Course Descriptions

Core Course

MSM900 Mathematical Research Methods

Rationale. We identify that it is important for our graduate students in mathematics to be equipped with the 21st century life skills. For mathematics students, these skills include their abilities to think about mathematics critically, solve mathematical problems, read and write mathematical arguments, communicate effectively these solutions, collaborate with others to solve problems and exploit Information Technology in research. Collectively, these skills are the Mathematical Research skills any proficient mathematician should possess. Above all, we recognize that it is the most important for students to acquire all these skills independently. Therefore, the rationale of this course is to immerse our graduate students in intentionally constructed learning experiences will which promote academic independence in acquiring and mastering the aforementioned Mathematical Research skills.

Aims and objectives. This course aims to equip students with Mathematical Research skills through a methods-based approach – hence, the title "Mathematical Research Methods" – with the belief that the mathematics students should be equipped with a set of research methods that are directly relevant to conducting research in mathematics.
By immersing graduate mathematics students in learning experiences that focus on the disciplinarity of mathematics, i.e., to think and behave as a mathematician, the objective of this course is to equip students with the abilities to:

(i) acquire mathematical knowledge and skills independently;
(ii) solve mathematical problems independently;
(iii) communicate mathematical ideas clearly, and to collaborate with others in mathematical research; and harness Information Technology in mathematical research.

Syllabus. This course equips students with fundamental skills needed for initiating a mathematical research. Together with knowledge gained in other courses, this course is a pre-requisite of the Elective Course MSM960 Mathematical Inquiry. It is also an ideal, though not mandatory, course for students who are pursuing Master of Science by Research or PhD (Mathematics).

These core Mathematical Research skills include:

1. Reading mathematical texts: making sense of definitions, giving examples and counterexamples, understanding the structure of theorem statements, understanding proof methods, constructing original proofs;
2. Performing literature search and review;
3. Formulating research questions and future directions;
4. Using computation for sense-making in mathematics and verifying instances of conjectures and mathematical statements;
5. Applying problem solving strategies and heuristics to give first few attempts at solving some of the proposed questions;
6. Carrying out a mathematical discourse with a mathematician or peer;
7. Using LaTeX for typing mathematical documents and presentation slides, and citation skills.

Elective Course (Foundation)

MSM910 Calculus and Analysis for Educators

Rationale. Calculus is the mathematical study of continuous change. Its importance is witnessed by its ubiquitous uses in science, engineering and economics. Whence, Calculus has long become part of modern mathematics education – a course in calculus is a gateway to more advanced courses in mathematics. In Singapore secondary schools and junior colleges, both differential and integral calculus are topics in the Additional Mathematics and H2 Mathematics syllabi. While learners of calculus, at this level, are only expected to be conversant with the algebraic processes of differentiation and integration and some simple applications, their teachers must possess a deep understanding of the two fundamental principles that underpin the central ideas of calculus, that of *functions* and *limits*. The rationale of this course is to equip teachers of calculus with rigorous understanding of:

1. functions and their properties;
2. concept of limits;
3. concepts of differentiation and integration;
4. applications of differential and integral calculus based on first principles, e.g., maximization/minimization problems, elemental strip, disc and shell methods, simple ordinary differential equations.

Aims and objectives. This course aims to equip calculus teachers with a rigorous understanding of important pre-calculus and calculus concepts by making an intentional link with those concepts taught in school calculus

Syllabus. Topics covered may include:

1. Sequence and Limits of a sequence
2. Recurrence relations
3. Series, Method of Summation
4. Convergence of Series, Sequence and series of variables (functions).
5. Maclaurin's series, Approximation of functions by polynomials.
6. Limits and continuity of functions.
7. Differentiation.
8. Rolle's Theorem, Mean Value Theorem,
9. Definite integral as a limit of sum.
10. Fundamental Theorem of Calculus.
11. Indefinite integrals.
12. Applications of differential and integral calculus.

MSM911 Ring Theory for Educators

Rationale. Algebra is one of the broad parts of mathematics, together with number theory, geometry and analysis. The central spirit of algebra is the use of mathematical symbols and rules for manipulating them. Thus, in its most general form, algebra becomes a unifying tool in most fields of mathematics. For Singapore Mathematics curriculum, algebra is taught to students in its most elementary form progressively from as early as in Primary Six through the solution of polynomial equations in the Secondary to obtaining the solution set of a system of linear equations using matrices at the Pre-university levels. In order to understand and appreciate the deeper structural meaning of school algebra, teachers of algebra must be equipped with the knowledge of algebra at an even higher level of abstraction. This is precisely where abstract algebra fits into the picture.

Aims and objectives. This course is intended for educators who have never had a course in modern abstract algebra. The set of integers, rational numbers, polynomials and matrices, which are mathematical entities studied in school mathematics, are concrete examples of rings with respect to the operations of addition and multiplication. This course will help high school educators to have an in-depth conceptual understanding of some topics in school mathematics such as number systems, polynomials, from an advanced and structural perspective of abstract algebraic systems.

