Technical Report M2010-02
September 2010
Mathematics and Mathematics Education
National Institute of Education
Singapore

When exactly is Scott sober?

Weng Kin Ho, Dongsheng Zhao
Mathematics and Mathematics Education, National Institute of Education,
Nanyang Technological University, Singapore
{wengkin.ho,dongsheng.zhao}@nie.edu.sg

September 23, 2010

Abstract

A topological space is sober if every nonempty irreducible closed set is the closure of a unique singleton set. Sobriety is precisely the topological property that allows one to recover completely a topological space from its frame of opens. Because every Hausdorff space is sober, sobriety is an overt, and hence unnamed, notion. Even in non-Hausdorff settings, sober spaces abound. A well-known instance of a sober space appears in domain theory: the Scott topology of a continuous dcpo is sober. The converse is false as witnessed by two counterexamples constructed in the early 1980's: the first by P.T. Johnstone and the second (a complete lattice) by J. Isbell. Since then, there has been limited progress in the quest for an order-theoretic characterization of those dcpo's for which their Scott topology is sober. This paper provides one answer to this open problem.

Subject classification: 06B30, 06B35, 06F30, 68Q55
Keywords: Scott topology, sober space, dcpo, dominated dcpo, H-continuous, H-algebraic, H-compact, $\flat$-stable, strongly H-algebraic

1 Introduction

A subset of a topological space is irreducible if it is not the union of two proper closed subsets. A topological space is sober if the singleton closures are the only nonempty irreducible closed sets. In the theory of Hausdorff topological spaces, the notion of sobriety is overtly invisible since every Hausdorff (also known as T_2) space is sober. In turn, every sober space is T_0. With regards to