The Effect of Exploratory Computer-Based Instruction on Secondary Four Students' Learning of Exponential and Logarithmic Curves

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The computer stand betwixt and between[1] the world of formal systems and physical things; it has the ability to make the abstract concrete.

Turkle and Papert (1990, p. 346)

[1] The phrase "betwixt and between" means "partly one and partly the other".

ABSTRACT

The study investigated the effect of exploratory computer-based instruction on pupils' conceptual and procedural knowledge of graphs, and the affective issues towards the use of computers in mathematics. Many previous studies compared the effect of computer-assisted instruction with traditional teacher-directed teaching and any difference in performance might be due to a different pedagogical approach instead of the use of information technology (IT). In this study, both the experimental and control classes were taught using a guided discovery method to explore the characteristics of the exponential and logarithmic curves. One class used an interactive computer algebra system called *LiveMath*, while the other did not have access to IT. The findings indicated a significant difference in pupils' conceptual and procedural knowledge although there was no significant difference in their affect towards mathematics in general and towards the topic in particular. The pupils in the experimental class also showed a moderately positive affect towards the use of IT. This seemed to suggest that there was an inherent advantage of using IT to explore mathematical concepts.

1. INTRODUCTION TO THE STUDY

This chapter introduces the study by looking at the background, purpose and significance of the research. Then it provides three theoretical frameworks and a research framework for the five research questions and nine null hypotheses.

1.1 Background of the Research

Technology has definitely changed our lives dramatically. The ways we communicate, interact and do business have revolved around the computer. With such an impact on our society, it is inevitable that educators desire to harness information and communication technology to develop their pupils to the fullest potential (Mariotti, 2002; Nickerson, 1988).

In the United Kingdom, the Cockcroft Report (1982) devoted a chapter to the use of computers in the teaching of mathematics. But there was no widespread adoption of the computer until the Department of Trade and Industry initiative put one in every school (Ernest, 1989). In the United States, the NCTM Curriculum and Evaluation Standards (1989) viewed the learner as actively participating in the exploration and construction of mathematical relationships with the help of information and communication technology and several billions were spent in the 1990s to install computers in every school (Noble, 1998). In Singapore, the vision of the Ministry of Education, “Thinking Schools, Learning Nation”, gave impetus for the infusion of the information technology (IT) initiative into the school curriculum (Ministry of Education, 1997).

However Noble (1998) pointed out that despite the fact that teachers in the United States celebrate the importance of computers in schools, most of them have never used one. There is a variety of reasons but one of them is the perception of the irrelevance of the use of computers in teaching and learning because of teachers' beliefs about the nature of mathematics and mathematics learning (Thompson & Thompson, 1996). A study by Norton and McRobbie (2000) revealed why a highly computer literate teacher refused to use computers in teaching. The teacher believed that mathematics is objective and logical, or what Ernest (1991) called the absolutist nature of mathematics. The teacher also believed that explanation and student practice was the most effective way of teaching mathematics. So there is no place for pupils to explore mathematical concepts and to construct their own knowledge. As a result, he refused to use computers in his teaching.

The local scene is similar. Research has suggested that teachers are not convinced of the relevance and benefits of the use of computers in education (Ang, 1999; Cheong, 2001; Ong-Chee, 2000). However, unlike their counterparts in the United Kingdom and the United States, the IT Masterplan requires all local teachers to use 30% of curriculum time for IT lessons. A survey by Ang (1999) suggests that many schools had yet to exert any pressure on their teachers to use IT in their lessons and so these teachers ended up using very little of their curriculum time for computer-based lessons. However, in other schools that emphasise the use of IT, their teachers have no choice but to find ways of clocking the required time.

The most pressing question is whether pupils benefit from such infusion of IT into the curriculum. It is the quality of the IT lessons that educators should be concerned with, not the quantity (Dimmock, 2000). Jensen and Williams (1993) observed that many educators exhibit a closed mindset when they use software that resembles electronic workbook pages. Roberts and Jones (2000) discussed the naïve model of online learning, which is characterised as "putting" lecture notes on the World Wide Web. In the classroom, this translates to converting transparencies to PowerPoint presentations. Is this the direction that mathematics teachers should go?

In Singapore, the Ministry of Education launched the second IT Masterplan (or mp2) in July 2002 to make better use of technology to stimulate thinking and creativity among the pupils (Shanmugaratnam, 2002). A key feature of mp2 is to integrate IT into the design of a more flexible and dynamic curriculum by taking into account new teaching methods that are made possible by technology (Ministry of Education, 2003). This is a different approach from the current practice of using IT to support an existing curriculum. The mp2 will also seek to change the current predominant pedagogy of a teacher-centred use of IT to a more pupil-centred strategy for learning.
However, does research show that such computer-based lessons actually benefit the students? First, research in this area is already very limited (Kaput & Thompson, 1994). Secondly, many of these studies are more anecdotal than conclusive because they did not control for other influences such as differences between teaching methods (Oppenheimer, 1997). Many local studies, such as Ho (1997), Ong (2002) and Tan (1987), found that pupils who used IT to explore mathematical concepts performed significantly better in achievement tests than pupils who were taught using traditional teacher-directed teaching. But this might be due to the use of a different

pedagogy rather than the use of technology (Oppenheimer, 1997). Therefore there is a need for more research studies that control these extraneous variables so that the findings will be more conclusive. Otherwise, the government will be wasting a lot of taxpayers' money on technology that fails to deliver the desired outcomes of education.

1.2 Purpose and Significance of the Study

Therefore the purpose of this study is to investigate whether an exploratory approach using computer technology helps pupils to construct knowledge better, in particular, conceptual and procedural knowledge (see Sections 2.8.3 & 2.8.4) in the area of graphs. It also seeks to find out whether such an approach helps students to improve their affect towards mathematics since it has been well established that there is a positive correlation between attitude and achievement in mathematics (see Section 2.6.1).

This study is significant in that it will control extraneous variables, such as employing the same pedagogy for both the experimental and the control classes, so that its findings will be more reliable. It is also significant in that the three dependent variables for the research (namely conceptual knowledge, procedural knowledge and affect) are relevant because they are important parts of the five inter-related components of the Pentagon model which provides the framework for the mathematics curriculum in Singapore since 1992 (Ministry of Education, 1998; Wong, 1991). If the study shows significant improvements in pupils' knowledge and affect, teachers may be convinced to use the computer more often.

1.3 Theoretical Frameworks

The theoretical frameworks for using the computer as a tool (see Section 2.1.1) for pupils to explore mathematical concepts are social constructivism and Bruner's discovery learning.

Constructivism is a theory that knowledge is constructed by learners as they attempt to make sense of their experiences (Driscoll, 2000). Learners, therefore, are not empty vessels waiting to be filled, but rather active organisms seeking meaning. Social constructivism was proposed by Ernest (1991) as a philosophy of mathematics where subjective knowledge of individuals becomes objective knowledge of the community when it is accepted by society. This will be discussed more fully in Section 2.2.1.

Constructivism has multiple roots in educational psychology and philosophy, for example, Bruner's social emphasis and discovery learning (Driscoll, 2000). Bruner (1961) defined discovery as "all forms of obtaining knowledge for oneself by the use of one's own mind" (p. 22). He believed that the process of discovery contributes significantly to intellectual development and that the heuristics of discovery can only be learned through problem solving. This will be discussed more fully in Section 2.2.2.

The third theoretical framework for this study is based on Mandler's schematic interruptions. Mandler (1984) believed that the interruption of a schema or an ongoing cognitive activity will produce a state of arousal that may develop into another emotional expression. Repeated interruptions will usually produce a more permanent and predictable response that comes to be characterised as an attitude. This will be discussed more fully in Section 2.6.3.

1.4 Research Framework

There are generally two areas of research in the use of computers in education. The first area focuses on issues related to the infusion of IT into a school curriculum. Examples are national and school policies, nature of curriculum and assessment, cost and availability of hardware and software, training of teachers, time constraint and teacher resistance to change. The second area focuses on the effect of the use of IT on pupils' learning. Dimmock (2000) pointed out that technology should not be seen as a final goal in itself but as a means of enhancing teaching and learning.

The two areas of research are not independent of each other. The findings of the second area may help to address the problems in the first area. For example, if it can be shown by research that computer-based lessons enhance teaching and learning, then teachers may be convinced to engage pupils in this manner.

The focus of this study is on the second area. The research framework for this dissertation is represented by Figure 1 below. The independent variables are the two methods of instructions: non-IT guided discovery approach and exploratory computer-based learning (see Sections 2.8.1 & 2.8.2). The dependent variables are conceptual knowledge, procedural knowledge and affective variables (see Sections 2.8.3 – 2.8.5). The arrows represent possible causation.