Syllabus. The topics covered may include:

1. Basic propositional logic; methods of proofs
2. Axiomatic definition of integers; Well-ordering Principle and the Principle of Mathematical Induction
3. Congruence and congruence classes of integers
4. The ring $\mathbb{Z}_n$ and its basic properties
5. Definition and examples of rings, including commutative rings and rings with identity, basic properties of rings, units and zero divisors in a ring, integral domains, fields (only definitions and examples), subrings
6. Isomorphism and homomorphism of rings, examples of isomorphic rings, properties of homomorphism, properties of rings preserved under isomorphism
7. Rings of polynomials over a field F, division algorithm in $F[x]$, Remainder Theorem and Factor Theorem, greatest common divisor, Euclidean algorithm
8. Irreducible polynomials, unique factorization in $F[x]$, Factorizations in $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$, Gauss Lemma, Eisenstein's Criterion
9. Congruence and congruence classes of polynomials over a field, congruence class arithmetic, the ring of congruence classes of polynomials $F[x]/\langle p(x)\rangle$, structure of this ring when $p(x)$ is irreducible in $F[x]$.
10. Ideals, principal ideals, quotient rings, concrete examples of quotient rings, kernel and image of homomorphism, First Isomorphism Theorem

MSM912 Discrete Mathematics for Educators

Rationale. Discrete Mathematics is a branch of mathematics which deals with finite or countable elements or processes. Discrete mathematics, together with calculus and abstract algebra, is one of the core components of mathematics at the undergraduate level. The mathematics of modern computer science is built almost entirely on discrete mathematics, in particular, combinatorics and graph theory. Discrete mathematics, in particular counting and probability, allows students to explore non-trivial "real world" problems that are challenging and interesting. Even students at A-level are exposed to basic counting principles and combinatorics; and thus, it is essential that mathematics teachers have a firm grounding in Discrete Mathematics.

Aims and objectives. This course aims to expose mathematics educators to counting principles which will enhance their pedagogical content knowledge of teaching permutations and combinations, as well as the elementary probability. Additionally, this course introduces a useful branch of discrete mathematics called graph theory which has many applications in modelling real-life contexts.

Syllabus. This course consists of two parts.

Part A (Counting – Its Principles and Techniques) covers:

1. addition principle,
2. multiplication principle,
3. divisors of natural numbers,
4. subsets and arrangements,
5. bijection principle,
6. binomial expansion, Pascal's triangle,
7. principle of inclusion and exclusion.

Part B (Graph Theory) covers:

1. mathematical modelling using graphs,
2. foundation of graph theory,
3. handshaking Lemma,
4. isomorphism,
5. subgraphs and
6. self-complementary graphs.

MSM913 Computing and Programming Techniques for Educators

Rationale. One of the most called-for 21st Century Skills is the ability to code, i.e., to script computer programs to handle computations and data of all kinds. Unlike the past century, computational thinking -- the mode of thinking that enables one to think how a computer program works -- weighs far more than the ability to invoke a computer application. Writing computer programs to do specified jobs is not just a science but it is an art to be mastered by most in the future. As professionals preparing for the next generation, it is more urgent than ever that teachers must equip themselves with this crucial life-skill. The ability to write computer programs requires active problem solving and compartmentalizing, water-tight logical reasoning, meticulous planning, and careful program verification. In essence, these are all mathematics-related skills. Viewed as a natural and useful application of mathematics, computing and programming techniques ought to be an integral part of a graduate mathematics course, and especially so for mathematics educators.

Aims and objectives. The aim of this course is to provide an introduction to programming using a common programming language such as Haskell, Excel VBA, C, FORTRAN or advanced CAS like Maple or Mathematica. The focus will be on writing computer programs for mathematical computations.

Syllabus. Topics may include some of the following:

1. computer basics,
2. statements,
3. control flow and structures,
4. arrays, lists,
5. functions and subroutines,
6. recursive techniques,
7. testing and debugging.

MSM914 Statistical Theory for Educators

Rationale. Questions: For a normal population, when σ in $\frac{\bar{X}-\mu}{\sigma}$ is replaced by *S*, why do we get a *t*-distribution? What is *t*-distribution in the first place? Why is the degree of freedom $n-1$ (and not *n*)? For a non-normal population, when σ is replaced by *S* from a large sample, can we still say that by the Central Limit Theorem,

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim N(0,1) \text{ approximately?}$$

These are some questions that a statistics educator may ponder when teaching *z*-test and *t*-test. Answers to these and other higher-order questions require a good understanding of the underlying statistical theory, and this course can help to bridge the gap in understanding.

