

BARD CELEBRATING 18 - 19 NOVEMBER 2013 LEE PENG YEE SYMPOSIUM MATHEMATICS

Programme Booklet





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Welcome Note

It gives me great honour and pleasure to chair the Local Organizing Committee for the 3rd Lee Peng Yee Symposium, and on behalf of the Mathematics and Mathematics Education Academic Group, National Institute of Education, we would like to bid all of you a warm welcome. Prof. Lee Peng Yee has a special place in the hearts of many mathematicians and mathematics educators, both locally and internationally. This symposium is organized in honour of Prof. Lee's enormous contribution to the academia of mathematics and mathematics education, as well as in celebration of his 75th birthday this year. Such an event presents an intersection of these two communities, and as such this year's theme is "Celebrating Mathematics" - a joyous occasion for mathematicians and mathematics educators, students and teachers alike to celebrate the vibrancy of mathematics and mathematics education. The 3rd Lee Peng Yee Symposium not only offers a platform for school students to showcase their mathematics research and for teachers to share their rich experience in classroom teaching, but also creates a channel for mathematicians and mathematics educators to disseminate their recent research findings. Once again, I take this opportunity to thank one

Asst/P Ho Weng Kin

It gives me immense pleasure to welcome all delegates and participants to the third Lee Peng Yee Symposium, organized and hosted by the Mathematics and Mathematics Education (MME) academic group of the National Institute of Education (NIE). The inaugural Lee Peng Yee Symposium was held in 2008, while the second symposium held in 2010 was organized in conjunction with MME's mathematical modelling outreach event. The success of the past two meetings has encouraged MME to continue with the series and enlarge the scale. This year, the symposium organizers have put together a scientific programme that promises to be both stimulating and inclusive. Paper presentations from mathematicians, mathematics educators, teachers and school students form a major component of the programme. On this note, I'd like to extend a special welcome to students from our local schools and hope that you will continue to be actively involved in mathematics for many years to come. Apart from the academic programme, let us not forget that this year's symposium coincides with the 75th birthday of our beloved colleague and mentor, Professor Lee Peng Yee. As we celebrate the joys of discovering, learning and teaching mathematics at this event, let us also celebrate this happy occasion with Prof Lee. There is much to look forward to, and on behalf of MME, may I wish you a fruitful experience and a memorable time at this symposium.

A/P Ang Keng Cheng

Local Organising Committee

Asst/P Ho Weng Kin	Chair
A/P Ang Keng Cheng	Advisor
A/P Zhao Dongsheng	
Asst/P Toh Pee Choon	
Dr Chan Chun Ming Eric	
Dr Cheang Wai Kwong	
Mdm Lim Li Gek Pearlyn	Committee Members
Mdm Nai Yuan Ting	

The following staff members of MME have helped make the symposium possible as well:

Asst/P Soon Wan Mei Amanda	Mrs Low Chwee Tee
Mdm Quek Siew Eng Michelle	Mdm Rodziah Binte Abdul Rahman
Mr Ang Kheng Yeong	Mr Mohamed Fadzli Bin Mohamed Ibrahim
Mr Ngo Hong Tat	

General Information

Venue Codes

LT1 Lecture Theatre 1 TR 714 Tutorial Room 714

Secretariat Room

TR 714

Registration and Opening Ceremony on 18 Nov 13 LT1

Registration on 19 Nov 13 LT1

Prayer Room for Muslims Block 3, Basement 1

Meals

Lunch – Linkway between Block 5 and NIE Canteen, Basement 1 Tea – Foyer @ LT2

Internet Descartes Lab, NIE 7-B1-11 (Block 7-Basement 1-Unit 11)

Local participants:

- (1) Send SMS text "hotspot" to 9004 3379 and wait for reply.
- (2) Connect notebook to WiFi network "eduwin".
- (3) Launch browser and login using information from SMS.

Overseas participants:

Follow instructions on separate printed slip

Symposium Dinner

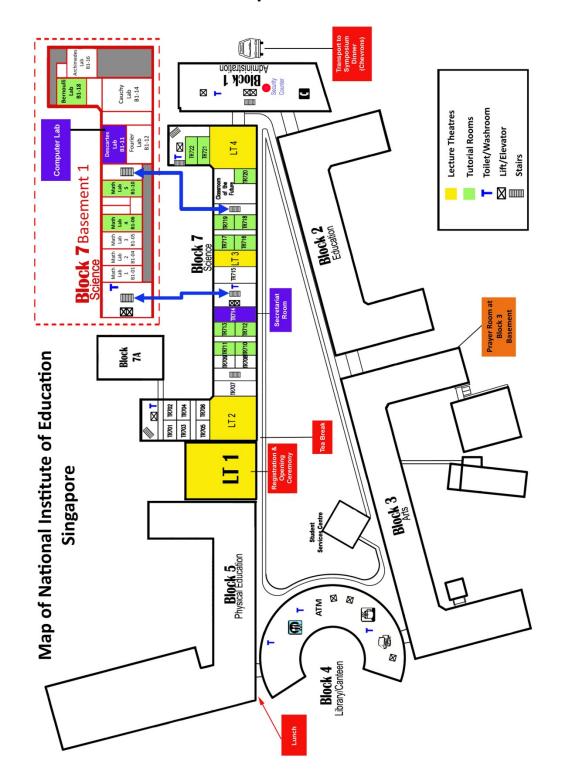
Tuesday, 19 Nov 2013 @ 7.30 p.m. Venue: Chevrons, 48 Boon Lay Way, Singapore 609961

Transportation Details to Dinner Venue

Bus leaves from NIE, Block 1, lobby, 7 at 6.15 pm and from Nanyang Executive Centre (NEC) at 6.30 pm to Chevrons.

