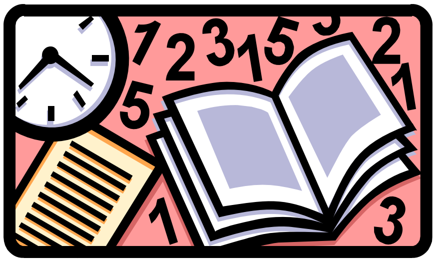


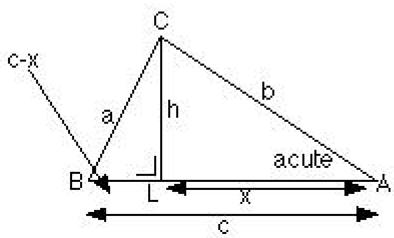
Exploring Cosine Rule Using Geometer Sketch Pad

Name:____ Index Number:____

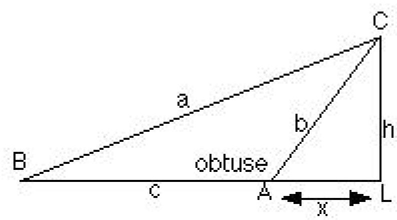
Class: Secondary_____________________________________ Date:____________

Activity 1:

Given a triangle, ΔABC with $\angle A$ acute (case 1) or obtuse (case 2), CL can be drawn perpendicular to AB (or AB produced.). Let a, b, c denotes the sides opposite angles A, B, C respectively. Let | CL | = h and | AL | = x.



Case 1



Case 2

Step 1: For the triangle in Case 1, prove the following in sequence.

1.1. Using Pythagoras Theorem for ΔCBL, express a in terms of c, x and h. [1 mark]

1.2. Similarly for ΔCLA, express b in terms of x and h. [1 mark]

1.3. Using your answers in **1.1** and **1.2**, find an expression for a, b, c and x by eliminating h [1 mark]

1.4. For ΔCLA, derive an expression for x in terms of b and $\angle$ A [0.5 mark]

1.5. Finally, use your answer in **1.3** and **1.4**, derive the following expression [1.5 mark]

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Step 2: For the triangle in Case 2, prove the following in sequence.

2.1. Using Pythagoras Theorem for ΔCBL, express a in terms of c, x and h. [1 mark]

2.2. Similarly for ΔCLA, express b in terms of x and h. [1 mark]

2.3. Using your answers in **1.1** and **1.2**, find an expression for a, b, c and x by eliminating h [1 mark]

2.4. For ΔCLA, derive an expression for x in terms of b and $\angle$ A [0.5 mark]

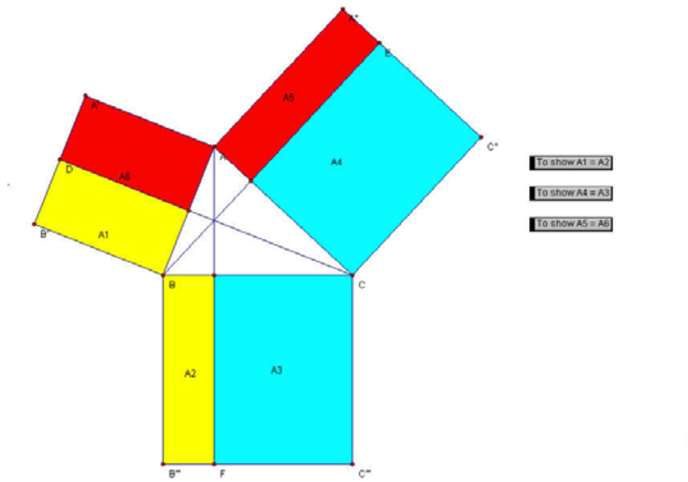
2.5. Finally, use your answer in **2.3** and **2.4**, derive the following expression [1.5 mark]

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Activity 2:
Using GSP to explore Cosine Rule…

Please proceed in the following steps:

1. Open the program "GSP 4.01" on your computer
2. Within the GSP environment, open the file '*Cosine_rule_Proof1.gsp*'
3. You should see on your screen something like the following:



4. You can **vary the size of ΔABC** by firstly clicking on any of the points A, B or C, and then holding down the left-mouse button, drag the point any suitable position.
5. Click on
   a. [To show A1 = A2] to see a demonstration why area A1 is equal to A2,
   b. [To show A4 = A3] to see a demonstration why area A4 is equal to A3,
   c. [To show A5 = A6] to see a demonstration why area A5 is equal to A6,

   through a series of transformations.

Let us now prove **Cosine Rule**:
Showing your working clearly in each part,
1.1. Show that A1 = A2.

Fill in the blank: A1 = A2 = ___________[1]
1.2. Show that A3 = A4.

Fill in the blank: A3 = A4 = ___________[1]

1.3. Show that A5 = A6.

Fill in the blank: A5 = A6 = ___________[1]

1.4. Complete the following by filling in the blanks. [2]

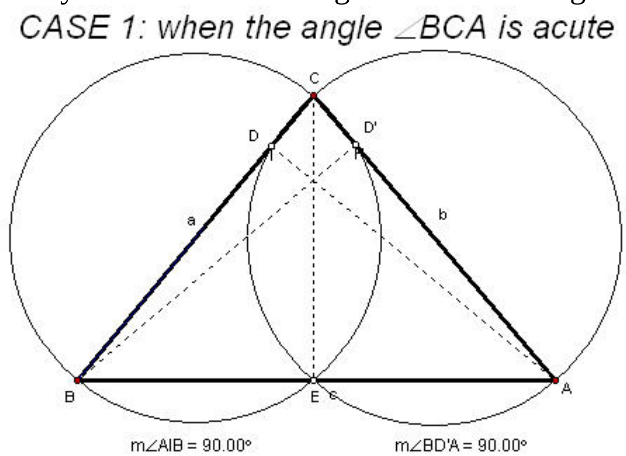
$$a^2 = A2 + A3 = \_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_$$
$$= \_\_\_\_\_\_\_\_\_\_\_$$