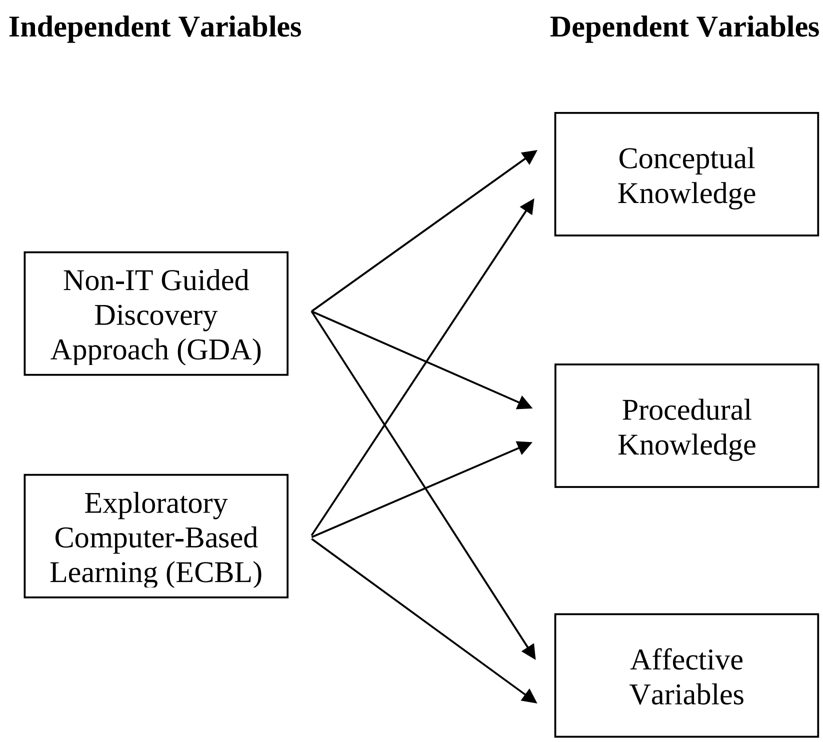

Figure 1. Research Framework

1.5 Research Questions

There are five research questions for this study.

(1) What is the effect of exploratory computer-based learning on pupils' understanding of exponential and logarithmic curves?

(2) What is the effect of exploratory computer-based learning on developing pupils' procedural skills?

(3) What is the effect of exploratory computer-based learning on pupils' affect towards mathematics?

(4) What is the effect of exploratory computer-based learning on pupils' affect towards exponential and logarithmic curves?

(5) What is the affect of pupils in the experiment group towards the use of computers in learning exponential and logarithmic curves?

1.6 Null Hypotheses

To study the first four research questions, nine statistical null hypotheses were formulated as shown below. The last research question could only be answered using descriptive statistics since there was only one set of data[2]. It may be helpful to note that the Conceptual Knowledge Test (CT), the Procedural Knowledge Test (PT) and the Questionnaire on Affect towards Graphs of Exponential and Logarithmic Functions (ATG) will be administered only after the experiment, but the Questionnaire on Affect towards Mathematics (ATM) will be administered before and after the experiment (see Section 3.7.6).

Null hypotheses 1 to 3 address Research Question 1:

(1) There is no significant difference in the mean scores of the school's Secondary Three Elementary Mathematics Final Examination (EME) between pupils in the experimental group and the control group before the experiment.

(2) There is no significant difference in the mean scores of the school's Secondary Three Additional Mathematics Final Examination (AME) between pupils in the experimental group and the control group before the experiment.

[2] Although it is possible to use a one-sample t-test to compare the mean of the set of data with some mid-value, others may question the arbitrariness of this value. So it was decided not to use a one-sample t-test but to use descriptive statistics.

(3) There is no significant difference in the mean scores of the Conceptual Knowledge Test (CT) between pupils in the experimental group and the control group after the experiment.

Null Hypothesis 4 addresses Research Question 2:

(4) There is no significant difference in the mean scores of the Procedural Knowledge Test (PT) between pupils in the experimental group and the control group after the experiment.

Null hypotheses 5 to 8 address Research Question 3:

(5) There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics (ATM) between pupils in the experimental group and the control group before the experiment.

(6) There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics (ATM) between pupils in the experimental group and the control group after the experiment.

(7) There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics (ATM) between pupils in the experimental group before and after the experiment.

(8) There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics (ATM) between pupils in the control group before and after the experiment.

Null Hypothesis 9 addresses Research Question 4:

(9) There is no significant difference in the mean scores of the Questionnaire on Affect towards Graphs of Exponential and Logarithmic Functions (ATG) between pupils in the experimental group and the control group after the experiment.

2. LITERATURE REVIEW

Sections 2.1 to 2.4 contain a review of IT-related literature. But there is also a need to review the various types of learning outcomes (Section 2.5) and the affective variables (Section 2.6) as these are contained in the research questions. This chapter concludes with a clarification of some terminologies used in the research.

2.1 Various Approaches to the Use of Computers in Education

There are a few models on the use of computers for the teaching and learning of mathematics, for example, Taylor's (1980) tutor, tool and tutee modes, Streibel's (1988) three approaches, Jonassen's (2000) mindtools, and Roberts' and Jones' (2000) four models of online teaching.

2.1.1 *Taylor's Tutor, Tool and Tutee Modes*

The most frequently used mode since the 1970s (Jensen & Williams, 1993; Manoucherhri, 1999) is the tutor mode (Taylor, 1980) where pupils learn *from* computers. This involves using computer-assisted instruction (CAI) to tutor and drill pupils in procedural skills. An example of such software used locally is the *Math Explorer* (Seow, 2002), which contrary to its name, does not allow pupils to explore mathematics. Research studies have suggested that CAI was more effective when used as a supplement to traditional instruction than the latter alone (Jensen & Williams, 1993). But when CAI was used as the sole basis for instruction, results were mixed.

The tool mode (Taylor, 1980) enables pupils to learn *with* computers. This was the most promising and exciting mode because it contained the widest range of applications and was the most active area for research and development (Jensen & Williams, 1993). It involves using application software such as *LOGO* to explore mathematical ideas. Designed by Seymour Papert and others in 1967, *LOGO* programming is simple enough for children to learn and yet powerful enough for expressing important intellectual content (U.S. Congress, Office of Technology Assessment, 1988). It uses a different style of geometry, called Turtle geometry, just as Euclid's axiomatic style is different from Descartes' analytic style (Papert, 1980). The software consists of a turtle and the user has to programme it to move.

The tutee mode (Taylor, 1980) enables pupils to learn *through* programming computers. There are two approaches. The first approach is to learn a programming language like *BASIC*. This is what Jonassen (2000) called learning *about* computers (see Section 2.1.3). Advocates of this approach believed that programming was an important life skill that should be included in the curriculum for its own sake. But Jensen and Williams (1993) believed it was unwise to spend so much curriculum time studying a programming language that is constantly changing. The second approach to the tutee mode is to learn some simple programming, like *LOGO*, and then use it to explore mathematical concepts and ideas. Jensen and Williams (1993) believed this was a more useful approach.

It may be useful to realise that some approaches encompass more than one of the three modes. For example, *LOGO* programming encompasses both the tool and tutee modes.

2.1.2 *Streibel's Three Approaches*

Streibel (1998) analysed three approaches to the use of computers in education: drill and practice, computer tutoring, and simulation and programming. The first two approaches are similar to Taylor's (1980) tutor mode. But Streibel's (1998) idea of computer tutoring goes beyond drill and practice: it has a more sophisticated form of dialogue between computer and learner. This approach seems to be the new generation of interactive drill and practice software whilst the first approach refers to the old generation of non-interactive software. The third approach of simulation and programming is similar to a combination of Taylor's (1980) tool and tutee modes. Streibel (1998) also referred this approach as the use of computers as intellectual tools, a term that is similar to Jonassen's (2000) mindtools (see Section 2.1.3).

Streibel's (1998) model demonstrates that Taylor's (1980) model of tutor, tool and tutee modes is still relevant today, even after two decades of advances in technology. But with new technology, there are better tools to make the three modes more exciting and there are also some new approaches (see Section 2.1.4).

2.1.3 *Jonassen's Mindtools*

Jonassen (2000) observed that there were previously two approaches to the use of computers in education. The first approach was learning *from* computers. This is similar to Taylor's (1980) tutor mode. The second approach was learning *about* computers or computer literacy which is similar to the first approach of Taylor's tutee mode: programming for its own sake.

Jonassen (2000) did not mention about Taylor's other modes of computer instruction in the previous two approaches probably because these other modes were not major approaches that most educators engaged in. So when he advocated a third approach, learning *with* computers or mindtools for constructive learning, it is rather similar to a combination of Taylor's tool mode and the second approach to his tutee mode. Jonassen defined his mindtools as "computer-based tools and learning environments that have been adapted or developed to function as intellectual partners with the learner in order to engage and facilitate critical thinking and higher order learning" (p. 9). These tools include, but are not limited to, databases, semantic networks, spreadsheets, systems modeling tools, microworlds, intentional information search engines, multimedia publishing tools, live conversation environments and asynchronous computer conferencing.

Jonassen (2000) believed that learning *from* and *about* computers was not helpful to engage pupils in meaningful learning because he subscribed to the constructivist view that knowledge could not be transmitted from teacher to learner but constructed by the learner himself. His mindtools helped learners to construct such knowledge by the building of models. Jonassen (2002) made a comparison amongst the learning power of four modes of instruction: remembering (5%), arguing (45%), model using (50%) and model building (90%). He believed that model building using mindtools would enable pupils to learn better and retain more. However he had no objection to model using, as long as computers were not used as a presentation tool or for drill and practice.