Transportation Details from Dinner Venue

Bus leaves from Chevrons to: Jurong East MRT station at 9.45 pm and Jurong East MRT station and Nanyang Executive Centre (NEC) at 10 pm Map of NIE



Breakout Session Codes

Day 1 (18 Nov 2013)

SR01-01: Students' Research Session 01 (morning) – Presentation 01 TS01-01: Teachers' Sharing Session 01(morning) -Presentation 01 TW02: Teachers' Workshop (afternoon)

Day 2 (19 Nov 2013)

MR01-01: Mathematics Research 01 (morning) – Presentation 01 MRH-01: Mathematics Research on Henstock-Kurzweil Integral 01 (morning) – Presentation 01 MER01-01: Mathematics Education Research 01 (morning) – Presentation 01

Note: Afternoon sessions are annotated with a "02", e.g. SR02-01.

Programme

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Time	18/11	(Mon)	Time	19/11(Tue)
Morning			Morning	
0900-0930	Regist	ration	0900-0930	Registration
0930-1000	Opening	Ceremony		Symposium Lecture 2
1000-1015	Video	Tribute	0930-1030	Emmanuel Cabral
1015-1145		n Lecture 1 Ing Yee	1030-1200	Research Sessions - Mathematics Session 1
1145-1200	Symposiu	m Updates	1030-1200	- Henstock-Kurzweil Integral Session 1 - Mathematics Education Session 1
1200-1300	Lu	nch	1200-1300	Lunch
Afternoon			Afternoon	
1300-1400	Students'	Talk for Teachers		Research Sessions - Mathematics Session 2
1400-1500	Research Session 1	Teachers' Sharing Session 1	1300-1500	- Henstock-Kurzweil Integral Session 2 - Mathematics Education Session 2
1500-1530	Aftern	oon Tea	1500-1530	Afternoon Tea
		Teachers'	1530-1630	Symposium Lecture 3 Lim-Teo Suat Khoh
1530-1730	Students' Research Session 2	Research Session 2	1630-1700	Panel Discussion Lee Peng Yee, Emmanuel Cabral, Lim-Teo Suat Khoh
			1700-1730	Closing Ceremony
			1930-2130	Symposium Dinner

Symposium Lecture

18 Nov 2013 1015-1145 Lecture Theatre 1

Symposium Lecture 1

CELEBRATING MATHEMATICS

A/P Lee Peng Yee

National Institute of Education, Nanyang Technological University, Singapore

Abstract

The speaker will review mathematics education in Singapore for the past 50 years under the titles: textbook, syllabus, mathematics, and classroom. The discussion covers the new role of textbook, the fat pentagon, teacher mathematics, and essence of teacher training. There are also reflections of many years of teaching and some memories. In addition, three connections between Singapore and mathematics are given.

Bio

Lee Peng Yee, age 75, currently teaching at the National Institute of Education, Singapore. He is a graduate (first batch) of Nanyang University. He graduated with PhD (1965) from Queens University of Belfast, Northern Ireland, the United Kingdom. He taught mathematics for 50 years in Africa, New Zealand, the Philippines, and Singapore. He has supervised over 20 PhD students. He was formerly head of Department of Mathematics, Nanyang University, and head of Mathematics and Mathematics Education Academic Group, National Institute of Education. He served as vice president of International Commission on Mathematical Instruction (1987-1990, 1991-1994).

Father Bienvenido Nebres (Professor of Mathematics, Ateneo de Manila University, Philippines) will give a commentary following this symposium lecture.

Symposium Lecture

19 Nov 2013 0930-1030 Lecture Theatre 2

Symposium Lecture 2

THE HENSTOCK APPROACH TO THE ITO STOCHASTIC INTEGRAL

A/P Emmanuel Cabral

Ateneo de Manila University, Philippines

Abstract

Henstock defines his integral *A* of a function *f* over [a,b] as follows: For every $\varepsilon > 0$, there exists a function $\delta(\cdot) > 0$ such that

$$\left| (D) \sum f(x) |I| - A \right| < \varepsilon$$

whenever *D* is δ - fine division of [*a*,*b*]. For the Riemann integral, δ is a constant while for the Henstock integral δ is a function. This seemingly simple modification of the Riemann integral has many interesting and far-reaching consequences. For one, the set of integrable functions under the new definition now includes highly oscillatory functions, which the Riemann nor Lebesgue integral could not integrate. The Henstock integral therefore includes the Lebesgue integral, whose definition uses measurable sets thereby making it less accessible to students with little background on Lebesgue measure theory. The Henstock theory on the other hand uses point-interval pairs. The Lebesgue integral as a special case of the Henstock integral can therefore be formulated using point-interval pairs. This formulation was given by McShane when he introduced the McShane integral.

One successful application of the Henstock approach is that on the Ito stochastic integral, which is an essential tool in financial mathematics. This lecture will present what has been done along this direction. The works of Toh and Chew, Arcede and Cabral will be highlighted here.

Bio

Dr. Emmanuel A. Cabral is an associate professor at the Ateneo de Manila University in the Philippines. He obtained his PhD degree in mathematics in 2002 under the supervision of Professor Lee Peng Yee of the Mathematics and Mathematics Education Group (MME). His thesis was on the fundamental theorem of calculus for the Henstock integral in multidimensional space. Professor Cabral's research interests include Henstock integration, Baire-one functions, stochastic integration and financial mathematics.

