**Speaker:** Song Heng Chan

**Title:** Inequalities for ranks of partitions and the first moment of ranks and cranks of partitions

**Abstract:** A partition of an integer $n$ is a nonincreasing sequence of positive integers whose sum is $n$, and the partition function, $p(n)$, counts the number of partitions of $n$. We give a brief introduction to ranks and cranks of partitions, two statistics that give combinatorial interpretations to Ramanujan's famous congruences for the partition function.

Let $M(m, n)$ and $N(m, n)$ denote the number of partitions of $n$ with crank $m$ and rank $m$, respectively. We prove two fundamental inequalities for $N(m, n)$, the number of partitions of $n$ with rank $m$, namely,

$$N(m, n) \geq N(m + 2, n)$$

for any nonnegative integers $m$ and $n$, and

$$N(m, n + 1) \geq N(m, n)$$

for any nonnegative integers $m$ and $n$ such that $n \geq 12$, $n \neq m + 2$. These two inequalities further lead to inequalities satisfied by the first positive rank crank moment function, $\mathrm{ospt}(n)$. In particular, we show that for $n \geq 8$,

$$\frac{p(n)}{4} + \frac{N(0, n)}{2} - \frac{M(0, n)}{4} < \mathrm{ospt}(n) < \frac{p(n)}{4} + \frac{N(0, n)}{2} - \frac{M(0, n)}{4} + \frac{N(1, n)}{2}.$$

This improves a previous inequality by G.E. Andrews, B. Kim, and the speaker, who proved that $\mathrm{ospt}(n) > 0$, and an asymptotic result by K. Bringmann and K. Mahlburg where they proved $\mathrm{ospt}(n) \sim p(n)/4$.

This is joint work with R. Mao.