2.1.4 *Online Learning*

With the advance of the Internet and communication technology in the 1990s, there is now another mode for using computers in education. Roberts and Jones (2000) discussed four models of online teaching: naïve, standard, evolutionary and radical. The naïve model is the most widely used and is characterized as “putting lecture notes on the World Wide Web” with no opportunities for interaction or feedback. The standard model allows some degree of communication using electronic mails. The evolutionary model goes one step further: the content and delivery will evolve from time to time, based on student feedback. In the radical model, pupils are assigned to groups and they learn by interacting among themselves using existing Web-based resources with guidance from teachers only when it is required.

The radical model of online teaching is the closest to Jonassen’s (2000) mindtools using intentional information search engines, live conversation environments and asynchronous computer conferencing, where pupils collaborate and construct knowledge on their own with some guidance from teachers. But there are also Internet portals where teachers put up lecture notes and pupils are drilled on examination-type questions. An example of such portal used locally is *Postkid.com* (Lee Y. K., 2002).

To conclude this section, there are various approaches to the use of computers in education. However mathematics educators may prefer an approach over the others because of what they believe in the nature of mathematical learning (see Section 2.2). For example, Taylor (1980) preferred the tutee mode and Jonassen (2000) preferred his mindtools to build models. But due to the time constraint in this study, the pupils did not build the models themselves. Instead, they used pre-built models as a tool to explore the graphs of exponential and logarithmic functions.

2.2 Theoretical Bases for Tool Mode

If an educator believes that knowledge is transmitted from teacher to learner, then he (or she) will most likely engage in the tutor mode of computer-based instruction. But if he believes that knowledge is constructed by the learner, he will most likely engage in the tool or tutee mode where the learner explores and constructs his own mathematical knowledge. In this study, the theoretical bases for pupils to use the tool mode for exploration are social constructivism and Bruner's discovery learning.

2.2.1 *Social Constructivism*

Constructivism is a theory that knowledge is constructed by learners as they attempt to make sense of their experiences (Driscoll, 2000). Learners, therefore, are not empty vessels waiting to be filled, but rather active organisms seeking meaning. This theory has multiple roots in educational psychology and philosophy, for example, in Piaget's cognitive and developmental perspectives, Bruner's and Vygotsky's social emphasis and Dewey's philosophy of reflective thinking. von Glasersfeld (1987, 1990, 1994) has had a considerable influence on constructivist thinking in mathematics and science education.

Many people have misunderstood that constructivists subscribe to the view that all knowledge is equally valid because it is personally constructed (Jonassen, Peck, & Wilson, 1999). The litmus test for the knowledge that individuals construct is its viability. Within any knowledge-building community, shared ideas are accepted and agreed upon. If individual ideas are discrepant from community standards, they are not regarded as viable unless new evidence supporting their viability is provided.

Ernest (1991) proposed social constructivism as a philosophy of mathematics where subjective knowledge of individuals becomes objective knowledge of the community when it is accepted by society. And when individuals try to construct the socially-accepted objective knowledge in their own minds, it becomes subjective knowledge to them. This is how subjective and objective knowledge interplay to become a full cycle. If the subjective knowledge that individuals have constructed is different from the socially-accepted objective knowledge, then the former is not viable.

2.2.2 *Bruner's Discovery Learning*

Bruner's (1973) idea of discovery learning is quite similar to the theory of constructivism because he believed that discovery teaching "generally involves not so much the process of leading students to discover what is 'out there', but rather, their discovering what is in their own heads" (p. 72). Bruner (1961) defined discovery as "all forms of obtaining knowledge for oneself by the use of one's own mind" (p. 22). He believed that the process of discovery contributes significantly to intellectual development and that the heuristics of discovery can only be learned through problem solving.

Bruner (1973) believed that discovery is not haphazard but is guided by a model. He, Goodwin and Austin proposed in 1956 a concept attainment model that exemplifies this notion of discovery teaching (cited in Driscoll, 2000). Concepts are rules for organizing the regularities of experience and so they stand as models of the world to be constructed and internalized. Learners acquire concepts by setting forth tentative hypotheses about the attributes that seem to define a concept and then testing specific instances against these hypotheses. Discovery of a concept proceeds from systematic comparisons of instances for what distinguishes examples from non-examples. To promote concept discovery, the teacher presents the set of instances that will best help learners to develop an appropriate model of the concept. Guided practice in inquiry and sufficient prior knowledge constitute minimum conditions for discovery learning to be successful. Other useful processes include reflection, and contrast that leads to cognitive conflicts.

A guided discovery approach means that there is a need for some telling and explanation (Jaworski, 1994). How much to tell depends on the teachers' sensitivity to the needs of their pupils. Noddings (1990) believed that straightforward telling may sometimes facilitate genuine learning if it is clear that performance errors are getting in the way of concentrating on more important problems. Teong (2002) questioned the need for children to rediscover formulae that mathematicians take years to construct. Although she strongly believed in the investigative approach, she agreed that teachers should just use the expository approach to teach their pupils what these mathematical conventions are because there is ultimately only one way of representing conventions accepted by the mathematics community.

This study employed a guided-discovery approach where pupils used pre-built computer-based models to explore and construct mathematical concepts of the graphs of exponential and logarithmic functions. The models were built using a computer algebra system called *LiveMath*.

2.3 Computer Algebra Systems

A computer algebra system is a type of software that can perform algebraic manipulations such as factorization, differentiation and integration. It usually includes graph plotting utilities. Common examples of CAS are *Maple*, *Mathematica* and *Derive*. There are also handheld CAS or algebraic calculators such as *TI-89*. Asp and McCrae (2000) observed that mathematicians and engineers had always used CAS as symbolic manipulators to solve complex algebraic and calculus problems. They believed that CAS, when used in school, would challenge the emphasis placed on algebraic manipulations.

There are three main issues in the use of CAS in school. First, does the use of CAS result in a loss of by-hand algebra skills and conceptual understanding? Second, how much do pupils need to know by-hand algebra skills? Third, what are the benefits of using CAS in school?

In the first issue, critics are concerned that the use of CAS may result in a loss of pupils' by-hand algebra skills and conceptual understanding. But proponents cited research to show that the use of CAS not only did not reduce pupils' symbolic computational skills but had a positive impact on their conceptual learning. For example, Heid and Baylor (1993) cited five studies which suggested that there was no loss in by-hand algebra skills for pupils using CAS. One of these studies by Heid (1988) on the effects of concentrating initial and primary attention on the concepts and applications of calculus while relegating execution of routine procedures to the computer found that the experimental classes showed better conceptual understanding than traditional classes, without noticeable loss in by-hand algebra. Another study by Palmiter (1991) found that pupils using CAS for homework could learn integration concepts in substantially less time than traditional classes. For more studies, please refer to Asp and McCrae (2000), Kaput (1992) and Monaghan (1994).

However critics are quick to point out that pupils from the experimental class actually performed worse in by-hand algebra skills than those from the control class although the researchers might claim that these detrimental effects were not significant. To address this problem, it is helpful to examine the purpose of learning symbolic computational skills. This leads us to the second issue: how much do pupils need to know by-hand algebra skills?

The purpose of learning by-hand algebra skills was because there was previously no other way to perform symbolic manipulations such as the differentiation of a product or quotient of two functions. But now that we have a CAS to do these tedious jobs, why do we spend mindless hours practising these skills?

Monaghan (1994) made a comparison between the use of CAS and the use of scientific calculators in school. When the latter were first introduced, there were concerns that their use would reduce pupils' paper-and-pencil arithmetic skills.

However calculators are now widely used in schools to extract square roots, find logarithms and perform tedious arithmetic evaluations so that pupils can concentrate on solving problems, although they are still expected to perform simple mental arithmetic. It will be to the detriment of mathematics if pupils use a calculator to simplify $12 \div 3$. The same can be said of by-hand algebra skills. It will be to the detriment of mathematics if pupils use a CAS to simplify $(2x + 4xy) / 2x$. Advocates of CAS believe that pupils should learn how to perform simple by-hand algebra skills but not mindless practice of more complex algebraic skills. The purpose of a CAS is to free up the time spent on tedious symbolic manipulations so that pupils can concentrate on other more important tasks.

This leads us to the third issue: what are these important tasks or what are the benefits of using CAS in school? First, researchers have consistently found that CAS helped the problem solver focus on generating ideas and trying out various approaches rather than expending energy on its computational aspects (Jensen & Williams, 1993). Secondly, pupils can use a CAS as an exploratory tool to form and develop mathematical concepts. For example, Hunter and Monaghan (1993) used a CAS for two weeks with a Year 10 group to explore quadratic graphs, graphical solutions of quadratic equations, expanding and factorizing expressions and solving equations. Monaghan (1994) suggested using a CAS to expand and factorize quadratic expressions so that pupils could discover and later prove the rules for themselves. Pupils can also learn quadratics via the transformation of functions and graphs (Oldknow & Taylor, 2000). This type of approach is a departure from the traditional way of using CAS as a symbolic manipulator to solve problems (Asp & McCrae, 2000). It is more of a discovery approach for concept formation and development.