Symposium Lecture

19 Nov 2013 1530-1630 Lecture Theatre 2

Symposium Lecture 3

IMPORTANT ATTRIBUTES OF MATHEMATICS EDUCATORS

A/P Lim-Teo Suat Khoh

National Institute of Education, Nanyang Technological University, Singapore

Abstract

The mathematics community of the 20th century has recognized the importance of mathematics education and, in the second half of the century, the abundance of research into how students learn mathematics has made mathematics education a significant field of study, separate from mathematics and more aligned to educational research. Within this, the critical role played by mathematics educators has also been recognized and hence the study of mathematics teachers and their education has grown as a crucial area of research, particularly in the past twenty years. This talk will describe the attributes of effective mathematics teachers and discuss issues in mathematics teacher education in the nurturing and development of such teachers.

Bio

Lim-Teo Suat Khoh is an Associate Professor in the Mathematics and Mathematics Education Academic Group of the National Institute of Education, Singapore. She has been a mathematics teacher educator for more than 25 years, teaching various mathematics pedagogy and content courses and has also carried various leadership appointments at NIE such as Academic Group Head, Associate Dean for pre-service programs and Dean of faculty affairs. While her earlier research interests were in various aspects of mathematics learning, her recent research interests are in the area of mathematics teacher education.

Talk for Teachers

18 Nov 2013 1300-1400 Lecture Theatre 2

THE SMAPP WAY TO ENGAGE SECONDARY SCHOOL STUDENTS IN MATHEMATICS TASKS WITH REAL-LIFE CONTEXTS

A/P Wong Khoon Yoong

National Institute of Education, Nanyang Technological University, Singapore

Abstract

The ability to solve mathematics problems with real-life contexts has been given stronger emphasis in the most recent version of the Singapore Mathematics Curriculum (2013). The SMAPP (Singapore Mathematics Assessment and Pedagogy Project) project has developed two types of tasks with real-life contexts for secondary school students: extended tasks and short tasks. Extended tasks can be delivered online using HTML, which exposes students to ICT-based mathematics assessment. Short tasks are similar to standard exercises and can be modified for school tests. Teachers can use these SMAPP tasks as learning experiences for various topics. The project also validated the Attitude toward Learning Mathematics questionnaire, which can be used in research about student attitudes. This presentation will provide a "tour" of these materials and report some findings about student performance to solve SMAPP tasks and perceptions about these tasks.

Bio

Dr. Wong Khoon Yoong is an associate professor at the Mathematics and Mathematics Education Academic Group at NIE. As a mathematics educator for the past 40 years, he has taught mathematics education courses, supervised graduate students, conducted research in mathematics education in Australia, Malaysia, Brunei Darussalam, and Singapore.

Panel Discussion

19 Nov 2013 1630-1700 Lecture Theatre 2

> In this session, A/P Lee Peng Yee, A/P Emmanuel Cabral and A/P Lim-Teo Suat-Khoh will discuss their perceptions on some of the thoughts based on the symposium lectures and answer questions like:

- 1. What, in your opinion, is the impact of research in mathematics/mathematics education on the Asian's education systems (particularly, in mathematics)?
- 2. What are some future directions in mathematics and mathematics education in Asian region, e.g., Singapore, Philippines?

Students' Research Session 18 Nov 13

Time	Breakout Session: Code and Venue	Name and Title
1300-1330	SR01-01 TR 715	Jia De Qian, Lim Kah Ye, Terrence Mak and Yeo Sze Ling (River Valley High School, Singapore) Pollard's rho Method for Integer Factorization: the Impact of the Function used on its Efficiency Abstract : This report sets out to understand the relationship between the function used to generate the sequence of random values in Pollard's rho method for integer factorization and the efficiency of the algorithm. It is found out that the degree of the function has a close relationship with the efficiency of the algorithm. If the degree shares a large common factor with $p - 1$, where p is a factor of the integer, it is likely that fewer steps are required.
1330-1400	SR01-02 TR 715	Kuang Qiangru, Ma Hongqiang, Xia Wenzong and Wu Ziqing (River Valley High School, Singapore) On the Analysis of the Nim Game Abstract: This paper presents an in-depth analysis of the Nim game, starting from the introduction of binary operators and the reduction of the game into a single number. The central theorem based on these concepts are proposed and proved in chapter 4. Practical strategies are given, followed by the study of a variation of the game. Monte Carlo method is used to investigate the relationship between the probability of winning and the two factors, namely the maximum number of piles and the maximum number of counters in each pile. The data is presented and analyzed in the last part.
1400-1430	SR01-03 TR 715	Jiang Jin Jing, Wang Xin, Yang Chen and Shi Hongzhou (River Valley High School, Singapore) I am learning – ipad and Education Abstract: Our report aims to discuss the effectiveness of using iPad in education by analyzing the data collected in River Valley High School from students, teachers and parents respectively. We conducted three surveys and two experiments to find out whether iPad is an effective educational tool in both objective and subjective ways. Detailed analysis of survey results show that the iPad program is supported by different groups of people and we also realized various benefits that iPad can bring in our teaching and learning practices. Based on our analysis of experiment results, iPad has helped students in the more effective understanding of relevant concepts and thus it has further enhanced their learning of content knowledge in the studying both Humanities and Science subjects.
1430-1500	SR01-04 TR 715	Liew Hui Min, Soh Boon Jun and Teo Qi Lin (River Valley High School, Singapore) Applications of 2-Dimensional Random Walks Abstract : In this report, we aim to study the application of 2-Dimensional (2-D) random walks in real life situation, by constructing a simulation model. In 2011, a student claimed that she fell off a Mass Rapid Transit (MRT) platform onto the tracks due to a "dizzy spell", resulting in her losing both legs. Using this incident as a motivation, we propose to use Visual Basics for Applications (VBA) as a programming tool to create a simulation model to investigate if she could be walking around randomly, without being noticed, and find out the average number

