one, which he tossed to a conveniently nearby monkey; and, secreting his pile, mixed up the others and went back to sleep. The second sailor did the same thing, and so did the third. In the morning, the remaining pile of coconuts (less one) is again divisible by 3. What is the smallest number of coconuts that the original pile could have contained?


Solution: The problem is intended for Secondary One students. First of all, before using any heuristics to solve the problem, the student is to acquire as much information as possible from the question and try to classify the information given. From the problem above, the student should be able to see that he have to solve four sub-questions arising from the problem before knowing the solution to the problem. Therefore, the student can use the before and after concept to simplify the problem by breaking it down into the four sub-questions as follows:

1. How many coconuts are left in the pile after the first sailor did his act?
2. How many coconuts are left in the pile after the second sailor did his act?
3. How many coconuts are left in the pile after the third sailor did his act?
4. How many coconuts does each of the sailors get from the remaining pile the next day?

Using these four sub-questions, the student should be able to find the solution to the problem as in the methods shown in the next few pages.

Heuristics used: before and after concept, simplify the problem. Thinking skills: observing, classifying.

Method 1: The student can use marbles or other manipulative to represent the coconuts and act out the situation in the problem. First of all, we can infer from the problem that a total of 4 coconuts were thrown out from the pile and in the morning, the remaining pile (less one) has to be divisible by 3. To start off, each sailor will get at least one coconut from the remaining pile the next day. Therefore, the student can start from 7 coconuts and act out the situation. After every failed attempt, add in 3 more marbles and re-act the situation till success.

Heuristics used: Act it out, guess and check. Thinking skills: classifying, observing, deduction.
**Method 2:** Using the same idea from method 1, instead of acting out, the student can *make a systematic list by using tabulation* as follows:

<table>
<thead>
<tr>
<th>Number of coconuts in original pile</th>
<th>Number of coconuts remaining after Sailor 1's act</th>
<th>Number of coconuts remaining after Sailor 2's act</th>
<th>Number of coconuts remaining after Sailor 3's act</th>
<th>Number of coconuts each sailor gets from the remaining pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
<td>2</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>?</td>
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<tr>
<td>79</td>
<td>52</td>
<td>34</td>
<td>22</td>
<td>7</td>
</tr>
</tbody>
</table>

Note that for every guess the student make, the student cannot continue when a question mark occurs. This is when the previous number in the row subtracted away 1 is indivisible by 3.

Heuristics used: *Use tabulation, guess and check, make a systematic list.*

Thinking skills: *observing, sequencing, deduction.*

**Method 3:** The student can reword the problem by *restating the question* as follows:

a) What is the smallest number of coconuts each sailor could get from the remaining pile?

b) Using the solution to (a), what is the smallest number of coconuts that the original pile could have contained?

The thinking process is to *tabulate* and *deduce* the solution as follows:

<table>
<thead>
<tr>
<th>Number of coconuts each sailor gets from the remaining pile</th>
<th>Number of coconuts remaining after Sailor 3’s act</th>
<th>Number of coconuts remaining after Sailor 2’s act</th>
<th>Number of coconuts remaining after Sailor 1’s act</th>
<th>Number of coconuts in original pile</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>7</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>?</td>
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</tr>
<tr>
<td>7</td>
<td>22</td>
<td>34</td>
<td>52</td>
<td>79</td>
</tr>
</tbody>
</table>

Similarly, for every guess the student makes, the student cannot continue when a question mark occurs. This is when the previous number in the row is indivisible by 2.

Heuristics used: *Restate the problem in another way, use tabulation, guess and check, make a systematic list.*

Thinking skills: *observing, sequencing, deduction.*
Method 4: First of all, the student can try using a model to illustrate the situation:

Original pile:  
After Sailor 1:  
After Sailor 2:  
After Sailor 3:  

Drawing the model does not seem to help. But the model can be used to help the student work backwards and form the following equations:

Let $x$ be the number of coconuts each sailor gets from the remaining pile.

After Sailor 3's act, the remaining coconuts in the pile $= 13 + x$

After Sailor 2's act, the remaining coconuts in the pile $= \frac{3}{2} (13 + x) + 1$

After Sailor 1's act, the remaining coconuts in the pile $= \frac{3}{2} \left( \frac{3}{2} (13 + x) + 1 \right) + 1$

The number of coconuts in the original pile $= \frac{3}{2} \left( \frac{3}{2} \left( \frac{3}{2} (13 + x) + 1 \right) + 1 \right) + 1$

$= \frac{81}{8} x + \frac{65}{8}$

$= 10 \frac{1}{8} x + 8 \frac{1}{8}$

Now, to find the smallest number of coconuts that the original pile could have contained, $x$ have to take the smallest value possible. And since by common sense, the number of coconuts in the original pile has to be a whole number, the student should be able to deduce that $x = 7$ from the last equation. Therefore since $x = 7$, using the last equation, the smallest number of coconuts that the original pile could have contained is 79.

Heuristics used: Using model, work backwards, use before and after concept, use equations.

Thinking skills: observing, sequencing, identifying attributes and components, deduction.

In solving the above problem, the student might have difficulty understanding and formulating the question in the problem. Therefore to help the student, the teacher can simplify the problem by formulating sub problems as illustrated in the first part of the solutions. In this way, the student can organise the information given and solve the sub problems and progress towards the solution.

After getting the answer to the problem, the student might forget to give the answer with the correct unit to accompany the numerical part of the answer or
state the answer in a complete sentence. Therefore the teacher must give constant practice and reminders for students to do so.

Next, since the above problem involves dividing up the pile of coconuts, the student should be able to determine whether or not the answer make sense. Therefore, in the solutions, the students should be able to stop where a question mark appears since further division will lead to a fraction of the coconut, which does not make sense. To help students see this, the teacher can use marbles and ask the students to act it out. In the process, the student will find it impossible to do the division in some cases. Therefore when visualising the problem, the student should also bring this common sense into his work.

Now, normally, given such a question, the student will want to organise and formulate the information into mathematical equations. But given such a complicated question, the students might formulate the wrong equation. For example, they might do the division and multiplication first and adding the four coconuts into the pile in the final step, thus leading to the wrong solution. Therefore to help the students overcome this difficulty, the teacher can encourage students to draw models and systematically link the models to formulate the equations.

Hence in conclusion, problem solving can be a frustrating task for the students. The students might give up too easily after failed attempts taken to solve the problem. Therefore to help the students develop the skills for problem solving, the teachers must help the students to organize their thinking process. With the right attitude and skills, the student will be able to overcome all the possible difficulties encountered in the process of problem solving.