Number Bases II

REVIEW

The cardinality of the set of stars {* * * * * * *} is **seven** (spoken symbol). In base ten the written symbol is 7. How is it written / spoken in base six? five? four? three? two?

In base six	**11_{six}**	**One-one base six**
In base five	**12_{five}**	**One-two base five**
In base four	**13_{four}**	**One-three base four**
In base three	**21_{three}**	**Two-one base three**
In base two	**111_{two}**	**One-one-one base two**

[For example, 12_{five} means 1 group of five and 2 ones and so it is 7 ones. It should not be read as 'twelve base five'.]

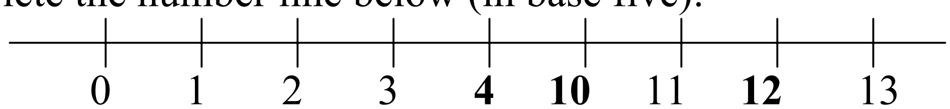
OPERATIONS IN OTHER BASES

Addition and subtraction

Check understanding:

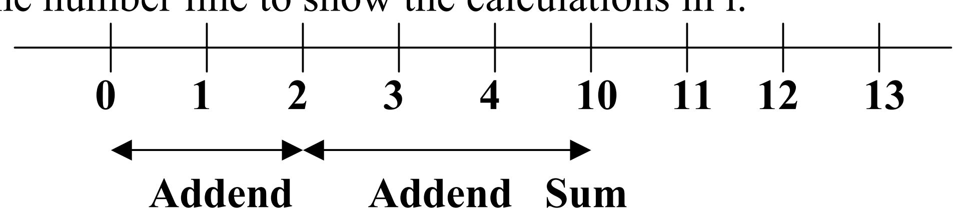
i. Use the Base-Five Addition Table p141 to find
$2_{five} + 3_{five}$, $4_{five} + 3_{five}$
[Ans: 10_{five} , 12_{five}]

ii. Complete the number line below (in base five):



iii. Use the number line to show the calculations in i.

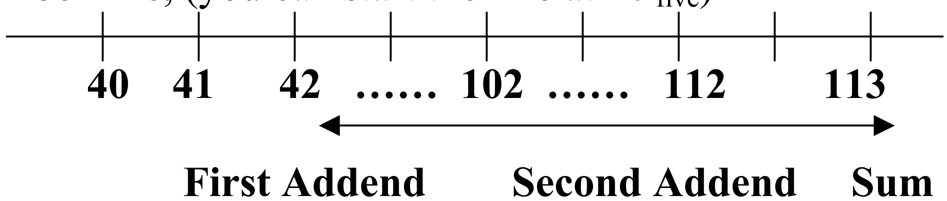
Base 5



iv. The computation below is done using a concrete model.
Write down the number represented in each set in base five.
Show the computation, in base five, using

(a) the number line, (you can start the line at 40_{five})

Base 5



(b) the expanded algorithm, (use the addition table)

(c) the standard algorithm.

(b)

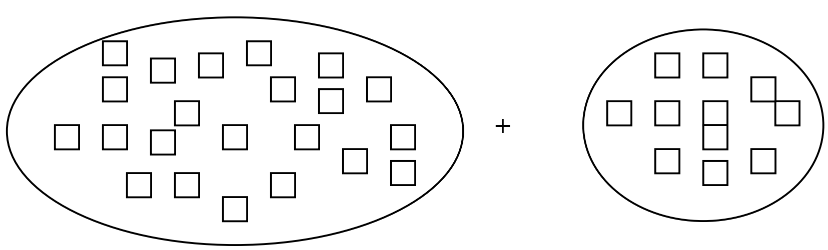
$$\begin{array}{r} 42_{\text{five}} \\ +\ 21_{\text{five}} \\ \hline 3 \\ +\ 110 \\ \hline 113_{\text{five}} \end{array}$$

(c)

$$\begin{array}{r} 42_{\text{five}} \\ +\ 21_{\text{five}} \\ \hline 113_{\text{five}} \end{array}$$

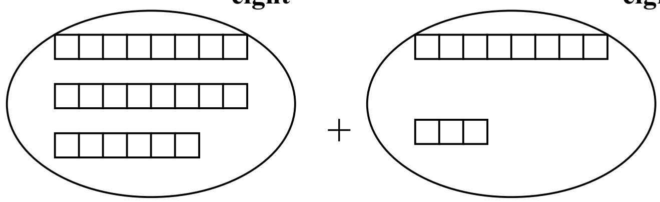
Try this: Construct the addition table for base eight.

+	0	1	2	4	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16



Group the squares in eights and write the numbers represented in the two sets in base eight.

$\mathbf{26_{eight}}$ $\mathbf{13_{eight}}$



Complete the model by drawing the set representing the sum in base eight.

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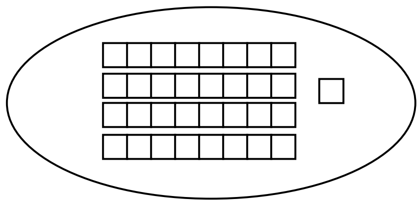
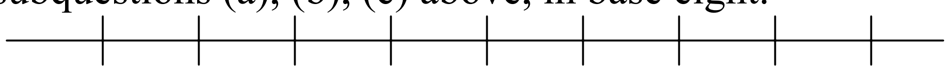


$\mathbf{41_{eight}}$

Repeat the subquestions (a), (b), (c) above, in base eight.