		of steps she takes before falling off the platform, to determine if there were other factors which caused her fall onto the tracks.
1530-1600	SR02-01 TR 715	Lee Bing Quan and Rachel Burns (Hwa Chong Institution, Singapore) Cancel Cancer Abstract: Breast cancer is one of the leading causes of female deaths, affecting every one in eight women. The HER2 gene has been found to promote tumour growth in 25% of these breast cancer cases, and thus drugs such as pertuzumab and trastuzumab emantasine have been developed to target this gene. Our project, Cancel Cancer, aims to explore the possibility of a combination therapy of these two drugs via mathematically modeling their effects on tumour growth and hence devise an optimal dosing regimen. To do this, we have used a pharmacokinetic- pharmacodynamic model to predict the interactions between the drugs and the body. Our final product is a unique model integrating the linear two compartment concentration model, the Koch tumour growth model and the Simeoni tumour growth model into one, specifically designed to study tumour growth. This model was replicated in both Microsoft Excel and Wolfram Mathematica using Euler's Method and the Runge-Kutta Order 4 Method respectively. The resultant models showed similar results, with trends pointing towards a long-term consistent drug dosing regimen as the optimal regimen.
1600-1630	SR02-02 TR 715	Liu Changshuo, Philip Ong Zheng Yang and Gao Yuan (Anglo-Chinese School, Independent, Singapore) Computer Solution to Convex 7-Gon Happy Ending Problem via Graph-to-Matrix Transformation Abstract : Happy Ending Problem aims to identify the minimum number of vertices f(N) with general position required on a plane to ensure that they contain a convex <i>N</i> -gon. Proofs for $f(5) = 9$ and $f(6) = 17$ have been published, with $f(7)$ and above unknown. In our investigation, we proposed Graph-to-matrix transformation to transform a graph into a matrix. The property of the matrix indicates whether the graph contains a convex 7-gon. We proposed a computer solution and realized the program.
1630-1700	SR02-03 TR 715	Ng Fu Tian Benny (National Institute of Education, Nanyang Technological University, Singapore) An Alternate Formula, The Partition of Integers Function Abstract : The paper introduces an formula for $p(m,k)$ which counts the total number of partitions of the integer m into exactly k parts. We will see that the partition of integers function is but a special case from this formula and hence, this formula serves as an alternate formula for the partition of integers function.

Teachers' Sharing Session 18 Nov 13

Time	Breakout Session: Code and Venue	Name and Title
1400- 1420	TS01-01 LT2	Khoo Hock Heng and Roland Chua (River Valley High School, Singapore) Use of iPads in the Teaching of Statistics: Probability and Correlation & Regression Abstract: The presenters will share on the use of iPads in the learning of least squares lines in the topic of Correlation and Regression. They will walk participants through an Action Research Project they had carried out, where selected students were taught using iPads and Geometer Sketchpads in the learning of Least Squares Lines complementing the use of GCs in normal lessons. The presenters will also share on the use of the application RVHS Stats in the teaching of probability. The project aims to implement technology-rich mathematical tasks in lessons with the Technological and Pedagogical Content Knowledge (TPACK) framework in mind.
1420- 1440	TS01-02 LT2	Chan Zi Ping, Mark John Cordiner and Khoo Bing Ting (River Valley High School, Singapore) Designing an Elective Mathematics Course using the iPad as a Leverage Abstract: River Valley High School (RVHS) has launched an iPad 1:1 Programme since 2012 to enhance teaching and learning in and beyond the classroom. An elective course on Cryptology was designed - in line with the Singapore Mathematics Framework (SMF) - with the iPad as the main teaching and learning tool. Cryptology allows students to see the application of Mathematics in the real world while the iPad was useful in content delivery, assessment and communication. Both the content and interactive nature of the course had made the lessons more engaging and effective. This presentation focuses on the rationale for choosing Cryptology, the role of the iPad, and the reflections of teachers and students.
1440- 1500	TS01-03 LT2	Leong Meng Wai and Lei Peishan (Compassvale Secondary School, Singapore) Teaching Algebra Through Learning Experiences Abstract : This lesson study documents the journey of a group of teachers in exploring if meaningful learning experiences can help students understand and apply better the algebraic rules to expand and simplify algebraic expressions. The research lesson incorporated learning experiences such as decoding exercise - to decode what some ancient symbols represent; activity for students to generalise basic algebraic rules through basic mathematical rules, 'Algetools' software and a 'Snakes and Ladders' game. Through this lesson study, it can be concluded that the use of learning experiences improves student development of conceptual understanding of Basic Algebra.
1530- 1555	TS02-01 LT2	Tuo Yeong Lee and Yuxuan Xiong (NUS High School, Singapore) An elementary calculus method for evaluating $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1}, \qquad \sum_{k=1}^{\infty} \frac{1}{k^2}, \qquad \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)^3}, \qquad \sum_{k=1}^{\infty} \frac{1}{k^4}, \dots$ Abstract: We use freshman calculus to prove that $\frac{d^n}{d\theta^n} \left(\cot \theta - \frac{1}{\theta} \right) = -\sum_{k=1}^{\infty} \frac{d^n}{d\theta^n} \left(\frac{1}{k\pi - \theta} - \frac{1}{k\pi + \theta} \right)$

