effectve practice in the classroom (see Recommendation 3 of the TE[21] report, Low et al., 2009).

While the importance of reflection is not new to education, the techniques of reflective practice are relatively recent. Here's a useful model for structuring your reflections (Low et al., 2009, p. 72):

- **Observe** *What happened?*
- **Reflect** *Why?*
- **Plan** *So what?*
- **Act** *Now what?*

This cyclical process facilitates reflection in action and on action. Practised over time, as a planned and structured exercise, it helps to build personal-professional knowledge.

As a couple, Gerald and Elaine also take time each day to interact with each other about their day and the challenges they face on the job. These "verbal reflections" have become part of their daily interactions.

They make a conscious effort draw conclusions and not just complain about their day. Though Elaine may have been teaching for longer, she says Gerald helps her to see issues from a different perspective.

A critical practice

In planned and structured reflection, what we do is try to mentally restructure an experience or a problem (Korthagen, 2001). Actively reflecting on our own practice can help us see where to improve.

"The reflection helps to anchor me down, to know which direction I should be moving towards," says Elaine. "Because you hear many voices from different stakeholders, different teachers, different parents—you need to sit down and reflect on what you're doing, on your principles."

For Gerald, as a relatively new teacher, he feels it is necessary to interact with someone as part of the process of reflection. "It gets you to organize your thoughts," he says. "Because as a new teacher, every experience is new, so you tend to be bombarded by a lot of new things."

Reflection may require quite a bit of time, but Gerald and Elaine believe it is time well spent—and they say it gets easier with more practice.

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Read more about the TE[21] recommendations in our previous issues of SingTeach.

MATH ED

Are You Game Enough?

by Joseph B. W. Yeo

Children love games! But it can be more than just fun and games when mathematically rich games are used in the classroom. Learn how games can help your students acquire the skills of mathematical investigation.

Article highlights

- How can mathematically rich games aid in learning?
- Do students know what and how to investigate when faced with a math problem?
- What are the thinking processes students use in investigating?

Many math educators believe in making math real to students. Playing mathematically rich games is one way to engage both their hearts and minds.

Such games are very real to students because the outcome—whether they win or lose—matters to them (Ainley, 1988). They may become more interested in looking for a way to win the game (Civil, 2002).

A winning strategy

The fact is, we all want to win! And when people don't know how to solve a problem, they will start investigating and exploring various solutions. Problem solving and investigation are essential skills in our daily lives (Carraher & Schliemann, 2002).

Likewise, finding a winning strategy for a game involves the application of problem-solving heuristics. For example, we can solve problems by working backwards or by considering all possible scenarios.

I distinguish between a *sure-win strategy*, which will ensure a win for a player, and a *winning strategy* that maximizes the chance of winning for a player if a sure-win strategy does not exist, such as in the game *Fifteen* (described below).

Investigating mathematical investigation

In a recent study, I used mathematical games to examine the nature and development of cognitive and metacognitive processes when students engage in mathematical investigation.

A group of 20 Secondary 2 students was presented with a game called Fifteen (Mason, Burton, & Stacey, 1985). The rules of the game are simple:

> *Place 9 discs marked with the digits 1 to 9 on the table. Two players take turns to pick one disc from the table. The first player to obtain the sum of 15 among any 3 of his discs wins.*

The students were tasked to explore and investigate. What they were *not* told is that there is a winning strategy.

I wanted to see if they knew what and how to investigate, and if they understood what a winning strategy or a sure-win strategy is, among other things.

Knowing how to start

When faced with this game, most of the students did not even know where to start. The idea of finding a winning or sure-win strategy was alien to most of them.

Many mathematics educators are surprised why students do not have a correct conception of a winning or a sure-win strategy. I suggest that this is because most of the games that students play in their daily lives have no such strategy, so such an idea contradicts with their real-life experiences.

This finding is in line with what Civil (2002) found out when she played another game called *Nim* with her students. In this game, there is a sure-win strategy for the player who starts first, but her students mistakenly thought that their ability to win depended on the other players' moves.