However most of the software available at that time required pupils to key in the algebraic expressions for all the cases they were exploring. This became tedious when the expressions were complicated. However, a little known interactive software at that time, called the *Theorist*, allowed pupils to set up a template by keying in the algebraic expressions once, and then explore by changing the values of certain quantities. It was so much easier than the other non-interactive CAS. Even Kaput (1992) found *Theorist* "intriguing because of a unique user interface that allows one to perform 'natural' algebraic maneuvers even more 'naturally' than one can achieve them on paper" (p. 534). The software has since changed its name to *MathView* and then *LiveMath*. This is the software used in the dissertation (see Section 3.3).

To sum up this section, the use of CAS appears to reduce pupils' by-hand algebra skills although most researchers claim that it is insignificant. But the advantages that come with the use of CAS, such as more focus for problem solving and more time for concept formation, outweigh the "not too adverse" loss in symbolic manipulations.

2.4 Local Research on the Use of Computers in Mathematics Education

Software vendors have tried unsuccessfully to sell CAS to local secondary schools because they advocate using it as a symbolic manipulator but pupils are not allowed to use them in their examinations. So this type of software is not very popular and there are currently no local research studies on the use of a CAS. Of the limited number of local research studies on IT, only fourteen deal with mathematics education and out of these fourteen, only nine are quasi-experimental designs (see Appendix A). The latter used software like *BASIC*, *LOGO*, *Graphmatica* and *Geometer's Sketchpad (GSP)* to explore topics like graphs, angles, transformation geometry and angle properties of circles.

There are a few similarities among the nine quasi-experimental research studies. First, most of the authors referred to the mode of instruction for the experimental class as "computer assisted instruction" or CAI (Ho, 1997; Lee-Leck, 1985; Ong, 2002; Woo-Tan, 1989; Yeo, 1995). This phrase is usually associated with the tutor mode (see Section 2.1.1). But the way they used the software is actually the tool mode. Therefore, to avoid confusion, this dissertation uses the term "computer-based instruction" to describe the tool mode of instruction (see Section 2.8.1).

Second, most of these studies used the traditional teacher-directed teaching for the control class (Ho, 1997; Lee-Leck, 1985; Ong, 2002; Tan, 1987; Woo-Tan, 1989; Yeo, 1995). But their experimental class employed the tool mode to explore the mathematical concepts. So any significant difference in the learning outcomes may be due to the difference in approaches rather than the use of IT. Therefore this dissertation uses the same exploratory approach for both the experimental and the control classes (see Section 3.5).

Third, most of these studies investigated the effect of IT lessons on pupils' academic achievement which is biased towards procedural skills (Ho, 1997; Lee-Leck, 1985; Lee C. M., 2002; Ong, 2002; Tan, 1987; Woo-Tan, 1989; Yeo, 1995). But conceptual knowledge is also very important (see Section 2.5.1). Therefore this dissertation investigates the effect of computer-based instruction on both conceptual and procedural knowledge.

Fourth, most of these studies found that computer-based learning enhances academic achievement (Ho, 1997; Ingham, 2001; Lee-Leck, 1985; Ong, 2002; Tan, 1987; Woo-Tan, 1989) although Yeo (1995) discovered that this applied only to pupils with medium ability. But the findings for pupils' affect towards the particular topic taught and towards mathematics in general were mixed.

To sum up this section, what is lacking in local research are quasi-experimental studies that use the same pedagogical approach for both experiment and control groups to find out whether computer-based learning enhances both conceptual and procedural knowledge.

2.5 Types of Knowledge and Understanding

It is important to know what we want our pupils to learn with the help of computers. There are many types of knowledge: skills, processes and learning outcomes. However this study focuses only on conceptual and procedural knowledge. In this section, the author will review literature on conceptual and procedural knowledge and the importance of conceptual or relational understanding.

2.5.1 *Conceptual and Procedural Knowledge*

Hiebert and Lefevre (1986) characterised conceptual knowledge as "knowledge that is rich in relationships [and] can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information" (pp. 3-4). The development of conceptual knowledge is achieved by the construction of relationships between pieces of information. This linking process can occur between two pieces of information that have already been stored in memory, or between an existing piece of knowledge and one that is newly learned. The former is called an insight when previously unrelated items are suddenly related in some way and this is the basis of discovery learning (Bruner, 1961). The latter is called understanding (Skemp, 1971) or meaningful learning (Greeno, 1983).

Hiebert and Lefevre (1986) divided procedural knowledge into two distinct parts. The first part consists of the formal language, or symbol representation system, of mathematics and an awareness of syntactic rules for writing symbols in an acceptable form. The second part consists of the algorithms, or rules, for completing mathematical tasks. A key feature of procedures is that they are executed in a linear sequence, unlike conceptual knowledge.

VanLehn's (1986) idea of schematic and teleological knowledge is rather similar to procedural and conceptual knowledge respectively. He used the procedure for making gravy as an illustration of the difference between these two types of knowledge. A novice cook often has only the schematic (or procedural) knowledge for the gravy recipe – which ingredients to add in which order. The expert cook usually realises that the order is crucial in some parts but arbitrary in others. He also knows the purposes of various parts of the recipe and so he can easily substitute any part with some other ingredients. This is called teleological (or conceptual) knowledge.

Carpenter (1986) discussed the importance of conceptual knowledge as the foundation of procedural knowledge. However he contended that connections between conceptual and procedural knowledge are established *gradually*. Advances in conceptual knowledge do not immediately result in advances in all related procedures. This link between procedural and conceptual knowledge is called utilization competence (Greeno, Riley & Gelman, 1984). It is the ability to access the performance requirements of a particular task in light of the constraints imposed by conceptual competence. A pupil who is not competent in utilization may not know what procedure to apply when faced with an unfamiliar task.

Silver (1986) maintained that it might be more reasonable and appropriate to observe that procedural knowledge can be quite limited unless it is connected to a conceptual base, rather than arguing that conceptual knowledge is the foundation of procedural knowledge. However it is reasonably clear that one can demonstrate procedural fluency without conceptual knowledge. This is called rote-learning.

2.5.2 *Relational and Instrumental Understanding*

Closely related to the notion of conceptual and procedural knowledge is the issue of meaningful versus rote learning (Hiebert & Lefevre, 1986). Meaningful learning develops conceptual knowledge. Skemp (1976) called it relational understanding because it builds relationships between pieces of information. It is knowing both what to do and why. Rote learning, on the other hand, produces knowledge that is absent in relationship and is tied closely to the context in which it is learned (Hiebert & Lefevre, 1986). Conceptual knowledge cannot be created by rote learning but procedures can be learned by rote. However a pupil who rote learns will face great difficulty when trying to apply procedural skills to solve unfamiliar problems. Skemp

(1976) called this instrumental understanding which is actually not an understanding at all because it is rules without reasons.

Sadly, there are many teachers who engage in teaching for instrumental understanding and many pupils who learn by rote (Skemp, 1976). Skemp went further to suggest that mathematics has now been taught in such a way that it could be called instrumental *mathematics*. Within its own context, instrumental mathematics is usually easier to *learn* and the rewards are more immediate. But it is less adaptable to new tasks and ironically harder to *remember* because it involves a multiplicity of separate rules. In contrast, there is more to learn in relational mathematics – the connections as well as the rules (Skemp, 1976). But it is more adaptable to new tasks and these are the types of problems that pupils will face when they go out to work. That is why there is going to be a shift in Singapore junior college (JC) curriculum from content to higher order thinking skills because the latter is ultimately what employers are looking for (Frances, 2002).

Evidence from research supports the importance of learning with understanding from the beginning (Carpenter & Lehrer, 1999). For example, a study by Sigurdson and Olson (1992) found that teaching with meaning increased student achievement as compared to algorithmic-practice teaching. Another study by Pesek and Kirshner (2002) also found that a group of pupils who received only relational instruction outperformed a group of students who received instrumental instruction prior to relational instruction. So we see the importance of conceptual knowledge.

Many local research studies investigated the effect of computer-based learning on academic achievement and the latter is usually biased towards procedural knowledge (see Section 2.4). But conceptual knowledge is also a very important component of the Pentagon model which provides the framework for the mathematics curriculum in Singapore (Ministry of Education, 1998; Wong, 1991). Therefore this dissertation investigated the effect of computer-based learning on both conceptual and procedural knowledge. Another important aspect of the Pentagon model which this study also investigated was the affective variables.

2.6 Research on Affective Variables

This section looks at the importance and complexity of research on affective variables. It concludes with a theoretical basis for such research.

2.6.1 *Importance of Research on Affect*

McLeod (1992) observed that teachers often talked about their pupils' enthusiasm or hostility towards mathematics and that students were just as likely to comment about their feelings towards the subject. These informal observations supported the view that affect played a significant role in mathematics learning and instruction. Moreover the NCTM Curriculum and Evaluation Standards (1989) reaffirmed the centrality of affective issues because the pupils' affective responses were going to be more intense if they were going to engage in higher order thinking. Reyes (1980) also emphasised the importance of research on student attitudes towards mathematics as such information would enable teachers to teach more effectively and successfully. A good attitude is very important because many research studies have suggested a positive correlation between attitude and achievement in mathematics learning (McLeod, 1992).