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		for $\theta \in (0, \pi)$ and $n = 0, 1, 2,;$ in particular, we obtain a simple unified method for evaluating the following infinite series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2k-1}, \qquad \sum_{k=1}^{\infty} \frac{1}{k^2}, \qquad \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)^3}, \qquad \sum_{k=1}^{\infty} \frac{1}{k^4},$
1555- 1640	TS02-02/03 LT2	Pow Tien Min Jaron and Ginny Tan (Millennia Institute, Singapore) Productive Failure in the A-level Classroom Abstract : Millennia Institute participated in a year-long study with NIE's Learning Sciences Laboratory that examined the implementation of the Productive Failure pedagogy (PF, Kapur, 2008) in the A-level Mathematics classroom. 3 specific topics were explored, namely Normalization, Hypothesis Testing and Mathematical Induction. This presentation focusses on (1) explaining the PF pedagogy, (2) sharing issues and benefits arising from the design, implementation and assessment process, and (3) sharing participating teachers' insights and reflections. The topic Hypothesis Testing will be used as an anchor for the sharing session.
		Pow Tien Min Jaron (Millennia Institute, Singapore) The Productive Failure Project Results Abstract: Pre-University Three classes (<i>n</i> =35) from Millennia Institute (MI) participated in a study that examined the design of Productive Failure (PF, Kapur, 2008) in the learning of the concept of Hypothesis testing. Co-designed by MI teachers, the PF unit was implemented on three intact classes and their performances on assessment and process measures compared respectively. The complex problem "Do Wala Chocolate Bars have Similar Weights?" designed for the unit managed to engage students and allowed them to produce a good amount of representations and solution methods (RSMs). Analyses of post assessments results revealed a levelling effect, suggesting that classes with different mathematical performance profiles are able to perfom equally well after being exposed to the PF learning design.
1640- 1705	TS02-04 LT2	Chua Seow Ling (Compassvale Secondary School, Singapore) Developing Mathematical Thinking in Students through a Four-Pronged Strategy to Solve Problems Abstract: Our lesson study involves the use of a four-pronged strategy to help students set a structure to develop a cognitive thinking process in the solving of a mathematical problem. This process helps them to understand and make meaning and connect information given in the problem set. Using what they have understood from the process, they will be able to devise a plan to help them solve the problem. After solving the problem, the students would also know how to check and reflect on their work for consolidation and extension of their learning, connecting to past learning experiences and possibly creating new knowledge. Data from lesson observations and students' and teachers' reflections were used to validate the effectiveness of the four-pronged strategy. The students found the strategy effective and useful in helping them to solve problems.
1705- 1730	TS02-05 LT2	Simmi Naresh Govindani and Albert Kang (Yishun Secondary School, Singapore) Enhancing the Teaching and Learning of Algebra through Teacher Language and Other Strategies Abstract: Teacher Language or "teacher-talk" has a tremendous impact on the success of teacher-students and students-students interactions. With an increasing emphasis on mathematical communication and metacognition, it is imperative for teachers to effectively communicate instructions and processes so that students

	develop a deep understanding of concepts, make connections and become confident learners. The presentation aims to discuss and present findings from an Algebra Diagnostic Test and to share the development and co-creation of a teacher language aimed towards empowering both the teacher and student to communicate mathematically. Post study findings, through lesson observations and formative assessments have shown favourable results.
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Teachers' Workshop 18 Nov 13

Time	Breakout Session: Code and Venue	Name and Title
1530- 1630	TW02 Math Lab 5 @ Block 7, Basement 1	Rosalind Phang, Vivian Yeo and Choo Swee Kim (Add-venture Learning, Singapore) Teach Less, Play more, Learn most Abstract : This workshop is inspired by Plato: 'Let early education be a manner of amusement. Young children learn by games. Compulsory education cannot remain in the soul'. and driven by the question: "Is it possible to teach less and learn more?" Using number and geometry games, this hands-on session aims to demonstrate the power of play. Participants will learn how to 'excite and direct the learner's mental powers through instructional games, while telling him nothing that he can learn himself'. The result of this meaningful play is learning which is both pleasurable and enduring.