Winning the game

So, what is the winning strategy for Fifteen?

To solve this game, the students needed to be familiar with the Magic Square and Tic Tac Toe—games they have all played in their childhood.

Magic Square

Magic Square is something that many of our students would have learnt in primary school. In a 3-by-3 Magic Square (see Fig. 1), all three numbers in each row, column and diagonal add up to 15.

8	1	6
3	5	7
4	9	2

Figure 1. Magic Square.

Some of the students did manage to link this game to the Magic Square, but they failed to consider all the possibilities: Are there anymore combinations of three numbers whose sum is 15, other than the eight combinations as shown on the Magic Square?

To win this game, you need to prove that there are no more combinations. Only then can you apply the winning strategy for Tic Tac Toe to the Magic Square. But none of the students were able to link this game to Tic Tac Toe. (In fact, many people do not realize that there is a winning strategy for Tic Tac Toe!)

In the game of Fifteen, a winning strategy for the player who starts first is as follows:

- **Turn 1**: Pick the number 8. If the second player does not pick 5, the first player wins!
- **Turn 2**: If the second player picks 4 the first player can win by picking 6, which will force the second player to pick 1 (to prevent the first player from winning by 8 + 6 + 1 = 15).
- **Turn 3**: The first player can then pick 2 and he will win in two ways: 8 + 2 + 5 = 15 *or* 6 + 2 + 7 = 15, which the second player cannot prevent.

This looks very confusing but it becomes clearer when you try the above moves as if playing Tic Tac Toe on the Magic Square (see Fig. 2).

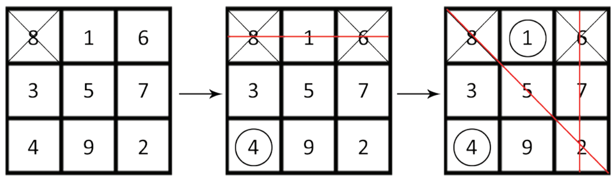

Figure 2. The game of Fifteen.

The challenge, of course, is to play this game without drawing a Magic Square in front of you, to prevent the other player from knowing the winning strategy. This makes the game a lot more complicated and interesting.

Processes for mathematical investigation

What are the thinking processes involved when students play mathematically rich games?

To investigate the winning strategy for a game, students have to start by examining specific scenarios or cases (*specializing*). The next step involves formulating hypotheses or conjectures (*conjecturing*) and testing them. If the conjectures are proven correct (*justifying*), they can then be considered as generalizations of the specific cases (*generalizing*).

These are the four core mathematical thinking processes described by Mason et al. (1985).

For the game Fifteen, if the students are somehow able to see a link between the game and the Magic Square, they would then have some conjecture of how they can win. By confirming it, they can then generalize this to a wider number of cases.

Playing mathematical games such as Fifteen is not just about winning and losing. It is a great way to enrich the learning of math. Plus, it's a lot of fun!

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Further readings and resources are available on the SingTeach *website.*

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LANGUAGE ED

Let's Talk About Teaching Tamil

Mandarin. Malay. Tamil. These three languages are often lumped together under the umbrella term of "mother tongue languages". One of these languages requires more attention to speaking, and another to reading. Do you know which? And are your teaching strategies and tasks appropriate to the needs of the particular language?

Article highlights

- Is one mother tongue language more difficult to learn than another?
- How can learning tasks be redesigned to suit the language?
- How can drama be used in the teaching of Tamil language?

Many assume that the same teaching and learning strategies can be applied to all mother tongue languages. As a result, language reforms in Singapore have tended to look to the Chinese language review (Ministry of Education, 2004) to "lead the way" (Lakshmi, Vaish, & Gopinathan, 2006).

While there are some commonalities to second language teaching, a uniform approach cannot work for all languages, says Dr Seetha Lakshmi, Associate Professor with the Asian Languages and Cultures Academic Group at NIE.

