Number theory, partitions, q-series and related research ($npqr^2$)

SEMINAR

Integrals involving products of elliptic integrals

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Date: 13th November 2013 (Wednesday)
Time: 2.00 – 3.00 pm
Venue: SR3, #04-04, S17, National University of Singapore
`http://math.nie.edu.sg/pctoh/S17_NUS_Map.pdf`

Abstract:

The complete elliptic integral of the first kind, K, is defined as

$$K(k) = \frac{\pi}{2}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{1}{2}\\ 1\end{matrix}\middle| k^2\right) = \frac{\pi}{2}\sum_{n=0}^{\infty}\binom{2n}{n}^2\left(\frac{k}{4}\right)^{2n} = \int_0^{\pi/2}\frac{\mathrm{d}t}{\sqrt{1-k^2\sin^2 t}}.$$

K is modular, and among other things, features heavily in Ramanujan's series for $1/\pi$. Integrals of products of K find physical applications in Feynman diagrams, lattice Green functions, and random walks on the plane.

Recently, integrals involving three or more K's have been evaluated in closed form. The first example,

$$\int_0^1 K(k)^3\,\mathrm{d}k = \frac{3\,\Gamma^8(1/4)}{1280\pi^2},$$

was discovered experimentally in 2010; since then many more have been found.

This talk will focus on how such evaluations are achieved. Chronologically, the first method involves L-values of modular forms, and has interesting connections with lattice sums. The second method exploits the fact that K is a Legendre function, and draws upon results from spherical harmonics. The third method relies on special transformations of Eisenstein series. The final method uses the Fourier coefficients of K and hypergeometric transforms.

The unexpected richness of these evaluations, and the apparent lack of unity among the methods, indicate the need for further research.

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