Title: Some new series for $1/\pi$

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Abstract: Much study has been devoted to Ramanujan-type series for $1/\pi$, the general form of which looks like

$$\sum_{n=0}^{\infty} H_n \left(a_0 n + b_0\right) z_0^n = \frac{1}{\pi},$$

where a_0, b_0, z_0 are algebraic numbers and H_n is an arithmetic sequence. (In Ramanujan's work H_n is a product of binomial coefficients, but in recent years more complicated terms have been used.)

This talk will focus on two new methods to generate series for $1/\pi$. One method relies on using computer algebra to guess when H_n has a hypergeometric generating function. A formula proven this way is

$$\sum_{n=0}^{\infty} \left\{ \sum_{k=0}^{n} \binom{2n-2k}{n-k} \binom{2k}{k}^2 \binom{2n}{n} 6^{-2k} \right\} (12n+1) \left(\frac{3}{20}\right)^{2n} = \frac{75}{8\pi}.$$

The other method relies almost solely on Legendre's relation. It allows us to recover some of Ramanujan's original series without resorting to modular theory or creative telescoping, and is able to produce series that cannot be 'explained' by other theories, for instance

$$\sum_{n=0}^{\infty} \left\{ \binom{2n}{n}^2 P_n\left(\frac{1}{2}\right) \right\} (14n+3) \left(\frac{3}{128}\right)^n = \frac{8\sqrt{2}}{\pi},$$

where $P_n(\cdot)$ denotes a Legendre polynomial.