Aims and objectives. This course aims to develop an understanding of the statistical theory underlying important key concepts in the school probability and statistics curriculum, and extend the knowledge beyond school level in both theory and applications. Topics will be selected from exploratory data analysis, probability distributions, statistical inference and simple regression.

Syllabus. Some of the following topics are covered:

1. Exploratory Data Analysis

- Line and scatter plots
- Box-and-whisker plots
- Empirical and Chebychev's rules
- Introduction to R (or equivalent software): R as a graphical tool

2. Probability Distributions

- Discrete distributions

  (e.g., relation between geometric and negative binomial)
- Continuous distributions (e.g., relation between *t* and χ^2)
- Moment generating functions
- Convergence in distribution: Slutsky theorem, simulation
- Central Limit Theorem: Proof and simulation

3. Statistical Inference

- Point and interval estimation
- Confidence intervals: Theory and simulation
- Hypothesis testing: Theory and relation to confidence interval

4. Simple Regression

- Least squares estimation
- Confidence intervals and testing of slope and intercept

Elective courses (Advanced)

MSM921 Real Analysis

Rationale. The rationale of this real analysis course is solely for deepening the understanding of the concepts of continuity and integrability of functions and their connections via measure theory. Extending from a typical foundational course on Calculus calls for an in-depth study of the property of continuity of functions in relation to the sets inhabiting in the real-line. The technicalities of continuity, uniform continuity, Lipchitz continuity all center around the Euclidean topology on the real line, i.e., roughly speaking, the structure of the open intervals of the real line. More precisely, a function is continuous if the inverse image of an Euclidean open is still Euclidean open. Another important area of in-depth study is that of the Riemann integral. The technicalities of this Riemann integrability center around the concept of measure. Like in the case of the Euclidean topology on the real line, the salient subsets in measure theory are exactly the measurable sets. The parallel notion of continuous functions in measure theory is that of measurable functions, i.e., a function is measurable if the inverse image of a measurable set is still measurable.

Aims and objectives. This course aims to give an in-depth treatment of functions, touching on the basic Euclidean topology of the real line, its connection with the concept of continuity, uniform continuity, and different variants of continuity. It also introduces measure theory is a fundamental study of the aspects of integration of real-valued functions. The most basic concept of integrability, i.e., Riemann integrability, will be studied thoroughly, and its connection with sets of measure zero will be made explicit. Parallel to the concept of continuous functions is that of measurable functions. The course brings the students deeper to the core of integration theory via the measure-theoretic approach. The ultimate learning objective will be that the student is able to look at calculus concepts learnt and taught in schools at a higher vantage point.

Syllabus. Some of the following topics are covered:

1. Continuity of functions

- Basic Euclidean topology on the real line
- Definition of functional continuity via Euclidean opens
- Properties of continuous functions
- Uniform continuity, Lipchitz continuity, etc.

2. Riemann integrability

- Partition, upper and lower Riemann sums
- Upper and lower Darboux integrals
- Equivalence of Riemann integrability and Darboux integrability

3. Lebesgue outer measure

- Introduction to outer measure
- Sets of measure zero
- Characterization of Riemann integrability in terms of sets of measure zero, i.e., a function is R-integrable if and only if the points of discontinuity of the function is of measure zero.
- Further properties of Riemann integrable functions

4. Measure theory

- Sigma-algebra of sets

- Measure: non-negativity, sub-additivity, etc.
- Measurable sets and measurable functions
- Properties of measurable functions

5. Convergence Theorems
6. Other notions of integrability
    - Lebesgue integrable functions
    - Henstock integrable functions

MSM922 Theory and Applications of Differential Equations

Rationale. Differential Equation find its application in many real world phenomena. It is an important tool for mathematical modelling, which is the trend of mathematics education worldwide today. Hence the inclusion of this module is extremely important. The course will cover both the analytic and qualitative study of the various types of differential equations (selected from Ordinary Differential Equation, Partial Differential Equation, Stochastic differential equations, and Functional Differential Equations). This course will also introduce the application of differential equations in real-world, modelling and dynamical systems.

Aims and objectives. This course will consist of both theoretical treatment as well as practical applications of various classes and types of differential equations.

Syllabus. Topics may include:

1. Analytic solutions
2. Numerical methods and qualitative analysis for differential equations (selected from ordinary, partial, stochastic or functional differential equations)
3. Systems of differential equations.
4. Applications in mathematical modelling, dynamical systems.

MSM923 Topology

Rationale. Topology is concerned with the properties of spaces that are preserved under continuous deformations, such as stretching, crumpling, bending but not tearing or gluing. The technicality of topology exploits naïve set theory in that it considers a particular kind of collection of subsets of the underlying space, called the open sets, with the special feature that it is closed under arbitrary union and finite intersections. Topologists are interested in properties that are maintained under the image or inverse image of continuous mappings, e.g., the continuous image of a compact set is compact; the continuous image of a connected set is connected; and the inverse continuous image of a closed set is closed.