2.6.2 *Complexity of Research on Affect*

The study of the affective domain is very complicated partly because there is no common agreement on the definitions of terms. Aiken (1972) used the term attitude to mean "approximately the same thing as *enjoyment, interest,* and to some extent, *level of anxiety*" (p. 229). Haladyna, Shaughnessy and Shaughnessy (1983) used the word attitude as "a general emotional disposition toward the school subject of mathematics" (p. 20). Hart (1989) used the word attitude towards an object as a general term to refer to emotional (or affective) reactions to the object, behaviour towards the object and beliefs about the object. This is similar to Rajecki's definition in 1982 (cited in Hart, 1989). Oppenheim (1992) also subscribed to this view. He believed that attitudes are reinforced by beliefs (the cognitive component) and often attract strong feelings (the emotional component) which may lead to particular behavioural intents (the action tendency component).

But Simon suggested the use of affect as a more general term in 1982 (cited in McLeod, 1992). McLeod (1989) expanded this view of affect by dividing the affective variables into the general rubric of beliefs, attitudes and emotions, and he (1992) tried to fit related concepts from the affective domain, such as confidence, anxiety and motivation, into this rubric. It was decided that this research study will use the word “affect” as the more general term since McLeod’s (1992) work on the new view of the affective variables was widely recognised.

Another reason why the study of the affective domain is complicated is because affective constructs are more difficult to describe and measure than cognition (McLeod, 1992). Gardner observed that past researchers had often avoided studying affective issues because of their desire to avoid complexity (cited in McLeod, 1992). As a result, the impact of Bloom’s taxonomy for the affective domain on education (Krathwohl, Bloom & Masia, 1964) was very minimal compared to that of his taxonomy for the cognitive domain (McLeod, 1992).

2.6.3 *Theoretical Basis for Research on Affect*

The traditional paradigm for research on affect is very practical as it relies on questionnaires and quantitative methods (McLeod, 1992). It does not attempt to present a theoretical framework for the assessment of affect. But theories are important to understand the affective domain and McLeod (1989) emphasised the usefulness of integrating research on affect into cognitive theories.

An alternative paradigm has grown out of the work of developmental and cognitive psychology. Mandler (1984) believed that the interruption of a schema or an ongoing cognitive activity will produce a state of arousal that may develop into another emotional expression, depending on factors such as one’s beliefs and knowledge. This emotional expression may be intense but temporary. However repeated interruptions will usually produce a more permanent and predictable response that comes to be characterised as an attitude. McLeod (1992) suggested that another way an attitude could develop is the assignment of an already existing attitude from one schema to a new but related schema.

Marshall (1989) believed that affect could enter schema knowledge in two ways. The first one is simultaneous encoding with other features of the schema. This happens when a student first starts to learn something new. The second way is posterior encoding after a schema is formed. This happens when a new affect node is attached to an existing schema. It was observed that the second way is quite similar to McLeod's (1992) transfer of an existing attitude from one schema to another existing schema.

Only when we understand how attitudes are formed can we attempt to study how they can be changed. That is why theories are important for research on the affective variables.

2.7 Summary of Literature Review

The review of literature on the use of computers in education suggests that most educators still engage in the tutor mode to teach their pupils (Manoucherhri, 1999). With the emergence of constructivism as a popular belief (Driscoll, 2000), using computers as a tool to explore mathematics is an exciting area that teachers may like to experiment with, especially when *GSP* and *LiveMath* provide educators with a comprehensive range of interactive tools for geometry, algebra, calculus and graphs, which cover almost all of the secondary school mathematics syllabus (Yeo, 2002). Moreover research studies do provide some evidence of the benefits of computer-based learning (Asp & McCrae, 2000).

However there are very few local studies on the use of computer-based learning in schools. Many of these studies compared the tool mode of exploration with the traditional teacher-expository method and any significant difference in result might be due to the differences in pedagogy (Oppenheimer, 1997). Many of these studies also investigated the effect of computer-based learning on students' procedural knowledge but not conceptual knowledge. Moreover with the advance of technology, some of the software used in past research studies have become obsolete. So there exist not only ample opportunities but also great necessity and importance to investigate how beneficial the use of computer-based learning in mathematics education is.

2.8 Definitions of Terms

After the review of literature, this section seeks to clarify the meaning of the terms used in this study.

2.8.1 *Computer-Based Learning*

The phrase "computer-based learning" (CBL), when used in a general way, refers to the various approaches (see Section 2.1) of using computers to learn mathematics. But when used in conjunction with the subjects in this study, CBL refers to a particular way of using computers to learn mathematics, namely the tool mode to explore mathematics (see Section 3.5). Sometimes the term "exploratory computer-based learning" (ECBL) will be used more specifically to describe this mode of learning.

2.8.2 *Guided Discovery Approach*

The phrase "guided discovery approach" (GDA), when used in a general way, refers to lessons where teachers guide pupils to discover a mathematical formula or to develop an appropriate model of a mathematical concept by presenting examples for pupils to explore (Bruner, et al., cited in Driscoll, 2000). This may or may not involve the use of IT to facilitate the exploration. However when used in conjunction with the subjects in this study, GDA refers to a non-IT approach using graphs printed in worksheets to guide pupils to discover certain characteristics of exponential and logarithmic curves (see Section 3.5).

2.8.3 *Conceptual Knowledge*

The phrase "conceptual knowledge" refers to a connected web of knowledge that is rich in relationships (Hiebert & Lefevre, 1986). The instrument "Conceptual Knowledge Test" was designed to measure pupil's conceptual knowledge of the graphs of exponential and logarithmic functions (see Section 3.7.1) and so an operational definition of conceptual knowledge for this study is the construct measured by the instrument "Conceptual Knowledge Test".

2.8.4 *Procedural Knowledge*

The phrase "procedural knowledge" refers to the knowledge of mathematical symbols and algorithms for completing mathematical tasks (Hiebert & Lefevre, 1986). The instrument "Procedural Knowledge Test" was designed to measure pupil's procedural knowledge of the graphs of exponential and logarithmic functions (see Section 3.7.1) and so an operational definition of procedural knowledge for this study is the construct measured by the instrument "Procedural Knowledge Test".

2.8.5 *Affect and Attitude*

This study uses the phrase "affect" as a more general term to describe a rubric of beliefs, attitudes and emotions (McLeod, 1992). However the word "attitude" is sometimes used in this study as the more general term when literature writers, especially those before the 1990s, used this word in that way.

Three instruments were designed to measure pupils' affect towards mathematics in general, towards the particular topic learnt, and towards the use of computers in learning this topic (see Section 3.7.6). So an operational definition of affect towards mathematics for this study is the construct measured by the instrument "Questionnaire on Affect towards Mathematics"; an operational definition of affect towards graphs of exponential and logarithmic functions is the construct measured by the instrument "Questionnaire on Affect towards Graphs of Exponential and Logarithmic Functions"; and an operational definition of affect towards the use of computers in learning exponential and logarithmic curves is the construct measured by the instrument "Questionnaire on Affect towards Computer-based Learning".

3. RESEARCH METHODOLOGY

This chapter describes the research methodology used in the study. It includes a description of the research design, the sample, the instructional design, and the course materials; the rationale for choosing the software and the topic; and the pilot test for the test instruments (including their validity and reliability).

3.1 Research Design

This study was carried out using a quasi-experimental pretest-posttest control group design. Although an experimental design is one of the strongest logical models for inferring casual relations, it is a common problem in educational research that the researcher cannot randomly assign individuals to comparison groups (Frankfort-Nachmias & Nachmias, 1996). Therefore a quasi-experimental design with intact comparison groups was used for this study. But certain modifications were available to safeguard against the intrusion of influences other than that of the independent variable. For example, a pretest was conducted to test for comparability between the two contrasted groups (see Section 3.2) before casual inferences were drawn.

3.2 Population and Sample

The population was secondary school pupils in Singapore. As it was beyond the scope of this study to select a large representative sample, the research was based on a sample of 65 Secondary Four pupils from two middle-ability Express classes in an independent boys' school. The two intact classes were selected based on their Secondary Three Elementary (EME) and Additional Mathematics Final Examinations (AME) results. Two t-tests were run to show that the classes selected had no significant difference in their academic achievement in mathematics before the treatment (see Section 4.1.1). Although the topic for this study was in the Additional Mathematics syllabus, it was preferable that both classes had similar abilities in both Additional and Elementary Mathematics.

It was decided not to set a pretest for both classes based on the topic, as some researchers did, because a "difficult" test would not differentiate between pupils with different abilities. As most of them would not know how to answer the questions because they had not learnt the topic yet, it was unlikely that such a pretest would bring out any significant difference in the abilities of the two classes even if such difference existed. Moreover the general mathematical abilities of the two classes should not be determined by the performance in only one topic. There were also some moral issues with setting such a pretest where most pupils would fail: their confidence might be shattered and they might then lose interest in mathematics. Another idea was to set a pretest based on some general topics. But there were a few problems. A short test would not have enough questions to examine pupils' general abilities in mathematics; a long test would be too time consuming to administer. Moreover pupils were usually more serious when taking a final examination than a test. So such a pretest might not truly reflect the pupils' abilities. In the end, it was decided to use the EME and AME scores as the pretest.

