

# **NATIONAL INSTITUTE OF EDUCATION**

## **Postgraduate Diploma in Education (Secondary)**

**July 2006 Intake**



## **QCM520 Teaching and Learning Mathematics I**

### **Problem Solving Assignment**

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**Problem:**

A 4 x 4 antimagic square is an arrangement in a square of the numbers from 1 to 16 so that the totals of each of the four rows and four columns and two diagonals are ten consecutive numbers in some order. The diagram shows an incomplete antimagic square. When it is completed, what number will replace the asterisk?

		*	14
	9	3	7
	12	13	5
10	11	6	4

**Intended level:** Secondary 4 Express

**Solutions:**

**Method 1**

Step1 Heuristics: Solve part of the problem

Thinking Skills: Deduction

To facilitate presentation, we label the table as follows:

	C1	C2	C3	C4
R1	W	z	*	14
R2	X	9	3	7
R3	Y	12	13	5
R4	10	11	6	4

Let the diagonal, D1, be from R1C1 to R4C4 and the other diagonal, D2, be from R4C1 to R1C4.

Information we can deduce so far:

$$R4 = 10 + 11 + 6 + 4 = 31$$

$$C4 = 14 + 7 + 5 + 4 = 30$$

$$D2 = 10 + 12 + 3 + 14 = 39$$

Since the sums are ten consecutive numbers and we already have 30 and 39, we can conclude that the ten sums are 30, 31, 32, ..., 38, and 39.

Step2 Heuristics: Solve part of the problem

Thinking skills: Deduction

We use the newly obtained information to deduce the possible value(s) for each of  $w$ ,  $x$ ,  $y$ ,  $z$  and  $*$ .

(1) Now,  $R3 = 30 + y$  and  $R3 < 40$ .

Thus, we can conclude that  $y < 10$ .

Also,  $y \neq 1$  and  $y \neq 9$  since the numbers 31 and 39 are already the sum for  $R4$  and  $D2$  respectively.

Possible values for  $y = 2$  or  $8$  (since 3, 4, 5, 6, 7 are already in the square)

(2)  $R2 = x + 19$  and  $31 < R2 < 39$ .

Thus,  $12 < x < 20$ .

Possible values for  $x = 15$  or  $16$  (since 13 and 14 are already appeared in the square and the biggest number is 16.)

(3)  $D1 = 26 + w$  and  $31 < D1 < 39$ .

Thus,  $5 < w < 13$ .

Conclusion:  $w = 8$  (since 6, 7, 9, 10, 11 and 12 already appeared in the square)

(4)  $C2 = 32 + z$  and  $32 < C2 < 39$ .

Thus,  $0 < z < 7$ .

Possible values for  $z = 1$  or  $2$  (since 3, 4, 5 and 6 already appeared in the square)

(5)  $C3 = 22 + *$  and  $31 < C3 < 39$ .

Thus,  $9 < * < 17$ .

Possible values for  $* = 15$  or  $16$  (since 10, 11, 12, 13 and 14 already appeared in the square)

	C1	C2	C3	C4	39
R1	w	z	*	14	14
R2	x	9	3	7	19
R3	y	12	13	5	30
R4	10	11	6	4	31
	10	32	22	30	26

Step3 Heuristics: Solve part of the problem

Thinking skills: Deduction

From step2, we conclude that  $w = 8$ .

Since  $y = 2$  or  $8$ , we conclude that  $y = 2$

Since  $z = 1$  or  $2$ , we conclude that  $z = 1$ .

(Note: updated values in the table are highlighted in bold.)

	C1	C2	C3	C4	39
R1	<b>8</b>	<b>1</b>	*	14	<b>23</b>
R2	x	9	3	7	19
R3	<b>2</b>	12	13	5	<b>32</b>
R4	10	11	6	4	31
	<b>20</b>	<b>33</b>	22	30	<b>34</b>

Step4 Heuristics: Guess and check, make a systematic list

Thinking skills: Deduction

Case 1:  $x = 15$ ,  $* = 16$

$$R1 = 23 + 16 = 39 \text{ (contradiction to D1)}$$

Case 2:  $x = 16$ ,  $* = 15$

$$R1 = 23 + 15 = 38$$

$$C3 = 22 + 15 = 37$$

$$R2 = 19 + 16 = 35$$

$$C1 = 20 + 16 = 36$$

Thus,  $* = 15$  and the antimagic square should look like:

	C1	C2	C3	C4	39
R1	8	1	15	14	<b>38</b>
R2	16	9	3	7	<b>35</b>
R3	2	12	13	5	<b>32</b>
R4	10	11	6	4	<b>31</b>
	<b>36</b>	<b>33</b>	<b>37</b>	<b>30</b>	<b>34</b>



## **Method 2**

Heuristics: Make a systematic list, guess and check

Thinking skills: deduction

The available numbers are 1, 2, 8, 15 and 16.

From Method 1 step 1, we can similarly deduce that the ten consecutive numbers are from 30 to 39.

We can then exhaust all the possible combinations until we get the correct answer.

There are a total of  $5! = 120$  cases.

Due to the tediousness of this method (though straightforward), I shall not illustrate it here.

## **Possible difficulties faced by students**

When students see this question for the first time, they might have difficulties in understanding the question itself. As this question is not the usual problem that they have come across in their textbooks or ten-year series, they might have difficulties in trying to come up with an appropriate approach to handle this problem.

Some students might also try to solve the problem using Method 2 (guess and check) as described above. However, due to the large number of cases involved, they might be discouraged or think that the problem is “unsolvable” when they fail to get the right answer in a few attempts.

Even when using method 1, students need to be very systematic in applying logical deductions in solving part of the problem. The large number of cases that need to be considered can be very mind-boggling to the students.

## **Suggested Help**

In order to help students overcome the problems mentioned above, teachers can expose students to a wide variety of challenging problems to introduce to them the usefulness of the various heuristics and to train them in deducing from existing information further relevant information logically. When students get more practices at solving “unseen” problems, their mind will be more accustomed to problem solving and this will help them in solving problems directly related to the actual mathematics syllabus. They will also be more confident in trying to find different ways to tackle a problem.

For a start, students could be allowed to work in groups. Co-operative learning might help in making the students feel less threatened by challenging problems and might even make such activities more interesting. As they discussed the problems with their peers, they will be able to clarify their doubts.

When faced with problems that are more “wordy”, teachers should try to identify (via body language, feedback etc) if the students are having trouble in interpreting the question itself. If so, teachers should go through the questions with the students. Students can be taught to jot down or highlight important information presented in the question. Model / diagrams might also aid in their understanding also. Alternatively, teachers may present a similar but simpler problem to the students first before letting them solve the higher dimension problem. If the students cannot even understand the question in the first place, they would not be able to apply any heuristics or thinking skills at all!