Topology has many applications. Two notable ones are worth mentioning. The first is domain theory which can be seen as topology on ordered structures. In this setting, the topology of concern is the Scott topology which is generally non-Hausdorff, and can be applied to manufacture denotational models for functional programming languages. The other, more recent application, is in Physics, where Nobel Laureates, Kosterilitz and Thousless made use of topology to study and explain unusual phases or states of matter, such as superconductors, superfluids and thin magnetic films.

This course is an introductory one for mathematics educators, creating an awareness for advanced mathematics and their applications.

Aims and objectives. This course introduces point-set topology, staring with metric spaces and ending up with non-Hausdorff topologies, e.g., Alexdandroff, Scott and upper topologies on partial orders. A deeper understanding of continuous functions of real variables is made possible by making an abstraction of the underlying space, first in terms of structure of metric distance, and then in terms of the collection of open sets. A new perspective of traditional calculus theorems is obtained through the topological lens. A key example of this is the proof of the Intermediate Value Theorem which relies crucially on the fact that the continuous image of a connected set is connected, and the continuous image of a compact set is compact. The main learning objective is for mathematics educators to gain a deeper insight into familiar phenomena encountered in calculus courses at a higher level.

Syllabus. Topics may include:

1. Metric spaces
2. Topological spaces
3. Continuous functions
4. Separation axioms and countability axioms
5. Compactness and connectedness
6. Non-Hausdorff topologies
7. Introduction to domain theory

MSM924 Euclidean and non-Euclidean Geometry

Rationale. Geometry is one of the most fundamental and important topics in mathematics. The modern Euclidean geometry was built as an axiomatic system. Most learners of geometry do not have the opportunity to learn geometry from the axiomatic approach and as a result, they do not have a clear view of the hierarchical structure of geometry. Because of this, they do not know the correct definitions of many of the basic geometric concepts (they may take an equivalent condition as the definition) and are not clear which results/theorems are dependent on which other results/theorems which sometimes lead to circular reasoning. This course will present a complete rigorous axiomatic system of Euclidean plane geometry and present the rigorous definitions of all the fundamental geometric concepts. The proofs of the fundamental theorems, equivalence of triangle congruency/similarity tests will be given. Non-Euclidean geometries will also be briefly introduced

Aims and objectives. This course presents a complete axiomatic system for Euclidean geometry. By taking this course, students will gain a clear picture of the whole hierarchical structure of geometry. They will learn the rigorous definitions of the fundamental geometry concepts, such as angles, triangles, rays, congruent/similar triangles. They will also learn the formal proofs of the fundamental results in geometry, such as the equivalence of various different triangle congruency (similarity) tests, Angle Sum Theorem and Exterior Angle Theorem as well as the Midpoint theorem. The course will also cover briefly the non-Euclidean geometries so that students can see the major difference among the different types of geometries.

Syllabus. The topics which may be covered are:

1. General axiomatic systems
2. Neutral geometry
3. Euclidean parallel postulation
4. Triangles; plane separation axiom
5. Angle measurement
6. Congruent triangles and congruency tests
7. Similar triangles and similarity tests; the Fundamental Theorem on similar triangles
8. Hyperbolic geometry
9. Transformational geometry; introduction to projective geometry

MSM925 Contemporary Topics in Analysis, Geometry and Topology

Rationale. The advancement of Mathematics, whether Pure or Applied, is fast-paced and constantly taking place. Mathematics educators in particular must keep themselves abreast with the recent development in both the theory and application of modern mathematics. Because modern mathematics is multi-faceted and often inter-disciplinary, a course covering contemporary topics in Analysis, Geometry and topology is ever so important in conveying this very nature of mathematics in relation to other disciplines, e.g., computer science, economics, financial mathematics, biology, physics, chemistry, engineering, etc. The significance of this course is to bring to attention that Mathematics, the Queen of all sciences, is still as relevant in the present as it was in the past; even more so it holds great promises for the future.

Aims and objectives. This course creates the golden opportunity for specialists in the more exoteric topics or fields of mathematics to share with mathematics educators the cutting edge developments and applications of modern mathematics, with particular focus in the areas related to analysis, geometry and/or topology. It is aimed that mathematics educators have a glimpse of interesting real-life applications of certain fields of mathematics that do not fit exactly into any of existing courses within the strand.

Syllabus. The topics covered any contemporary topics within Analysis, Geometry and/or Topology:

1. Fractal geometry
2. Differential geometry
3. Econometrics and financial mathematics
4. Game theory and its applications
5. Domain theory and denotational semantics
6. Functional analysis and its applications

MSM931 Number Theory

Rationale. The integers are the most fundamental mathematical objects encountered in school mathematics. Students are taught to assume important properties like the Fundamental Theorem of Arithmetic or the infinitude of primes. It is important that their teachers know why such results are true and understand these fundamental concepts from a higher standpoint.