One class was randomly assigned the experiment group whilst the other class was the control group. The experimental (or CBL) group refers to the class of pupils who were taught using exploratory computer-based lessons. The control (or GDA) group refers to the class of pupils who were taught using the guided discovery approach.

3.3 Selection of Software

The author decided to choose a piece of software first, before deciding on a topic, because there were not many types of interactive software suitable for mathematical exploration. As there was already a number of local research studies on the use of a geometry software (see Section 2.4), the author decided to use another type of software, one that dealt with algebra. Although there were many types of computer algebra systems (CAS), *LiveMath* was chosen because it was interactive (see Sections 2.3) and so was suitable for exploring mathematical ideas.

The diagram below shows a pre-designed *LiveMath* template used to explore the effect of the constant c on the exponential function of the form $y = e^x + c$. The user just needs to change the value of c and the graph and its equation will change instantaneously. This interactive feature of *LiveMath* allows the user to observe the effect immediately without having to re-type the whole equation and re-plot the corresponding graph.

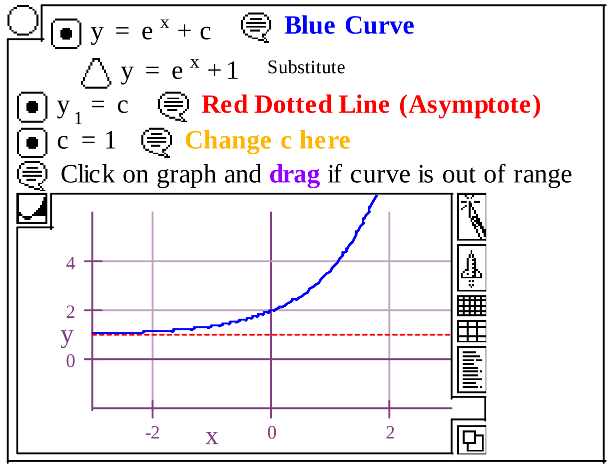

Figure 2. Example of an Interactive LiveMath Template

The diagram below shows another pre-designed *LiveMath* template that allows the user to animate the graph of the exponential function of the form $y = e^x + c$ where c increases from –2 to 4 in steps of 1. Such animation helps the user to visualise the effect of c on the graph of the exponential function.

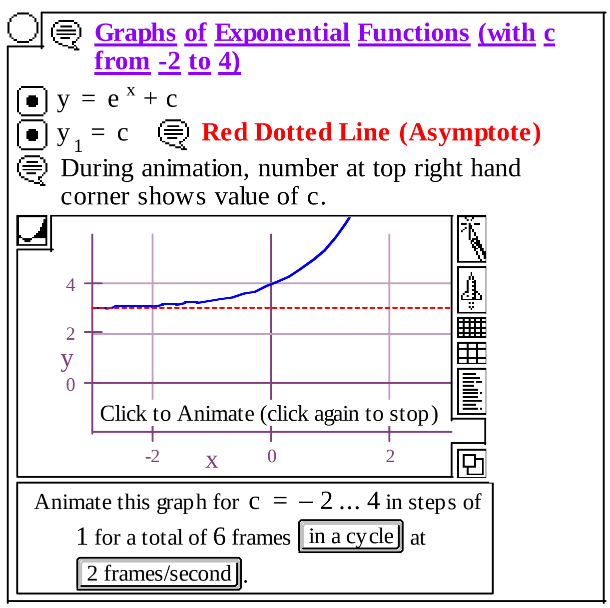

Figure 3. LiveMath Template showing Animation

3.4 Selection of Topic

After choosing the software, it was decided that a suitable topic for using *LiveMath* to explore was the graphs of functions. As explained in Section 3.3, the interactive feature of *LiveMath* facilitated the ease of exploring the relationship between the algebraic and the graphical representations of a function. The subtopic of the graphs of exponential and logarithmic functions was selected out of convenience because the author taught only Secondary Four pupils and this was the only type of graphs they studied at that level.

3.5 Instructional Strategies

Both classes were taught by the author for two weeks. There were a total of ten 40-minute periods of intervention for each class. All were single periods. Based on the literature review in Section 2.4, it was decided that both groups used the same guided exploratory approach, one with IT and one without.

The experimental class learned the topic using the *LiveMath* software and worksheets designed by the author (see Section 3.6.1). These computer-based lessons (CBL) occupied a total of five periods and were conducted in a computer laboratory where every pupil had access to a computer. As the pupils were able to arrive at the computer laboratory on time, they were able to have about 35 minutes of lesson each time. The other five periods were used for clarification and consolidation of concepts and for some practice of textbook-type of questions. A typical computer-based lesson (35 minutes) consisted of a review of the previous lesson (5 minutes), a short demonstration of what was required for the lesson (5 minutes), pupils' exploration using the software (20 minutes) and discussions of their findings (5 minutes). There was about five minutes' allowance for the students to go to the computer laboratory.

The control class learned this topic using the same guided discovery approach (GDA) but without the use of a computer (see Section 3.6.1). These lessons also took a total of five periods but they were conducted in the classroom. Since this approach was less time consuming than CBL, the pupils had more time to clarify their doubts. The other five periods were used for further consolidation of concepts and for some practice of textbook-type of questions. A typical GDA lesson (40 minutes) consisted of a review of the previous lesson (5 minutes), pupils' exploration using the graphs printed in their worksheets (25 minutes) and discussions of their findings (10 minutes).

Both classes were also given similar questions for homework. It was important to keep all the other factors or extraneous variables in both the experimental and control groups constant before casual inferences were drawn (Wiersma, 2000).

3.6 Course Materials

Both classes used the textbook *New Additional Mathematics* by Ho and Khor (2001). Questions for class practice and homework were taken from the textbook. Homework also included a few questions from past 'O' level examination papers (Lee L. K., 2002).

3.6.1 *Description of Course Materials*

The courseware for the computer-based lessons was based on the book *Maths Online* published by the author (Yeo, 2001). It came with a CD-ROM for each pupil and a Teacher's Guide which contained a description of what was in the CD-ROM and a set of transparency masters for the answers to the worksheets. This study used two worksheets, one for exponential curves (3 periods) and one for logarithmic curves (2 periods).

Appendices B and C show the lessons for the exponential and the logarithmic curves respectively. The first portion shows what the CD-ROM contained: pre-designed *LiveMath* templates for exploring the characteristics of the curves. The pupils did not need to have the technical know-how about *LiveMath*. They just changed the values of the parameters, and the corresponding equations and graphs would change automatically (see Section 3.3). They could also perform pre-designed animations with a click of the mouse. The second portion shows the worksheet. The last portion shows the transparency masters for the answers to the worksheet.

The worksheets for the guided discovery lessons were the same as those used for the experimental class, except that all the necessary graphs were printed in these worksheets because the pupils did not have a computer to explore these graphs (see Appendices D & E).

3.6.2 *Pilot Test for Course Materials*

The author had been teaching this topic using the same textbook for the past three years. He had also tried out the computer-based courseware for the past one year and no modification was needed based on verbal feedback from the students. But there was no opportunity to pilot test the worksheets for the control class.

3.7 Test Instruments

There were a total of five self-designed instruments: two achievement tests and three questionnaires on affective variables. Two more instruments used in this study were the school's Secondary Three Elementary (EME) and Additional Mathematics Final Examinations (AME) which were used as pretests for achievement (see Section 3.2).

3.7.1 *Design of Achievement Tests*

There was a need to classify the items in the achievement posttest according to some taxonomy. The author had reviewed the Bloom Taxonomy (Bloom, Engelhart, Furst, Hill & Krathwohl, 1956), the SOLO Taxonomy (Biggs & Collis, 1982), the MATH Taxonomy (Smith, Wood, Coupland & Stephenson, 1996) which was a modification of the Bloom Taxonomy for mathematics, and the NAEP Matrix (Corbitt, 1981).

It was observed that the SOLO Taxonomy was good for classifying pupils' responses according to their conceptual knowledge. But it would be a problem for classifying questions that test only procedural knowledge. The MATH Taxonomy or the NAEP

Matrix would be a better choice for classifying mathematical assessment tasks than the Bloom Taxonomy. However it was decided that it was beyond the scope of this study to really break down and analyse the various components of conceptual and procedural knowledge according to the MATH Taxonomy or the NAEP Matrix. So it was decided to classify the test items into two simple categories: those that attempted to test conceptual knowledge and those that attempted to test procedural knowledge.

Both were one-period 25-marks tests, administered over two single periods. The procedural knowledge test (PT) was administered first because it was supposed to be an easier test. The pupils might lose confidence if the supposedly more-difficult conceptual knowledge test (CT) was administered first and this might affect their performance in the procedural knowledge test. In order not to confuse pupils with unnecessary terminology, the tests were labelled simply as Part I and Part II (see Appendices F & G).

