WARMING-UP ACTIVITY 1: Check Understanding

Decimal (base Ten) System

i. Note that $10^0 = 1$. Why?
See (ii) in Extend Knowledge.

ii. What is the place value of 4 in 4107? What is its face value?
The place value of 4 is "thousand" and its face value is four.

iii. What is the actual value of the 4 in 4107
The actual value of 4 is 4 thousands.

iv. What is the role of the 0 digit in 4107?
As a place holder.

v. Describe the number obtained when we multiply 4107 by ten, by one hundred, by one thousand.
The number is obtained by adding one zero, two zeros and three zeros to the right of 4107 respectively, that is, by shifting one, two and three place values respectively.

Extend knowledge:

i. Find the meaning of 'deci'.
The word decimal comes from the Latin decem, meaning ten. "Deci" means a tenth part of any quantity.

ii For any $a \neq 0$, $a^0 = 1$. Why?
For positive real number a and positive integer m, we write a^m to mean a.a.a…a (m copies of a) and so we have $a^m a^n = a^{m+n}$ (one of the laws of indices). To extend the notation and this law of indices for all integers, we need a^0 to be 1.

iii. Find out the origin of the decimal point. What notation was used before?

Other symbols are/were used in some countries. Do you know of any?
Read textbook pg 294, Historical Note for some useful information. Alternatively, refer to an extremely useful site provided by University of Melbourne:
http://online.edfac.unimelb.edu.au/485219/DecProj/backinfo/metric.htm

COUNTRY	*NOTATION*
Australia today	*12 345.67*
Australia pre 1970's	*12,345.67*
Italy, Argentina, Portugal, Vietnam	*12.345,67*
Philippines	*12345,67*
Greece	*12,345.67*
Hong Kong, Singapore	*12,345.67*
France	*12 345,67*
International system of units (SI units)	*12 345.67*

Base Five System

i. Thinking in the 'One-Hand System' on pg 130, what does the numeral 20_{five} represent?
20_{five} *represents two hands and no fingers or two groups of five and 0 units. Note that the 'One-Hand System' is equivalent to a base five system. So* $20_{five} = 2 \times 5 + 0 = 10$.

ii. How is 8 represented in the 'One-Hand System'? Write it in base five.
1 hand and 3 fingers ($8 = 13_{five}$*).*

ii. Continue the lists in Figure 3-4 (pg 130) for 4 more lines. Check that you write 30_{five} in your last line.

22_{five}	xxxxx	xxxxx	xx	2 hands and 2 fingers
23_{five}	xxxxx	xxxxx	xxx	2 hands and 3 fingers
24_{five}	xxxxx	xxxxx	xxxx	2 hands and 4 fingers
30_{five}	xxxxx	xxxxx	xxxxx	3 hands and 0 fingers

iv. How many symbols are used in base five? What are they?
5 symbols in base five. They are: 0,1,2,3,4.

v. Write out 44012_{fivc} in expanded form.
$44012_{fivc} = 4 x 5^4 + 4 x 5^3 + 0 x 5^2 + 1 x 5^1 + 2 x 5^0$

Number Representations in Various Bases

i. In the base seven system, you will not see the numerals 7, 8 or 9. Explain why this is so.
In base 7, the number seven will be expressed as 10_{seven} (1 group of seven and 0 ones) and so on.

ii. Without expressing both in base ten, decide which is larger, 1433_{seven} or 1433_{six}. Explain why.
1433_{seven} is larger. Though both have the same digits 1433, but each digit in the former expression (base seven) has a bigger place value than that in the latter (base six)

iii. Draw a number line for base six, marking down the first eighteen counting numbers.
The first eighteen numbers from left to right are:
1 ,2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 30
(each number should have a subscript "six" attached, since they are base six numbers)

iv. If the system is in base thirteen, is it possible to use the normal symbols 10, 11 and 12 to express ten, eleven and twelve respectively? Why or why not?

In base thirteen, the symbol"10" represents one group of thirteen and 0 ones. So we need a symbol, say T, to represent ten.

v. Explain why two single digit numbers in different bases carry the same value e.g. $7_{nine} = 7_{eight}$, $4_{eight} = 4_{seven} = 4_{five}$.
Because the single digit in each of these numbers has the same place value, namely 'one'. ($7_{nine} = 7 \times 9^0 = 7 \times 1$, $7_{eight} = 7 \times 8^0 = 7 \times 1$)

vi. Can one be chosen as a base? Why or why not?
If 1 is chosen as a base, then the only number we could represent in this system is zero.