Number theory also has many applications that impact our everyday lives. For example, check digits are implemented in our National Registration Identity Card (NRIC) numbers as well as credit card numbers; cryptography is used to secure our online transactions. Teachers who are aware of such applications can better bring across the importance of mathematics to their students.

Aims and objectives. The course aims to expose mathematics educators to a rigorous development of elementary number theory. Many concepts and properties of integers that are currently taught in schools will be revisited from a higher standpoint. For example, a complete proof of the Fundamental Theorem of Arithmetic will be discussed. More advanced topics like representations as sums of squares, or partitions of integers will also be included to provide educators a broad view of number theory.

Syllabus. The topics may include:

1. Divisibility, Division Theorem, Greatest Common Divisor, Least Common Multiple;
2. Linear Diophantine Equations in Two Variables, Extended Euclidean Algorithm, Euclid's Lemma;
3. Infinitude of Primes, Fundamental Theorem of Arithmetic, Applications of Unique Factorization;
4. Linear Congruences, Chinese Remainder Theorem, Application of Congruences to Divisibility Tests and Check Digits;
5. Fermat's Little Theorem, Euler's Phi function, RSA Encryption;
6. Quadratic Reciprocity;
7. Representations as Sums of Squares;
8. Partition of Integers;

MSM932 Commutative Ring Theory

Rationale. Commutative Ring Theory can be seen as a course for deepening the knowledge about algebraic structures encountered in an introductory algebra course by zooming into the theory of commutative rings and modules over these.

Aims and objectives. This course aims at a further deepening of the knowledge about algebraic structures which have been acquired from the foundation level course on algebra. The area of specialization that we focus in this course is that of commutative ring theory. Commutative ring theory investigates commutative rings, their ideals and modules over such rings. Prominent examples of commutative rings include polynomial rings, rings of algebraic integers which include ordinary integers, *p*-adic integers, etc.

The main learning objective is for mathematics teachers to see algebra from a modern mathematical perspective at a higher vantage point.

Syllabus. The topics may include:

1. Ideals of rings: maximal ideals, prime ideals, radical ideals, Jacobson radicals
2. Unique Factorization Domains
3. R-modules and module homomorphisms
4. Localization (at prime ideals); local rings
5. Spectrum of a ring and Zariski topology; Dedekind Domains; Hilbert's Nullstellensatz
6. Hom and tensor product; exact sequences
7. Projective modules, free modules
8. Finitely generated modules, Nakayama's Lemma;
9. Noetherian and Artinian rings

MSM933 Topics in Applied Algebra

Rationale. Undergraduate algebra course typically focus on theoretical developments and very few students have the opportunity to see the links between the theory and practical world in a systematic way. Knowing the applied part of algebra would deepen students' understanding of the theoretical part and motivate students' active learning. For mathematics educators this is particular important. This course will discuss the practical applications of algebra in many different areas.

Aims and objectives.

This course intends to provide students with the opportunities to learn the applications of algebra in various different areas. By completing this course, students will see how the algebra theory they learned in their undergraduate studies can be used to solve different types of practical problems. For mathematics educators, such applications can be used to motivate their students' learning and design suitable project work topics for students.

Syllabus. The topics may include:

1. Constructing curves and surfaces through given points
2. The assignment problem
3. Games of strategy
4. Leontief Economic Models
5. Fractals
6. Chaos
7. Cryptography
8. Genetics
9. Wavelets
10. Computer graphics and computed Tomographics
11. Geometric linear programming

MSM934 Group Theory

Rationale. If you wonder what is the mathematics behind the rubik's cube, then you should look no further than the abstract mathematics of group theory. In group theory, one studies the algebraic structure called groups which realize the idea of transformation by means of only one operation! Indeed the primary example of a group is that of a permutation group. Furthermore, Cayley exhibited any finite group as a permutation group via the notion of group action – the earliest form of representation theory in groups. Groups are applied within mathematics, e.g., in Galois theory, in algebraic topology, etc, and physical sciences such as physics, chemistry and material science, and in statistical mechanics.

Aims and objectives. This course demonstrates that even with one algebraic operation, it is still possible to develop a great deal of algebraic results that are impactful to other fields of mathematics as well as other disciplines. Any proficient mathematics educator who claims to have a decent mathematics education at the graduate level must have encountered groups and their manifestation in different areas of mathematics. The learning objective of this course is that the student will be acquainted with one of the oldest branch of modern algebra, and at the same time, be amazed at the number of real-life applications group theory has to offer.