Procedural Knowledge Test (PT)

There were six questions. The first five were of the same structure. The aim was to test pupils in their skills of curve sketching and types of transformation. Questions 4 and 5 were each worth one mark more than each of the first three questions because it was more tedious to find the x-intercept or the asymptote. The last question aimed to test pupils' ability in graphical solutions of exponential or logarithmic equations. It was decided to provide the graphs of $y = e^x$ and $y = \ln x$ in the test paper so that pupils who forgot these basic shapes were not disadvantaged, since this test was not meant to test recall of memory.

Conceptual Knowledge Test (CT)

There were essentially three parts. The first part, consisting of Questions 7 and 8, aimed to test pupils' understanding of the nature of the asymptote. The second part, consisting of Questions 9 to 14, also aimed to test pupils' understanding of the nature of the asymptote but in a real-life context: cooling curve of a cup of hot coffee. The third part, consisting of Questions 15 to 21, aimed to test pupils' understanding on the nature of the exponential curve itself: how it increases rapidly, as compared to the graph of a polynomial function (in this case, a straight line). This was also set within a real-life context.

It was not easy to design a test purely on conceptual knowledge. It was inevitable that there were some related questions that tested more on procedural knowledge. These were necessary to guide pupils to the more difficult questions that tested on conceptual knowledge. But overall the test aimed to test conceptual knowledge and this was validated by experienced teachers (see Section 3.7.2). Another important point was that the pupils had not seen these types of questions before. Otherwise these questions might just test recall of memory.

3.7.2 *Validity of Achievement Tests*

The achievement tests were given to three experienced teachers to see whether they had met the specific instructional objectives (SIO) specified in the school's scheme of work (SOW) and whether they had met the research goals of testing both the pupils' conceptual and procedural knowledge. The relevant SIO are attached in Appendix H.

Feedback from the teachers was that the Procedural Knowledge Test had met the SIO but the Conceptual Knowledge Test went beyond the SIO to test pupils' conceptual knowledge. It was agreed that the research goals for both tests were met, thus establishing the validity of the achievement tests.

3.7.3 *Pilot of Achievement Tests*

The achievement tests were piloted using a previous batch of 52 Secondary Four pupils from the same school. The tests were analysed qualitatively based on the questions asked by these students during the administration of the test and on their answers. Certain test questions were rephrased because some pupils had problems understanding them. All the pupils finished the test within the time allocated.

The tests were also analysed quantitatively in terms of reliability and facility. Reliability was based on the value of Cronbach's alpha and facility on the mean score of the test. For ease of comparison, it was decided to include the reliability and facility of the tests used for both the pilot test and the main study in the next two sections.

3.7.4 *Reliability of Achievement Tests*

The table below shows the values of the Cronbach's alpha for CT, PT and overall (both CT and PT combined) for the pilot test and main study.

Table 1. Cronbach's Alpha for Conceptual and Procedural Knowledge Tests

	Pilot Test	Main Study
Sample Size	52	65
CT	0.710	0.739
PT	0.580	0.501
Overall	0.748	0.684

Since all the values of the Cronbach's alpha were 0.5 and above, all the two tests passed the reliability test for achievement tests (Nunnally & Bernstein, 1994).

3.7.5 *Facility of Achievement Tests*

The table below shows the mean scores of CT, PT and overall for both classes combined during the pilot test and main study.

Table 2. Mean Scores of Conceptual and Procedural Knowledge Tests

	Pilot Test	Main Study
Sample Size	52	65
CT /25	17.8	18.2
PT /25	19.3	18.8
Overall /50	37.1	37.0

Since the mean scores were about 74%, both the conceptual and procedural knowledge tests were of average difficulty.

3.7.6 *Survey of Affective Variables*

There were three questionnaires on affective variables.

(1) The *Questionnaire on Affect towards Mathematics* (ATM) attempted to measure pupils' affect towards mathematics in general (see Appendix I).

(2) The *Questionnaire on Affect towards Graphs of Exponential and Logarithmic Functions* (ATG) attempted to measure pupils' affect towards this topic (see Appendix J).

(3) The *Questionnaire on Affect towards Computer-based Learning* (ATC) attempted to measure pupils' affect towards the use of computers in learning this topic (see Appendix K).

The first questionnaire, ATM, was administered before and after the treatment; the other two questionnaires, ATG and ATC, were administered only after the treatment. The operational definitions of the affective variables for this study were listed in Section 2.8.5.

3.7.7 *Design of Questionnaires*

Bell (1987), Burns (2000), Oppenheim (1992) and Wiersma (2000) suggested some guidelines for the design of questionnaires. Items in a good questionnaire should be clear and unambiguous. Double-barrelled questions and double negatives should be avoided. But this did not mean that an item should not be worded in a reverse direction. Burns (2000) suggested that a questionnaire should include a few such items to stop people filling in the scale carelessly by going down in one column. But these items could still be phrased without using the word “not”. For example, “Mathematics is useless” was better than “Mathematics is not useful” although both were phrased in a reverse direction. Other guidelines included using shorter and simpler items, asking only for relevant information, using “soft” words, no leading questions, no hypothetical questions, no offensive questions, no questions covering sensitive issues, and no personal questions.

With these guidelines in mind, the author designed the three questionnaires for this study by modifying traditional sources such as Aiken (1972), Aiken (1974), Sandman (1979), Haladyna, Shaughnessy and Shaughnessy (1983), and a few local master’s dissertations such as Ho (1997), Leong (2001), Tan (1987), Woo-Tan (1989), and Yeo (1995). The three scales used in the questionnaires were enjoyment, value and confidence in mathematics in general, in the particular topic taught and in the use of computers. These were the most commonly researched scales (Kulm, 1980).

The scores for all the three questionnaires were based on a 5-point Likert scale. For questions worded negatively, their scores were reversed using formulae such as q01r = 6 – q01 where q01 was the original score for question one and q01r was the corresponding final reversed score. The mean of each pupil’s questionnaire was then calculated using the mean of the final scores of all the questions.

3.7.8 *Validity of Questionnaires*

The questionnaires were given to three experienced teachers to test for their face and content validity. If the experts look at the items in a questionnaire and immediately say that the items do not measure the construct they claim to measure, then the instrument fails the face validity test (Burns, 2000). If the items look reasonable at face value but a closer look at their content by the experts reveal that they are not, then the instrument fails the content validity test. The three teachers agreed that the questionnaires measured what they claimed, thus establishing the validity of the questionnaires. There was no need to change any item but some items were rephrased for greater clarity.

3.7.9 *Pilot of Questionnaires*

The questionnaires were piloted using a previous batch of 52 Secondary Four pupils from the same school. These pupils had been taught the same topic by the author using computer-based learning and so the questionnaires were relevant for them as well.

The questionnaires were analysed qualitatively based on the questions asked by these students during the administration of the test and on their answers. There was no need to change or rephrase any item. The questionnaires were also analysed quantitatively in terms of reliability. For ease of comparison, it was decided to include the reliability of the tests used for both the pilot test and the main study in the section below.

3.7.10 *Reliability of Questionnaires*

The table below shows the values of the Cronbach's alpha for the Questionnaire on Affect towards Mathematics (ATM), the Questionnaire on Affect towards Graphs of Exponential and Logarithmic Functions (ATG) and the Questionnaire on Affect towards Computer-based Learning (ATC) for the pilot test, pretest (if applicable) and posttest.

Table 3. Cronbach's Alpha for the Three Questionnaires

	Pilot Test	Pretest	Posttest
ATM	0.930	0.920	0.914
ATG	0.905	–	0.894
ATC	0.914	–	0.838

Since all the values of the Cronbach's alpha were 0.8 and above, all the three questionnaires passed the reliability test for basic research (Nunnally & Bernstein, 1994).

4. FINDINGS AND DISCUSSIONS OF RESULTS

This chapter addresses the first four research questions by testing the nine null hypotheses using appropriate statistical tests. All the tests were run using the statistical software SPSS (Statistical Package for the Social Sciences). The last research question will be analysed using descriptive statistics. The chapter then concludes with a summary of the data analysis.

4.1 Hypotheses Testing

Parametric tests, such as the t-tests, are used to analyse the data if the data can be assumed to be from a normally distributed population (Field, 2000). Otherwise non-parametric tests, such as the Mann-Whitney (M-W) test and the Wilcoxon Signed-Rank test (W-S), are used. Therefore the first step is to test whether all the different sets of data are close enough to normality using the Shapiro-Wilk (S-W) test. If the test is significant ($p < 0.05$), then the distribution of the data is significantly different from a normal distribution. Otherwise, the data is close enough to normality.

The table below summarises the results of the S-W test for normality.