"Each language has its own unique characteristics," explains Seetha. "The process of familiarization of each language script is not same."

Comparing the mother tongue languages

The Malay language has a Romanized written script. This means that Malay words are essentially transliterations into English letters. "Once you know the alphabet, it's easy to write anything," says Seetha.

Chinese and Tamil scripts, however, are not Romanized. Chinese students tend to have more difficulties with reading and writing, compared with speaking, because of the *logographic* nature of the language. "In Chinese, you can't just spell out the word," explains Seetha. "Students have to memorize the whole picture that represents the word."

As a result, the *Report of the Chinese Language Curriculum and Pedagogy Review Committee* recommended an "early *reading* approach" for Chinese (MOE, 2004, p. vii). With the early use of *hanyu pinyin*, a Romanized phonetic system, Chinese students now have a headstart in learning to read and write as this helps them to recognize more Chinese characters at an early age.

But for Tamil, it's actually the converse. Tamil students tend not to have as many issues with reading and writing as with speaking. Thus, Seetha recommends an "early *speaking* approach" to teaching the Tamil language.

Focusing on speaking

It seems that our Tamil students are not proficient in the language because they do not speak it enough. They score well on their Tamil written exams as they are trained to be exam-smart, says Seetha. However, when talking with friends and family members, they tend to switch to English rather than speak in Tamil.

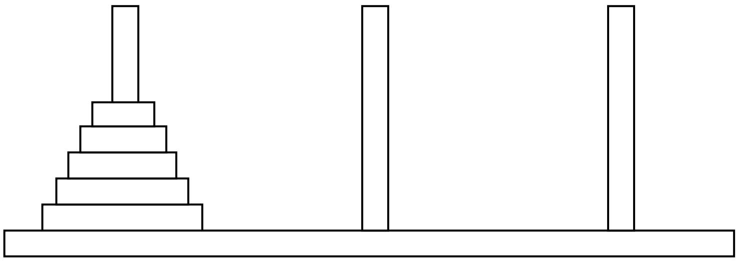
Mathematically Rich Games

by Joseph B. W. Yeo

Here are some mathematically rich games you can play in your math classroom for your students to develop the skills of problem solving and investigation.

1. Tower of Hanoi

The objective of the game is to move all the discs from one pole to another (there are a total of three poles) using the smallest number of moves possible. You can only move one disc at a time, and you cannot place a disc on top of a smaller one.



Heuristics:	Simplify the problem (start with a smaller number of discs) Solve part of the problem Working backwards Looking for patterns Logical reasoning
Thinking processes:	Specializing Conjecturing Justifying Generalizing
Extension:	What happens if there are four or more poles?

There is actually a mathematical formula for the minimum number of moves. You can play this game online at www.mazeworks.com/hanoi

2. Nim

There are many versions of this game. One version involves two players and a pile of 12 discs. Each player takes turn to pick either 1, 2 or 3 discs from the pile. The player who picks the last disc wins. This version has a sure-win strategy for the second player.

Heuristics:	Working backwards Looking for patterns Logical reasoning
Thinking processes:	Specializing Conjecturing Justifying Generalizing
Extension:	What happens if each player can pick 1, 2, 3 or 4 discs? What happens if there are a total of *n* discs and each player can

pick 1, 2, 3, … or *m* discs?

What happens if there are *p* piles of discs where each pile contains *n* discs each and each player can pick *any* number of discs from any one pile at any one time?

What happens if the *p* piles do not contain equal numbers of discs in each pile?

What happens if a pile of discs can be separated into two piles, depending on how a player picks a disc?

This last scenario is actually what happens in the following version of *Nim*, which is commonly played in schools. Each player can cancel any number of sticks in any one row at any one time. If the player cancels two sticks as shown below, the last row has effectively split into two rows, and a player cannot cancel “across the cancelled sticks”.

This version also has a sure-win strategy for either the first or the second player, depending on how many starting rows of sticks there are. If there are 7 rows, as shown below, the second player will win if he knows the sure-win strategy.