Syllabus. The topics covered include:

1. Group axioms; subgroups
2. Group isomorphisms, and automorphisms
3. Cyclic groups; normal subgroups; conjugacy classes
4. Three isomorphism theorems
5. Permutation groups; group actions
6. Sylow's theorems and their applications

MSM935 Contemporary Topics in Algebra and Number Theory

Rationale. The advancement of Mathematics, whether Pure or Applied, is fast-paced and constantly taking place. Mathematics educators in particular must keep themselves abreast with the recent development in both the theory and application of modern mathematics. Because modern mathematics is multi-faceted and often inter-disciplinary, a course covering contemporary topics in Algebra and Number Theory is ever so important in conveying this very nature of mathematics in relation to other disciplines, e.g., computer science, economics, financial mathematics, biology, physics, chemistry, engineering, etc. The significance of this course is to bring to attention that Mathematics, the Queen of all sciences, is still as relevant in the present as it was in the past; even more it holds great promises for the future.

Aims and objectives. There are many topics of Algebra and/or Number Theory that are so varied and colorful that they cannot all fit into the existing courses in this programme. To allow greater flexibility and to offer more interesting learning opportunities, this course is aimed at offering something different. Esoteric, and more advanced topics in Algebra and/or Number Theory, are offered in this course, e.g., coding theory, algebraic number theory, analytic number theory, category theory, etc. It is aimed that mathematics educators have a glimpse of interesting real-life applications of advanced mathematics that do not fit exactly into any of existing courses within the strand.

Syllabus. Topics that may be covered are:

1. Algebraic number theory
2. Analytic number theory
3. Coding theory
4. Category theory
5. Non-commutative and/or non-associative algebra

MSM941 Selected Topics in Graph Theory

Rationale. Graph Theory is an area in Discrete Mathematics which studies configurations involving a set V and set E, where each member of E is a pair of members in V. Graph Theory and its applications can be found not only in other areas of Mathematics, but also in scientific disciplines such as engineering, computer science, operational research, management sciences and life sciences. It has become an essential and powerful tool for engineers and applied scientists, especially, in the area of designing and analyzing algorithms for various problems.

Aims and objectives. This course is aimed at providing the mathematics educators an opportunity for a more in-depth study of graphs as a mathematical object. This deepening is enabled by an introduction, followed by more advanced graph theoretic concepts.

Syllabus. Topics may include:

1. Bipartite graphs, trees, spanning trees;
2. Vertex colouring, Brook's Theorem, greedy colouring algorithm, critical graphs and cliques, perfect graphs;
3. Planar graphs, vertex colouring of plane graphs, Four colour problem;
4. Matchings in bipartite graphs, Hall's Theorem, System of distinct representatives, Maximum matchings and perfect matchings, independence, coverings and Gallai identities;
5. Dominating sets, domination number, independent domination, Roman domination.

MSM942 Algorithms and Applications in Graph Theory

Rationale. An algorithm is any well-defined computational procedure that takes some value, or set of values, as input and produces some value, or set of values as output. Algorithms are very important in most scientific disciplines, especially in computer sciences. Algorithms are widely applied in graph theory to search for some special subgraphs or to determine some parameters of graphs or to be used for proving some open problems.

Aims and objectives. This course is aimed at providing mathematics educators a first encounter with the notion of algorithms from their most generic form to the more specific one applied in graph theory. Real-life applications of graphs will be introduced in this course, hence providing mathematics educators opportunities to be acquainted with problems in real-life contexts.

Syllabus. Topics may include:

1. Algorithm, complexity, NP-hard problems, NP-completeness
2. Algorithms in searching for spanning trees of connected graphs and solving the traveling salesman problem and the Chinese postman problem;
3. Digraphs, basic concepts, connectivity, tournaments, orientations
4. Structures of shortest paths in directed graphs, Algorithms for finding distances, Minimum diameter orientations, Kings in digraphs, Applications

MSM951 Numerical Mathematics and Applications

Rationale. Computers and computational thinking are becoming increasingly important in all aspects of our modern world. Consequently, the use of numerical methods in solving problems in mathematics, as well as in other disciplines such as the sciences, engineering, medicine and social sciences, has also gained popularity and importance. A course on numerical mathematics is an excellent platform to develop the relevant knowledge and skills required to keep students up to date with such methods of tackling problems in a modern digital world. The inclusion of using a computing tool in this course will not only facilitate the learning and practice of the skills involved, but also provide the opportunity for coding skills to be taught and used in problem solving.

Apart from practical applications, the theoretical and conceptual considerations in numerical methods, including areas such as analysis of errors, are also critical aspects that can lead to development in computational thinking.

Aims and objectives. This course aims to equip students with the understanding of the processes and skills involved in solving problems using numerical mathematics, and to provide students with the opportunity to apply these techniques in various contexts and situations.

Syllabus. Topics may include:

Topics may include:

1. Computer arithmetic, Number representation and errors;
2. Polynomial and function approximations; Approximation theory;
3. Numerical differentiation and numerical integration;
4. Numerical methods for ordinary differential equations;
5. Numerical methods for partial differential equations;
6. Monte Carlo methods and simulation;
7. Particle Swarm Optimisation techniques.