<u>WARMING-UP ACTIVITY 2: Group Work and Discussion, a follow-up on Check Understanding, pg 6, Tutorial Notes</u>

i. Students are to be divided into groups of six and each group is to be given 50 sticks/straws (or any other counters). Each group is to
- choose a number between forty and fifty and choose a base b between 2 and 6 inclusive
- express the number in the chosen base b using the two methods taught, both working out the procedures on paper
- <u>Link the procedure step-by-step with the concrete materials provided.</u> Compare the two methods.

ii. A given number is expressed in base two and in base seven. Which expression would you expect to have more digits and why? Explain your answer to each other in your group.
A number written with smaller base will be expected to have more digits – consider examples and it will be clear.

iii. In the example on pg 5 of the Tutorial Notes, we see that 2107 expressed in base four is 200323_{four}. Are these two numbers the same number in that they have the same value although they look different? Explain your answer to each other in your group.
The two represent the same "value" or "number", although the "numerals" are different.

TUTORIAL ACTIVITIES

Activity 1: The Duodecimal System (50 min)

Answers:

Task 1:

The digits in base twelve are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E
The numbers are written as 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, E, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1T, 1E, 20_{twelve} where 1T, for example, is read as 'one-T base twelve'.
Number line: Starting from left to right, we have
9, T, E, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1T, 1E, 20, 21, 22_{twelve}
Changing each of the above numbers into base ten, we have
9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 24, 26

Task 2:

(a) $TE07_{twelve} = 10x12^3+11x12^2+\underline{0x12}+7 = 18871$ (don't miss out the term involving 0)
(b) Using the procedure of long division, we have

$12^2=144$	1428	9
12^1	132	11 (denoted by E)
	0	

Hence the base twelve representation is $9E0_{twelve}$ (not $9E_{twelve}$!)

Task 3:

Let the missing digit be a, so that the number is $8aE_{twelve}$. *Now in base ten representation, it is*

$$8\times 12^2+a\times 12+11 = 1163 + 12a = 1199.$$

Hence 12a = 36 and so a=3. Thus the missing digit is 3.

Task 4:

i. EEE_{twelve}

ii. ETE_{twelve} *and* $EE1_{twelve}$

iii $T12_{twelve} = 10\times 12^2+x12+2 = 14$. *So nothing wrong. (Twelve in base twelve is* 10_{twelve}*)*

iv. *Use the two different methods discussed in Warming-up Activity II as well as the following:*

$12^3=1728$	*1963*	*1*
$12^2=144$	*235*	*1*
$12^1=12$	*91*	*7*
7	*7*	

The base twelve representation is 1177_{twelve}

v. (a) $92E_{twelve} = 9 \times 12^2 + 2 \times 12 + 11 = 1331$

(b) $T0E_{twelve} = 10 \times 12^2 + 0 \times 12 + 11 = 1451$

(c) $111_{twelve} = 1 \times 12^2 +1 \times 12 + 1 = 157$

(d) $111_{twelve} = 1 \times 12^2 +1 \times 12 + 1 = 157$

(e) $12^5 +25 \times 12^4 +23 =12^5 +(2(12)+1) \times 12^4 +23$

Activity 2: Mind-Reading Card Game (a) & (b) (20 min)

		D	*C*	*B*	*A*
Number	*Binary Expression*	*8*	*4*	*2*	*1*
1	*1*				√
2	*10*			√	
3	*11*			√	√
4	*100*		√		
5	*101*		√		√
6	*110*		√	√	
7	*111*		√	√	√
8	*1000*	√			
9	*1001*	√			√
10	*1010*	√		√	
11	*1011*	√		√	√
12	*1100*	√	√		
13	*1101*	√	√		√
14	*1110*	√	√	√	
15	*1111*	√	√	√	√

(c) (Optional)

The next size up will be from 1 to 31, and altogether 5 cards will be needed. To increase to the next place value, the largest possible number will be 11111_{two}*, and that will be 31.*