Mathematics Research Session – On Henstock-Kurzweil Integral 19 Nov 13

Time	Breakout Session: Code and Venue	Name and Title
1030- 1115	MRH01-01 LT3	Ye Guoju, Zhou Hao and Wang Ou (College of Science, Hohai University, Nanjing, China) The D_{HK} Integral and Applications Abstract : In this presentation, we introduce the D_{HK} integral called distributional Henstock-Kurzweil integral that is an important integral tool to deal with the differential and integral equations. We discuss the properties of such integral and establish the structure of the space of Henstock-Kurzweil integrable distributions, that is an ordered Banach space with a regular cone by using the convergence theorems of distributional Henstock-Kurzweil integral. In D_{HK} , we define an inner product so that it is an inner product space. We also give the fixed point theorems in D_{HK} which help us to prove the existence of solutions of the integral equation containing the distributional Henstock-Kurzweil integral.
1115- 1200	MRH01-02 LT3	Yang Haifeng (National Institute of Education, Nanyang Technological University, Singapore) On Henstock-Kurzweil Method to Stratonovich Stochastic Integral Abstract: The features of Henstock–Kurzweil integral makes it possible to redefine the classic stochastic integral using Riemann sums. Naturally, we may think that we could define the stratonovich integral with Henstock- Kurzweil method. However, the stratonovich integral is drastically different from the Itô integral. Since the point in stratonovich integral is not the left point, then the isometry and independence is no longer true. Therefore, we need to ensure the function is "smooth" enough in order to define the integral. Although the most excellent properties of Itô integral do not hold, the stratonovich integral is valuable to explore. It is more natural to use the Stratonovich integral in connection with stochastic differential equations on manifolds, given that the Stratonovich formula is more "natural" than that for the classical Itô integral. In the paper, we will define the stratonovich integral using the Henstock–Kurzweil approach with respect to continuous L^2 -martingale and establish the properties of the stratonovich integral.
1300- 1340	MRH02-01 LT3	Karlo S. Orge and Julius V. Benitez (MSU – Iligan Institute of Technology, Philippines) Baire One-Stieltjes Integration Abstract: This paper introduces the concept of Baire One-Stieltjes integral. Basic properties and existence theorems for this Stieltjes integral are discussed.
1340- 1420	MRH02-02 LT3	Abraham P. Racca (Adventist University, Philippines) Emmanuel A. Cabral (Ateneo de Manila University, Philippines) Characterizing Convergence Conditions for Henstock-Kurzweil Integral in Euclidean Space Abstract: In this paper, we give some characterizing convergence conditions for Henstock-Kurzweil Integral in Euclidean Space. We also prove that for pointwise convergent sequence of Henstock-Kurzweil integrable functions, equi-integrability is equivalent to the UI ₂ condition.

1420-	MRH02-03	Lee Tuo Yeong (NUS High School, Singapore)
1500	LT3	Another convergence theorem for Henstock-Kurzweil integrals
		Abstract: We establish a convergence theorem for Henstock-Kurzweil integrals.
		As a result, we obtain new sufficient conditions for the Henstock-Kurzweil
		integrability of $f \otimes g$.

Mathematics Research Session 19 Nov 13

Time	Breakout Session: Code and Venue	Name and Title
1030-	MR01-01	Soeparna Darmawijaya (Gadjah Mada University, Indonesia)
1100	TR 715	Reisz Analysis
		Abstract : By <i>E</i> is a Riesz algebra we mean <i>E</i> is an algebra and as a real vector space <i>E</i> is Riesz space. In what follows we only consider <i>E</i> is a commutative Riesz algebra with unit element <i>e</i> .
		First of all, if $f, g \in E$ we define : (i) $[f, g] = \{h \in E : f \in h\}$ and $(g \in f] = \{h \in E : h \in f\}$
		(i) $[f, \infty] = \{h \in E : f \le h\}$ and $(-\infty, f] = \{h \in E : h \le f\}$. (ii) $f < g \Leftrightarrow g \in [f, \infty], g \ne f$ and $g \ne f \lor h$ for every $h \in E$ which is incomparable with f
		(i) $f < g \hookrightarrow g \in [f, \infty], g \neq f$ and $g \neq f < h$ for every $h \in E$ which is incomparable with $g \in (-\infty, f], f \neq g$, and $f \neq g \land h$ for every $h \in E$ which is incomparable with g .
		Because the function $ \cdot : E \rightarrow E$ has properties :
		N1. $ f \ge \theta$ for every $f \in E$, where θ is the null element of E .
		$ f = \theta \Leftrightarrow f = \theta.$
		N2. $ \alpha f = \alpha f $, for every $f \in E$ and real number α , and
		N3. $ f + g \le f + g $, for every f , $g \in E$,
		then we may call $ \cdot $ is a Riesz norm on E and (E, \cdot) is called a Riesz normed space or Riesz normed algebra. This space has behaviours nearly like those in the numbers system or at least usual normed space. For examples :
		(a) if $f \in E$ and a real number $\delta > 0$, the neighbourhood of f with radius δe is the set $N_{\delta}(f) = \{h \in E : f - h < \delta e\}$
		$= \{h \in E : f - \delta e < h < f + \delta e\}$ = $(f - \delta e, f + \delta e)$.
		(b) $U \subset E$ is open if and only if for every $f \in U$, there is $N_{\delta}(f)$ such that $N_{\delta}(f) \subset U$. The topology generated by the norm $ \cdot $ is an Hausdorff one. Therefore, we may develop an analysis on E like those in the real numbers system but in the Riesz language; only we observe that a bounded set may be has no supremum or infimum.
1100-	MR01-02	Jonald P. Fenecios (Ateneo de Davao University, Philippines)
1130	TR 715	Emmanuel A. Cabral (Ateneo de Manila University, Philippines)
	_	Another Proof of the $\varepsilon - \delta$ Characterization of Baire Class One Functions on R
		Abstract : An alternative proof for the $\varepsilon - \delta$ characterization of Baire class one
		functions is given without using the Baire Category theorem.
1130- 1200	MR01-03 TR 715	Julius V. Benitez and Ferdinand P. Jamil (MSU – Iligan Institute of Technology, Philippines) [Absent from actual session] ACG_{M}^{*} and ACG_{H}^{*} Functions
		Abstract: In this paper, we introduce the ACG_M^* and ACG_H^* properties and show that they are equivalent to the ACG^* property.
1300- 1325	MR02-01 TR 715	Polly W. Sy (University of the Philippines, Philippines) The Dynamics of a Two-Predator One-Prey Competition Model with a Beddington-
1020	11.710	DeAngelis Functional Response
		Abstract : We study a population system of two competing predator species that exploit their consumption of a single renewable prey. In this system, each predator-
		prey interaction is modelled by a Beddington-DeAngelis functional response. We

		investigate on how the parameters can determine the limiting behavior of any solution in the given system. We obtain parametric conditions for a given predator species to independently become extinct, hence being an "inadequate" competitor. We exhibit that competitive exclusion holds whenever exactly one competitor is inadequate and that the surviving competitor can stabilize its density with the prey. Given that at least one competitor becomes extinct, we analyze subsystems to calculate the limiting values of the solution. In case neither competitor is inadequate, we establish conditions where all three species survive. More specifically, in this case, we use a Lyapunov function to establish coexistence through a global stability of the unique interior equilibrium point and find that a competitor can survive at a very low density.
1325- 1350	MR02-02 TR 715	Fengming Dong, Dongsheng Zhao, and Weng Kin Ho (National Institute of Education, Nanyang Technological University, Singapore) On the Largest Outscribed Equilateral Triangle Abstract: An outscribed triangle of a triangle $\triangle ABC$ is a triangle $\triangle DEF$ such that each side of $\triangle DEF$ contains a vertex of $\triangle ABC$. In this article we study the equilateral outscribed triangles of an arbitrary triangle and determine the area of the largest such triangles. We prove that if each angle of $\triangle ABC$ is smaller than 120°, the largest outscribed equilateral triangle of $\triangle ABC$ can be constructed by ruler and compass and its area equals $\frac{a^2+b^2+c^2}{2\sqrt{3}} + 2S_{\triangle ABC}$ where $S_{\triangle ABC}$ denotes the area of $\triangle ABC$ and a, b, c are the lengths of the three sides of $\triangle ABC$.
1350- 1415	MR02-03 TR 715	 Weng Kin Ho (National Institute of Education, Nanyang Technological University, Singapore) Maximal Outbox Problem Abstract: In (Zhao 2010), Dongsheng Zhao posed and solved (using calculus) the maximal outbox problem that asked for the position and dimensions of the maximal outbox of a given convex quadrilateral, i.e., the largest rectangle amongst those such that each vertex of the given convex quadrilateral lies on one side of the rectangle and all vertices lie on different sides. Unknown to many, an earlier work by M. F. Mammana (Mammana 2008) already gave a comparatively more in-depth geometrical analysis that yielded a necessary and sufficient condition for the existence of maximal outbox problem via a novel result in geometry the Outbox Centroid Theorem. Interestingly, this new approach lends itself naturally to dynamic geometric exploration.
1415- 1440	MR02-04 TR 715	Nai Yuan Ting and Zhao Dongsheng (National Institute of Education, Nanyang Technological University, Singapore) On a Class of Semirings Abstract: We consider a class of semirings that includes the set of ideals of rings and all distributive lattices. First, we prove a new characterization of such semirings, and then show that the sub-category of such class of semirings is corrective in the category of additively idempotent semirings. The extension problem asks when a join semilattice can be extended to such a semiring by defining a multiplication on the semilattice. We shall prove a necessary and sufficient condition for a join semilattice to be extended to such a semiring using the semiring of endomorphisms on the join semilattice satisfying certain conditions. An example is constructed to show that not every semilattice can be extended to such a semiring.

1440-	MR02-05	Leong Chong Ming (Nanyang Junior College, Singapore)
1500	TR 715	The Monotone Class Theorem
		Abstract : The Monotone Class Theorem (MCT for short) is a key result in measure theory and probability which allows statements about particularly simple classes of functions to be generalized to arbitrary measurable and bounded functions. It is also used in proving the Fubini Theorem, a core result in real analysis. The proof of the result basically involves a technique called 'boot strapping'. A couple of definitions and lemmas precede the theorem. In many literatures, these are presented in a rather fragmented manner and in others, several gaps make the proof hard to grasp by even the beginning graduate student. It is therefore my intention in this article to present the proof of the MCT in as systematic and pedagogically coherent a fashion as possible and to fill in those gaps that stump even the very good students.

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Time	Breakout Session, Code and Venue	Name and Title
1030- 1100	MER01-01 LT4	方均斌 (College of mathematics & Information Science, Wenzhou University, Wenzhou, China) 当代东西方数学教育交流给我们的启示 Abstract : 在权威的 TIMSS 国际测验中,新加坡和东亚各国学生的数学成绩领先世界。 西方数学教育界开始关注东方的经验。最近新加坡数学教育的领军人物李秉彝先生来华 访问。笔者在和他交谈中得到许多启示。其中包括:谨慎地借鉴西方的数学教育观 念;多些嫁接,少些移植;"上通数学、下达课堂"以及数学教育要"跨界"等。 The presentation is accompanied by an English translation.
1100- 1130	MER01-02 LT4	Tan Liang Soon (National Institute of Education, Nanyang Technological University, Singapore) Developing teachers' competence in mathematical modelling instruction through a school-based professional development (SBPD) programme Abstract: Recent emphasis on applied learning in Singapore schools has made mathematical modelling even more relevant to the mathematics curriculum. With this in mind, a SBPD programme is designed and introduced to a group of teachers in a secondary school. This paper reports on how a teacher plan, design and carry out a modelling task through his participation in the SBPD programme. Aspects of the teacher's competence in modelling instruction that are developed are examined. Analysis of the teacher's instruction practice suggests that the SBPD programme positively influence the teacher's knowledge, goals, and orientations in modelling instruction.
1130- 1200	MER01-03 LT4	Ng Lay Keow (National Institute of Education, Nanyang Technological University, Singapore) Translation between Word Statements and Algebraic Equations Abstract : This study examined teachers' competency in forming and expressing algebraic equations by using analogous statements of the students-and-professors problem. Three equation-formulation tasks and three equation-interpretation tasks were developed and tested. The effects of teaching experience, context and checking prompts were assessed too. 24.1% of 36 in-service teachers and 32.4% of 69 pre-service elementary teachers had problems transiting from text to symbols with more faltering in the reverse operation. Although teaching experience did improve performance, contextual and checking cues did not.
1300- 1330	MER02-01 LT4	Tay Eng Guan (National Institute of Education, Nanyang Technological University, Singapore) Making Mathematical Problem Solving Practical Abstract: With regard to the direction of problem solving research, Alan Schoenfeld wrote in the 2007 special issue on problem solving of the journal ZDM that the current focus should lie in translating decades of theory building about problem solving into workable practices in the classroom: That body of research—for details and summary, see Lester (1994) and Schoenfeld (1985, 1992)—was robust and has stood the test of time. It represented significant progress on issues of problem solving, but it also

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		left some very important issues unresolved The theory had been worked out; all that needed to be done was the (hard and unglamorous) work of following through in practical terms. (Schoenfeld, 2007, p. 539). Our project <i>Mathematical Problem Solving for Everyone: Infusion and Diffusion</i> (MInD) is an attempt at doing the "hard and unglamorous" work of realising the ideals of mathematical problems solving—as envisioned to be at the heart of the Singapore mathematics curriculum—into the daily practices of mathematics classrooms. This involves three major steps: (1) <i>initialisation</i> of problem solving as an essential part of the mathematics curriculum in a school at a foundational year level; (2) <i>infusion</i> of problem solving as an embedded regular curricular and pedagogical practice across all year levels in the school; and (3) <i>diffusion</i> of this innovation from this school to the full range of schools in Singapore. In this talk, we describe our "Practical" approach and our progress and findings with regard to the curriculum, instructional practices, assessment, and teacher development. Be prepared also to solve at least one problem at the talk!
1330-	MER02-02	Deng Feng (National Institute of Education, Nanyang Technological University,
1400	LT4	Singapore)
		The influence of a problem solving curriculum on students' mathematics
		achievement: An explorative study Abstract: This study aimed to investigate the influence of a problem solving
		curriculum (i.e., Mathematical Problem Solving for Everyone, MProSE) on
		Singaporean Grade 7 students' mathematics achievement. Specifically, it examined
		whether students' beliefs about mathematical problem solving and their attitude
		towards MProSE contributed to their mathematical test scores. Four classes (n =
		142) were engaged in ten lessons with each comprising Polya's four stages: understand the problem, devise a plan, carry out the plan, and look back. Heuristics and metacognitive control were highlighted during students' mathematical problem solving. Results showed that the students' beliefs about self and mathematics during MPS and their attitude towards Polya's model were two significant predictors for their mathematics achievement. Implications of the findings for the implementation of MProSE in mathematics classroom are discussed.
1400- 1430	MER02-03 LT4	Romina Ann S. Yap (National Institute of Education, Nanyang Technological
1430	L14	University, Singapore) A Trajectory of Learning for Mathematics Teachers Engaged in a Video Club Abstract : A video club is a group of teachers who regularly view video excerpts of their own or their peers' classroom teaching and discuss certain aspects of teaching that the video may show. It is said to be able to "offer teachers the opportunity to examine teaching and learning in new ways and have the potential to foster the learning called for by reform" (Sherin & Han, 2003, p. 163). In this report, I will share preliminary results of a study that examined how a group of Secondary 1 mathematics teachers learned from a video club that was situated within a larger professional development program aimed at supporting the teachers in the teaching of mathematical problem solving. The findings will be presented in the form of a trajectory derived from tracking how objects of inquiry in the video club discussions were translated by the teachers in their instruction.
1430- 1500	MER02-03 LT4	Chia Hui Teng (Singapore Polytechnic, Singapore) Informal Inferential Reasoning and the Primary School Statistics Education in Singapore Abstract: Making inferences is the heart of statistics education. Since inferential

statistics is more cognitively demanding, the teaching and learning of formal inferential reasoning is usually reserved for pre-tertiary or tertiary level students. Although young children may not have access to formal inferential reasoning, many researchers believe that young children can engage in informal inferential reasoning (IIR). This paper is sequenced into three parts. It begins with a discussion on the role of IIR in statistics education and its importance during the early years. Subsequently, the second part of this paper takes a closer look at the primary school statistics education in Singapore. What learning opportunities are provided for young children in Singapore for meaningful development of their inferential reasoning in statistics? The focus and content of Singapore primary school statistics education are examined through an interrogation of curricular materials using a statistics education framework and two graph comprehension rubrics as guides. It is found that is a heavy focus on answering questions based on graphs, the holistic cycle of statistical investigation is profoundly absent and hence, there were very limited opportunities for young children in Singapore to be engaged in inferential reasoning. Based on these findings, the last part of this paper suggests one possible avenue of improvement and concludes with a reflection on teaching statistics as a component of the mathematics curriculum.