In addition, a computing tool will be introduced and used to support the learning of the above topics.

MSM952 Large Scale Systems in Operations Research

Rationale. The problems which face us in the modern world are of an increasingly complex nature, where we are attempting to achieve multiple objectives simultaneously, and where the constraints under which we are operating are of many different kinds.

The simple techniques typically taught in undergraduate courses in Operations Research, where a single objective function is optimized under constraints of a single type, are inadequate for solving problems with this kind of complexity.

Educational systems are a particularly rich source of complex problems of this kind, and educators with a strong background in mathematics are increasingly being asked to step up into management roles to tackle such large scale problems.

Aims and objectives. Participants will learn how to combine elementary Operations Research techniques into powerful tools capable of solving problems with multiple objectives, and multiple constraint types.

Participants will then learn how to apply these powerful tools to solve the kind of large scale real world problems of a practical type which face us in our increasingly complex modern society.

Participants will see educational systems facing many problems of this type, and by learning how to solve them will enhance their professional contribution to their own educational institutions.

Syllabus.

Content will be drawn from the following topics:

1. Integer Linear Programming: primal, dual simplex algorithms, branch and bound algorithms and branch and cut algorithms, application to mixed-integer and zero-one linear programming.
2. Stochastic Processes: modeling of random events, birth and death models, application to inventory systems, simple queueing systems, multi-server queues and general queueing networks.
3. Scheduling and Sequencing: critical path analysis in project management, application to flow shop and job shop scheduling, timetabling algorithms, bin packing and knapsack problems.
4. Network Optimization: minimal spanning trees, shortest paths, maximal flows, applications to least cost flows, transportation algorithms, travelling salesman and vehicle routing problems.
5. Dynamic Programming: recursive solution of problems with optimal substructure, Bellman's Equation, resource allocation, manpower and equipment planning, non-linear programming.
6. Simulation: computer modeling of complex systems, Monte- Carlo methods, non-deterministic and non-standard probability distributions, discrete event simulation and systems dynamics.

MSM953 Contemporary Topics in Applied Mathematics

Rationale. The rapid development of technology in recent decades has seen mathematics applied to solve entirely new problems, which in turn has driven the development of new solution techniques. While some of these techniques are entirely novel, the most powerful new techniques often result from taking the essential ideas behind classical techniques and then applying them in these new contexts. Mathematics educators seeking to update their knowledge of and teaching in applied mathematics must therefore develop a critical perspective, blending mastery of existing topics with the advancement of entirely new topics.

Aims and objectives. Participants will be given an overview of contemporary problems in applied mathematics which are driving the development of new techniques. Participants will learn that such techniques can be entirely novel, but that the most powerful are the result of applying the essential ideas behind classical techniques in new contexts. Participants will therefore develop a critical perspective on the blending of new topics with a renewed appreciation of existing topics, to enhance their professional contribution to both teaching and curriculum development in applied mathematics.

Syllabus.

Content will be drawn from the following thematic strands:

1. Transforms and Mathematical Analysis: special functions in classical and contemporary mathematical analysis, classical integral transforms and their modern generalizations, the role of symmetry in the solution of equations.
2. Mathematical Optimization: constrained optimization, convex and non-linear programming, Lagrange multipliers and the calculus of variations, steepest descent, quasi-Newtonian search methods, stochastic programming.
3. Stochastic Processes: concepts of randomness, random number generators, Monte-Carlo methods and simulation, random walks and diffusion, mathematical probability theory, dynamical systems and chaos theory.
4. Modern Information Processing: coding and cryptography, data compression and image processing, wireless and optical communication technologies, computational imaging, maximum likelihood methods, compressed sensing.

MSM954 Models of Computation

Rationale. What constitutes a computation? Alan Turing has a simple answer: take a piece of (infinitely long) paper called a tape, a pencil to write only a finite number of symbols within cells, move the tape forward and backward, erase symbols and re-write symbols – all done within a finite number of “mental” states. This instance of realizing a computation is an example of a computational model – called a Turing Machine. This prototypical model allows one to concretely study many aspects of computation, problems of computability (what can be computed and what not), problems of computational complexity (how fast can a computation be done) and most importantly, the semantics of computation (what is the meaning of the computation that is taking place). This course allows the mathematics educator to ponder what computation is all about, and the philosophy behind computation – computational thinking in its most abstract form.

Aims and objectives. This course presents what has been taught and learnt at the foundation level about computing and programming techniques at an abstract but yet transparent level. A model of computation is the definition of the set of allowable operations used in computation and their respective costs. It is an extremely important course for mathematics educators who wish to explore the topic of computation at an advanced level.

