THE HENSTOCK APPROACH TO THE ITO STOCHASTIC INTEGRAL

Emmanuel Cabral, Ateneo de Manila University

Henstock defines his integral A of a function f over $[a, b]$ as follows: For every $\varepsilon > 0$there exists a function $\delta(\cdot) > 0$ such that

$$|(D) \sum f(x)|I| - A| < \varepsilon$$

whenever D is $\delta-$fine division of $[a, b]$. For the Riemann integral, δ is a constant while for the Henstock integral δ is a function. This seemingly simple modification of the Riemann integral has many interesting and far-reaching consequences. For one, the set of integrable functions under the new definition now includes highly oscillatory functions, which the Riemann nor Lebesgue integral could not integrate. The Henstock integral therefore includes the Lebesgue integral, whose definition uses measurable sets thereby making it less accessible to students with little background on Lebesgue measure theory. The Henstock theory on the other hand uses point-interval pairs. The Lebesgue integral as a special case of the Henstock integral can therefore be formulated using point-interval pairs.This formulation was given by McShane when he introduced the McShane integral.

One successful application of the Henstock approach is that on the Ito stochastic integral, which is an essential tool in financial mathematics. This lecture will present what has been done along this direction. The works of Toh and Chew, Arcede and Cabral will be highlighted here.