(a)



25 26 27 30 31 …… 41

Base 8

First Addend Second Addend Sum

(b)

$$\begin{array}{r} 26_{eight} \\ +\ 13_{eight} \\ \hline 11 \\ +\ 30 \\ \hline 41_{eight} \end{array}$$

(c)

$$\begin{array}{r} {}^{1}26_{eight} \\ +\ 13_{eight} \\ \hline 41_{eight} \end{array}$$

Practice: Ongoing assessment 3-2
ex 12 a,c,e p143 **[Answers given on p. 773]**
ex 25 p144 **[Ans: 110_{five}]**

Extend knowledge: Study the first paragraph and figure 3-9 p142
Do: ‘Now try this 3-5’ p142

+	0	1
0	0	1
1	1	10

Try: 53_{eight} - 11_{eight} **[Ans: 42_{eight}]**

Now try: 53_{eight} - 15_{eight}

$$\begin{array}{r} {}^{4}\cancel{5}{}^{1}3_{eight} \\ -\ 1\ 5_{eight} \\ \hline 3\ 6_{eight} \end{array}$$

Practice: ex 12 b,d,f p143 **[Ans: b. 20_{five} ; d. 14_{five} ; f. 1010_{two}]**
ex 26 p144 **[Answer for (a) given on p. 774]**
For ex 26 (b):

$$\begin{array}{r} 20010_{three} \\ -\quad 2\underline{0}2\underline{2}_{three} \\ \hline 1\underline{0}2\underline{1}1_{three} \end{array}$$

Extra practice (optional)

(a) 24_{six} + 3_{six}

(b) 24_{six} - 5_{six}

(c) 13_{twelve} - 8_{twelve}

(d) T_{twelve} + 3_{twelve}

(e) 1021112_{three} - 21221_{three}

[Ans: (a) 31_{six} (b) 15_{six} (c) 7_{twleve} (d) 11_{twleve} (e) 222121_{three}]

Multiplication

Multiplication is repeated addition and again the processes are the same as for base ten. Since we have not memorised our multiplication tables in the various bases, we need to construct these tables before we can perform multiplication (or division). We can do this by using addition repeatedly or we can perform single digit multiplication in base ten (*because in base b, where b is lest than 10,* $i_b = i_{ten}$) and then convert.

For example, 4_{five} x 3_{five} = 12 = $22_{five.}$

Table 3-9 p150 is the Base-Five Multiplication Table.

Look at the last row and see that it is obtained by adding 4 successively (in base five).

So the addition table base five should help you complete most of the multiplication table.

Only three more computations are needed to complete the table. What are they? **[4_{five} x 3_{five} , 3_{five} x 4_{five} and 4_{five} x 4_{five}]**

Try this: Use your addition table base eight to construct the multiplication table base eight. Complete the table.

×	0	1	2	4	4	5	6	7
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
1	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
2	**0**	**2**	**4**	**6**	**10**	**12**	**14**	**16**
3	**0**	**3**	**6**	**11**	**14**	**17**	**22**	**25**
4	**0**	**4**	**10**	**14**	**20**	**24**	**30**	**34**
5	**0**	**5**	**12**	**17**	**24**	**31**	**36**	**43**
6	**0**	**6**	**14**	**22**	**30**	**36**	**44**	**52**
7	**0**	**7**	**16**	**25**	**34**	**43**	**52**	**61**

For multiplication of numbers with more than one digit, the algorithm works in exactly the same way as for base ten.

Study Figure 3-2 p147

Check understanding

i. Why is 13_{five} fives $= 130_{five}$?

[c.f. 13 tens is 130. Also note that
13_{five} fives $=[1(5)+3]x5=1(5^2)+3(5)+0(1)=130_{five}$.]

ii. Why do we write 2_{five} twenty-fives $= 200_{five}$?

[c.f. 2 hundreds is 200. Also note that
2_{five} twenty-fives $= 2x5^2 = 200_{five}$]

iii. Is it correct that $322 = 432_{five}$? Explain.

[It is incorrect. The two computations are done in base ten and base five respectively.]

iv. The last multiplication on p147 shows the standard algorithm for 23 x 14 in base ten. Write the multiplication in standard algorithm in base 5 for 23_{five} x 14_{five}

$$\begin{array}{r} 23_{five} \\ \times\ \ 14_{five} \\ \hline 202\phantom{_{five}} \\ +\ 23\phantom{0_{five}} \\ \hline 432_{five} \end{array}$$

Now try this 23_{eight} x 14_{eight}

$$\begin{array}{r} 23_{eight} \\ \times\ \ 14_{eight} \\ \hline 114\phantom{_{eight}} \\ +\ 23\phantom{0_{eight}} \\ \hline 344_{eight} \end{array}$$

Practice (a) 43_{five} x 3_{five} (b) 43_{five} x 23_{five}
(c) 53_{eight} x 23_{eight} (d) 11011_{eight} x 101_{eight}

[Ans: (a) 234_{five} (b) 2144_{five} (c) 1461_{eight} (d) 1112111_{eight}]