Syllabus. Topics may include:

1. Finite automata; finite state machines;
2. Turing machines;
3. Recursive functions;
4. Recursively enumerable sets;
5. Lambda Calculus; PFC; FPC; Haskell;
6. Denotational and operational semantics

MSM961 Multiple Linear Regression

Rationale. Multiple regression has been widely used in many disciplines. It provides a powerful tool to examine how multiple independent variables are related to a dependent variable and uses the obtained information to make predictions on new data. Successful applications of the methods require a sound understanding of the theory. This course helps our graduate students to develop the ability to interpret the results and critically evaluate the methods used, which is of paramount importance for advanced level of statistics learning.

Aims and objectives. This course aims to provide a good understanding of the concepts and methods of multiple regression at graduate level of statistics learning. The course will start with the extensions of univariate techniques to multivariate framework, such as multivariate normal distribution and simultaneous confidence intervals, followed by the presentation of statistical results for multiple regression in matrix form and the development of estimation, inferences and residual analysis in multiple regression analysis. Computations will be implemented in R or equivalent statistical software.

Syllabus. The following topics may be included:

1. Simple linear regression
   - Least squares estimation
   - Inferences under normality
   - Maximum likelihood estimation
   - Analysis of variance approach
   - Correlation
2. Simultaneous Inference
   - Joint estimation of slope and intercept
   - Simultaneous estimation of mean responses
   - Simultaneous prediction intervals for new observations
3. Diagnostic and remedial measures
   - Properties of residuals
   - Diagnostic for residuals
   - Departure from normality
   - Transformation
4. Simple linear regression with matrices
   - Random vectors and matrices
   - Matrix approach to simple linear regression analysis
5. Multiple regression I
   - Multiple linear regression model
   - Estimation of regression coefficients
   - Analysis of variance results
   - Inferences about regression parameters
   - Estimation of mean response and prediction of new observation
   - Diagnostics and remedial measures
6. Multiple regression II
   - Test of linear hypotheses
   - Least squares estimation with linear constraints
   - Extra sums of squares principle
   - Qualitative predictor variable

MSM962 Multivariate Methods

Rationale. The complexity of most phenomena in the real world requires an investigator to collect and analyze observations on many different variables instead of a single variable. Multivariate methods that extend common univariate statistical procedures to analogous multivariate techniques become essential for data analysis. Successful applications of the multivariate methods require a sound understanding of the theory. This course helps our graduate students to develop the ability to interpret the results and critically evaluate the methods used, and to stimulate more interest for further study and independent research in multivariate statistics.

Aims and objectives. This course aims to present the concepts and methods of multivariate analysis at graduate level of statistics learning. The focus of this course will be on both theory and applications of multivariate methods. Topics will be selected from multivariate analysis of variance (MANOVA), principal component analysis, factor analysis, canonical correlation analysis, discriminant analysis, cluster analysis and recent approaches to multivariate data analysis. R or equivalent statistical software will be used to give students hands-on experience of computations and data analysis with topics covered.

Syllabus. The following topics may be included:

1. Multivariate analysis of variance (MANOVA)
   - One-way MANOVA
   - Factorial MANOVA

2. Principal Components Analysis
   - Population Principal Components
   - Summarizing Sample Variation by Principal Components
3. Factor Analysis
   - Orthogonal factor model
   - Methods of estimation
   - Interpreting the factors
4. Canonical Correlation Analysis
   - Canonical Variates and Canonical Correlations
   - Interpreting the Population Canonical Variables
   - Sample Canonical Variates and Sample Canonical Correlations

5. Discriminant Analysis and classification
   - Classification with two multivariate normal populations
   - Fisher’s method for discriminating
   - Logistic regression and classification

6. Cluster Analysis
   - Hierarchical clustering
   - K-means clustering

MSM970 Mathematical Inquiry

Rationale. The scholarly experience of completing a Master of Science programme in Mathematics cannot be said to be complete if the candidate has not tasted the fruits of his or her own mathematical labour. The rationale of this course is to give the student a foretaste of what a mathematician does in his/her mathematics research: read relevant mathematics research papers, graduate textbooks in advanced mathematics, surveying a field of mathematics, posing research questions/problems, coming up with innovative solutions and algorithms to open problems, etc.

Aims and objectives. This course is about putting all the mathematics research skills and methods acquired in MSM900 Mathematics Research Methods to practice. There are a few possible ways in which mathematics research may be carried out:

1. Reading relevant mathematics research papers, graduate textbooks or selected chapters of graduate textbooks in advanced mathematics.
2. Identify mathematics problems and pose them in a concise manner.
3. Perform literature review and survey past works regarding the identified problem/task.
4. Apply mathematics problem solving skills to solve the problem partially or completely.

Syllabus. This course requires the student to simplify, construct, reconstruct and extend mathematics results gleaned from the readings. A good mathematics research student should be able to give significant contribution and/or insight into the problem he/she is studying. This course requires the student to work under the supervision of a mathematician. The student must submit a written report and give an oral presentation at the end of the course.