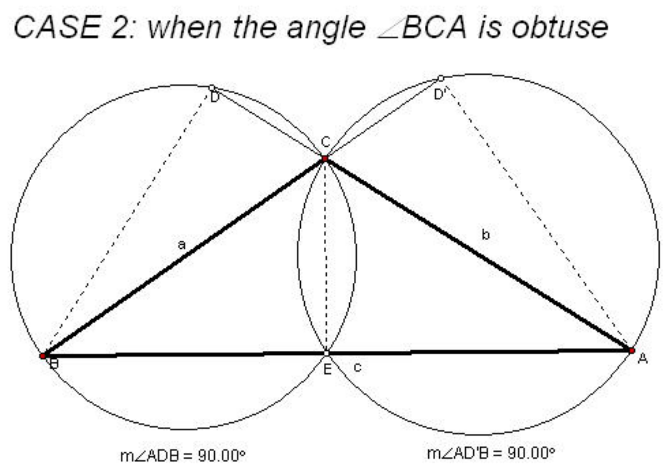
Activity 3:
Continuing to using GSP to explore Cosine Rule…

Please proceed in the following steps:

6. Within the GSP environment, open the file ‘*Cosine_rule_Proof2.gsp*’

You should see on your screen something like the following:





7. Once again, you can **vary the size of** Δ**ABC** by firstly clicking on any of the points A, B or C, and then holding down the left-mouse button, drag the point any suitable position.

For CASE 1: When the angle $\angle BCA$ is acute

Step 1. Using the ‘Distance’ button (under the ‘Measure’ menu), find the following distances: [1]

Different distances to be computed
1) BD
2) BC (length a)
3) BE
4) BA (length c)
5) AD’
6) AC (length b)
7) AE

Step 2. Using the ‘Calculate…’ button (under the ‘Measure’ menu), calculate the following values and include them in your gsp file ‘*Cosine_rule_Proof2.gsp*’. [2]

Different values to be computed
8) BE.c
9) BD.a
10) EA.c
11) D’A.b

Step 3. Deduce and write down below the relationship between the values BE.c, BD.a, EA.c and D’A.b. [1]
(Note: you will probably notice a pattern in the change in values of the above four values as you adjust the vertex C in the ***Geometer Sketchpad*** file.)

The following would probably be useful in Step 4.

1) Recall by Cosine ratio, we have the following property:

$$\cos \angle BCA = \frac{CD'}{a} = \frac{CD}{b}$$

2) BD = (a – DC) and D’A = (b – D’C)

Step 4. By adding BE.c and EA.c (as a first step), derive the Cosine Rule, mainly

$$c^2 = a^2 + b^2 - 2ab(\cos \angle BCA)$$

For CASE 2: When the angle $\angle BCA$ is obtuse

Step 1. Using the 'Distance' button (under the 'Measure' menu), find the following distances: [1]

Different distances to be computed
12) CD' 13) BC (length a) 14) BE 15) BA (length c) 16) CD 17) AC (length b) 18) AE

Step 2. Using the 'Calculate…' button (under the 'Measure' menu), calculate the following values and include them in your gsp file '*Cosine_rule_Proof2.gsp*'. [2]

Different values to be computed
19) BE.c 20) (a+CD').a 21) EA.c 22) (b+CD).b

Step 3. Deduce and write down below the relationship between the values BE.c, (a+CD').a, EA.c and (b+CD).b. [1]
(Note: you will probably notice a pattern in the change in values of the above four values as you adjust the vertex C in the ***Geometer Sketchpad*** file.)

The following would probably be useful in Step 4.

1) Recall by Cosine ratio, we have the following property:

$$-\cos \angle BCA = \cos(\pi - \angle BCA) = \frac{CD'}{b} = \frac{CD}{a}$$

Step 4. By adding BE.c and EA.c (as a first step), derive the Cosine Rule, mainly

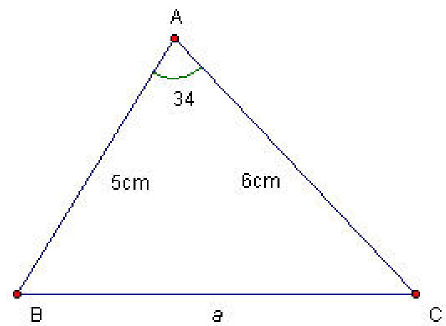
$$c^2 = a^2 + b^2 - 2ab(\cos \angle BCA)$$

Activity 4:

The cosine rule is needed to solve triangles when

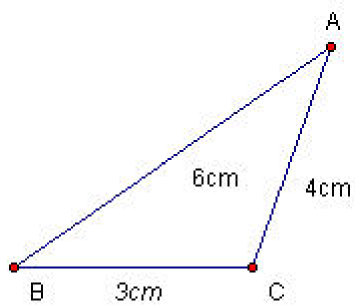
1. *Two sides and an included angle*, or
2. *Three sides are given.*

Exercise 1: (*Two sides and an angle given***.)**
Solve for length *a* if you are given the following figure: (Giving your answer correct to 1 decimal place) [2]



Solution:

Exercise 2: (*Three sides are given***.)**
Solve for $\angle BAC$ if you are given the following figure: (Giving your answer correct to 2 decimal place) [2]



Solution: