Mistakes in Mathematics

Mistake 1 $\infty = -1$.

Proof. Note that

$$\sum_{k=1}^{\infty}(k+1-k) = \sum_{k=1}^{\infty} 1 = \infty$$

and

$$\begin{aligned}
\sum_{k=1}^{\infty}(k+1-k) &= \sum_{k=1}^{\infty}(-k+(k+1)) \\
&= (-1+2)+(-2+3)+(-3+4)+\cdots \\
&= -1+(2-2)+(3-3)+(4-4)+\cdots \\
&= -1+0+0+0+\cdots \\
&= -1.
\end{aligned}$$

Therefore $\infty = -1$. □

Mistake 2 1 *is the smallest positive number.*

Proof. Let the smallest postive number be called x.
x squared is also positive, therefore

$$x^2 \geq x.$$

We can divide both sides by the positive number x. Therefore

$$x \geq 1.$$

□

Mistake 3 *All positive integers are equal.*

Proof. Sufficient to show that for any two positive integers, A and B, $A = B$.

Further, it is sufficient to show that for all $N > 0$, if A and B (positive integers) satisfy $(\max\{A, B\} = N)$ then $A = B$.

Proceed by induction.

If $N = 1$, then A and B, being positive integers, must both be 1. So $A = B$.

Assume that the theorem is true for some value k. Take A and B with $\max\{A, B\} = k + 1$. Then $\max\{A - 1, B - 1\} = k$. And hence

$$(A - 1) = (B - 1).$$

Consequently, $A = B$. □

Mistake 4 *All numbers are equal to zero.*

Proof. : Suppose that $a = b$. Then

$$a = b;$$

$$a^2 = ab;$$

$$a^2 - b^2 = ab - b^2;$$

$$(a + b)(a - b) = b(a - b);$$

$$a + b = b;$$

$$a = 0.$$

□

Mistake 5 $1\$ = 1c$.

Proof. The following gives you a sense of money disappearing...

$$1\$ = 100c = (10c)^2 = (0.1\$)^2 = 0.01\$ = 1c.$$

□

Mistake 6 $1\$ = 10c$.

Proof. We know that

$$\$1 = 100c.$$

Divide both sides by 100,

$$\$1/100 = 100/100c.$$

$$\$1/100 = 1c.$$

Take square root both side,

$$\sqrt{\$1/100} = \sqrt{1c}.$$

Thus

$$\$1/10 = 1c.$$

Multiply both side by 10,

$$\$1 = 10c.$$

□

Mistake 7 $1 = -1$.

Proof.

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{(-1)}\sqrt{(-1)} = -1.$$

□

Mistake 8

$$1 + 2 + 4 + 8 + 16 + ... = -1.$$

Proof. Let

$$x = 1 + 2 + 4 + 8 + 16 + ...$$

Then

$$2x = 2 + 4 + 8 + 16 + ...$$

Thus

$$x = 1 + 2x.$$

Therefore $x = -1$. □

Mistake 9 $\log(-1) = 0$.

Proof.

a)
$$\log[(-1)^2] = 2\log(-1).$$

b)
$$\log[(-1)^2] = \log(1) = 0.$$

Combining a) and b) gives:
$$2 \times \log(-1) = 0.$$

Divide both sides by 2:
$$\log(-1) = 0.$$

□

In the following are some wrong proofs given for correct results.

Result 1 *For any non-negative integer k,*

$$\int_0^\pi \frac{1-\cos kx}{1-\cos x}dx = k\pi.$$

Wrong Proof

Proof. As

$$\frac{1-\cos kx}{1-\cos x} = \frac{-\cos kx}{-\cos x} = \frac{\cos kx}{\cos x} = \frac{kx}{x} = k,$$

we have

$$\int_0^\pi \frac{1-\cos kx}{1-\cos x}dx = \int_0^\pi k dx = k\pi.$$

□

A Correct Proof

Proof. For any non-negative integer k, let

$$f_k = \int_0^\pi \frac{1-\cos kx}{1-\cos x}dx.$$

It is clear that $f_0 = 0$, $f_1 = \pi$ and

$$f_2 - f_1 = \int_0^\pi \frac{\cos x - \cos 2x}{1-\cos x}dx = \int_0^\pi (1 + 2\cos x)dx = \pi.$$

For any non-negative integer k,

$$\begin{aligned}
& \cos kx - \cos(k+1)x - \cos(k+2)x + \cos(k+3)x \\
= & \; 2\sin\frac{x}{2}\sin\frac{(2k+1)x}{2} - 2\sin\frac{x}{2}\sin\frac{(2k+5)x}{2} \\
= & \; 2\sin\frac{x}{2}\left(\sin\frac{(2k+1)x}{2} - \sin\frac{(2k+5)x}{2}\right) \\
= & \; -2\sin\frac{x}{2}\sin x\cos\frac{2k+3}{2}x \\
= & \; 2(\cos x - 1)\cos\frac{x}{2}\cos\frac{2k+3}{2}x \\
= & \; 2(\cos x - 1)\left[\cos(k+1)x + \cos(k+2)x\right].
\end{aligned}$$

Thus for any non-negative integer k,

$$\begin{aligned} & (f_k - f_{k+1}) - (f_{k+2} - f_{k+3}) \\ = & \int_0^\pi \frac{\cos kx - \cos(k+1)x - \cos(k+2)x + \cos(k+3)x}{1 - \cos x} dx \\ = & -2 \int_0^\pi (\cos(k+1)x + \cos(k+2)x) dx \\ = & \ 0, \end{aligned}$$

implying that for any non-negative integer k,

$$f_{k+3} - f_{k+2} = f_{k+1} - f_k.$$

This result further implies that for any $k \geq 1$, if k is even, then

$$f_{k+1} - f_k = f_1 - f_0 = \pi;$$

and if k is odd, then

$$f_{k+1} - f_k = f_2 - f_1 = \pi.$$

Hence $f_{k+1} - f_k = \pi$ for any non-negative integer k. Therefore

$$f_k = k\pi.$$

$\square$