On Real Roots of Flow Polynomials

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Abstract

This article studies the real roots of the flow polynomial $F(G, \lambda)$ of a bridgeless graph $G$. Let $W(G)$ be the set of vertices in $G$ of degrees larger than 3. For any integer $k \geq 0$, let $\xi_k$ be the largest real number in $(1, 2]$ such that $F(G, \lambda)$ has no real zeros in $(1, \xi_k)$ for all graphs $G$ with $|W(G)| \leq k$. We show that $\xi_k$ can be determined by considering a finite set of graphs and therefore deduce that $\xi_k = 2$ for $k \leq 2$, $\xi_3 = 1.430 \cdots$ and $\xi_4 = 1.361 \cdots$. We also show that for any bridgeless graph $G$, if $W(G)$ is dominated by some component of $G - W(G)$, then $F(G, \lambda)$ has no roots in $(1, 2)$. This result implies that $F(G, \lambda)$ has no zeros in $(1, 2)$ whenever $|W(G)| \leq 2$.

Keywords: matroid, graph, characteristic polynomial, chromatic polynomial, flow polynomial, root

1 Introduction

Following Tutte [14] (also see Brylawski and Oxley [2]), the flow polynomial of a graph $G = (V, E)$ is the polynomial defined as

$$F(G, \lambda) = \sum_{E' \subseteq E} (-1)^{|E| - |E'|} \lambda^{|E'| + c(E') - |V|},$$

where $c(E')$ is the number of components of the spanning subgraph $(V, E')$ of $G$. This definition is equivalent to the following basic properties of $F(G, \lambda)$ (see [14] also):

$$F(G, \lambda) = \begin{cases} 1, & \text{if } E = \emptyset; \\ F(G_1, \lambda)F(G_2, \lambda), & \text{if } G = G_1 \cup G_2; \\ 0, & \text{if } G \text{ has a bridge}; \\ (t - 1)F(G - e, \lambda), & \text{if } e \text{ is a loop}; \\ F(G/e, \lambda) - F(G - e, \lambda), & \text{otherwise}, \end{cases}$$

where $G/e$ and $G - e$ are the graphs obtained from $G$ by contracting $e$ and deleting $e$ respectively, and $G_1 \cup G_2$ is the disjoint union of graphs $G_1$ and $G_2$. 

*Partially supported by NIE AcRf funding (RI 5/06 DFM) of Singapore.
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