Table 4. Shapiro-Wilk Test for Normality

Data	Class	Statistic	df	p	Normal?
Sec 3 E. Math Final Exam (Achievement Pretest)	CBL	0.965	32	0.455	Normal
	GDA	0.961	33	0.390	Normal
Sec 3 A. Math Final Exam (Achievement Pretest)	CBL	0.968	32	0.512	Normal
	GDA	0.975	33	0.686	Normal
Conceptual Knowledge Test (Achievement Posttest)	CBL	0.937	32	0.081	Normal
	GDA	0.928	33	0.043	Not normal
Procedural Knowledge Test (Achievement Posttest)	CBL	0.924	32	0.039	Not normal
	GDA	0.964	33	0.446	Normal
Affect Towards Mathematics (Pretest)	CBL	0.895	32	0.010	Not normal
	GDA	0.960	33	0.372	Normal
Affect Towards Mathematics (Posttest)	CBL	0.897	32	0.010	Not normal
	GDA	0.932	33	0.055	Normal
Affect Towards Graphs of Exp & Log Functions	CBL	0.933	32	0.065	Normal
	GDA	0.908	33	0.012	Not normal

* CBL: Computer-Based Learning (Experimental Class)
GDA: Guided Discovery Approach (Control Class)
df: degrees of freedom (= sample size for Shapiro-Wilk Test)
p: probability of obtaining the test statistic

If both sets of data are close enough to normality, then an appropriate t-test is used: an independent-samples t-test when there are two independent samples; and a paired-samples t-test when comparing the means of the pretest and the posttest for the same sample. If a t-test is statistically significant ($p < 0.05$), then the null hypothesis is rejected. However there is another assumption for the independent-samples t-test: both samples should have homogeneity of variance. SPSS will automatically run Levene's test for equality of variances when conducting the independent-samples t-test. The outputs contain two rows of information, one for the case if equal variances are assumed ($p > 0.05$ for Levene's test), and the other for the case if equal variances are not assumed ($p < 0.05$ for Levene's test).

If one or both sets of data are not normal, then a non-parametric test is used: the Mann-Whitney (M-W) test when there are two different samples; and the Wilcoxon Signed-Rank test (W-S) when comparing the means of the pretest and the posttest for the same sample.

To summarise, Null Hypotheses 1, 2 and 8 were tested using an appropriate t-test, and Null Hypotheses 3-7 and 9 were tested using an appropriate non-parametric test.

However some researchers believe that a null hypothesis should not be rejected based only on a *statistically* significant test (Burns, 2000). So the effect size is also calculated to see whether it is *practically* significant to reject the null hypothesis. The effect size is "the *degree* to which the phenomenon is present in the population" (Burns, 2000, p. 167). It is used to determine the strength of the relationship between the independent and dependent variables that is independent of sample size. Meta-analysis of findings of quantitative research studies is based on the concept of effect size.

There are a few ways of estimating the effect size (Burns, 2000). In this study, the standardised mean difference, *d*, for two groups (experimental and control), is used to estimate the effect size:

- $d = t\sqrt{\frac{n_1 + n_2}{n_1 n_2}}$ for independent-samples t-test, where n_1 and n_2 are the sample sizes of the two groups;
- $d = \frac{t}{\sqrt{n}}$ for paired-samples t-test, where n is the sample size;
- $d = \frac{mean_E - mean_C}{SD}$ for non-parametric tests, where $mean_E$ and $mean_C$ are the means of the experimental and control groups respectively, and SD is the pooled within group's standard deviation given by $\sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}}$ where n_1 and n_2 are the sample sizes of the two groups.

The interpretation of the *d* value is as follows (Cohen, cited in Burns, 2000):

- *d* of ±0.8 = large effect,
- *d* of ±0.5 = medium effect,
- *d* of ±0.2 = small effect.

4.1.1 *Achievement Pretests*

In this section, the independent-samples t-test is used to find out whether there is any significant difference in the mathematical ability between the experimental class (CBL) and the control class (GDA) before the treatment. The effect size is also calculated. As explained in Section 3.2, this will be based on the school's Secondary Three Elementary Mathematics Final Examination (EME) and Additional Mathematics Final Examination (AME).

Null Hypothesis 1

There is no significant difference in the mean scores of the school's Secondary Three Elementary Mathematics Final Examination (EME) between pupils in the experimental group and the control group before the experiment. (The total possible score for this exam is 100 marks.)

The table below shows the independent samples t-test on EME for the CBL and GDA groups. "n" is the sample size or the number of pupils in each group. "Std Dev" stands for standard deviation. "df" stands for the degrees of freedom and is calculated using the formula $df = n_1 + n_2 - 2$, where n_1 and n_2 are the sample sizes of the two groups. The p-value is the probability of obtaining the t-value.

Table 4.1 Independent-Samples t-test on Sec Three E. Math Final Exam

Group	n	Mean	Std Dev	t-value	df	p-value	Effect Size
CBL	32	69.2	12.7	0.676	56.2	0.502	0.168
GDA	33	67.3	9.12				

* There is a significant difference (p = 0.024) in Levene's test for equality of variances. So equal variances are not assumed and the corresponding SPSS outputs for t-value, df and p-value are used.

The t-value of 0.676 with 56.2 degrees of freedom is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of EME between pupils in the experimental group and the control group before the experiment. The effect size of 0.168 is also small.

Null Hypothesis 2

There is no significant difference in the mean scores of the school's Secondary Three Additional Mathematics Final Examination (AME) between pupils in the experimental group and the control group before the experiment. (The total possible score for this exam is 100 marks.)

The table below shows the independent samples t-test on AME for the CBL and GDA groups.

Table 4.2 Independent-Samples t-test on Sec Three A. Math Final Exam

Group	n	Mean	Std Dev	t-value	df	p-value	Effect Size
CBL	32	66.5	14.2	0.032	63	0.974	0.00794
GDA	33	66.4	12.2				

* There is no significant difference (p = 0.339) in Levene's test for equality of variances. So equal variances are assumed and the corresponding SPSS outputs for t-value, df and p-value are used.

The t-value of 0.032 with 63 degrees of freedom is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of AME between pupils in the experimental group and the control group before the experiment. The effect size of 0.00794 is also very small.

Conclusion of Achievement Pretests

In conclusion, the t-tests suggest that there is no significant difference in the academic achievement of both classes before the treatment. In other words, both classes are very similar in their mathematical abilities before the experiment. Any difference after the experiment may be due to the treatment itself (assuming that there are no other extraneous variables).

4.1.2 *Achievement Posttests*

In this section, the Mann-Whitney test is used to find out whether there is any significant difference in the conceptual and procedural knowledge of the particular topic between the experimental class (CBL) and the control class (GDA) after the treatment. The effect size is also calculated. This will answer Research Questions 1 and 2.

Null Hypothesis 3

There is no significant difference in the mean scores of the Conceptual Knowledge Test (CT) between pupils in the experimental group and the control group after the experiment. (The total possible score for this test is 25 marks.)

The table below shows the Mann-Whitney Test on CT for the CBL and GDA groups. The p-value is the probability of obtaining the Z-value.

Table 4.3 Mann-Whitney Test on Conceptual Knowledge Test

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
CBL	32	20.7	2.95	–4.238	< 0.001	1.28
GDA	33	15.9	4.42			

The Z-value of –4.238 is significant at the 0.05 level. Hence the null hypothesis can be rejected. This suggests that there is a significant difference in the mean scores of CT between pupils in the experimental group and the control group after the experiment. The effect size of 1.28 also suggests that the experimental approach has a very large effect on test performance.

Null Hypothesis 4

There is no significant difference in the mean scores of the Procedural Knowledge Test (PT) between pupils in the experimental group and the control group after the experiment. (The total possible score for this test is 25 marks.)

The table below shows the Mann-Whitney Test on PT for the CBL and GDA groups.

Table 4.4 Mann-Whitney Test on Procedural Knowledge Test

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
CBL	32	20.2	3.77	–2.532	0.011	0.705
GDA	33	17.4	4.23			

The Z-value of –2.532 is significant at the 0.05 level. Hence the null hypothesis can be rejected. This suggests that there is a significant difference in the mean scores of PT between pupils in the experimental group and the control group after the experiment. The effect size of 0.705 also suggests that the experimental approach has a large effect on test performance.

Conclusion of Achievement Posttests

In conclusion, the Mann-Whitney tests suggest that there is a significant difference in both conceptual and procedural knowledge of both classes after the treatment. Moreover the effect size suggests that exploratory computer-based learning has a large effect on pupils' conceptual and procedural knowledge of exponential and logarithmic curves. A closer look suggests that the effect is greater in terms of gain in conceptual than procedural knowledge.

4.1.3 *Affect towards Mathematics between the Two Groups*

In this section, the Mann-Whitney test is used to find out whether there is any significant difference in the pupils' affect towards mathematics between the experimental class (CBL) and the control class (GDA) before the treatment, and between the two groups after the treatment. The effect size is also calculated. This will answer Research Question 3.

All the three questionnaires on affect were based on the 5-point Likert scale and the scores for questions worded negatively were reversed before calculating the mean (see Section 3.7.7).

Null Hypothesis 5

There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics between pupils in the experimental group and the control group before the experiment. (This questionnaire was based on a 5-point Likert scale.)

The table below shows the Mann-Whitney Test on Questionnaire on Affect towards Mathematics (ATM) for the CBL and GDA groups before the experiment.

Table 4.5 Mann-Whitney Test on Affect towards Mathematics before Treatment

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
CBL	32	3.48	0.721	–0.230	0.818	–0.220
GDA	33	3.62	0.527			

The Z-value of –0.230 is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of ATM between pupils in the experimental group and the control group before the experiment. The effect size of –0.220 is also small. In other words, both groups have similar affect towards mathematics before the experiment. Any difference after the experiment may be due to the treatment itself (assuming that there are no other extraneous variables).

Null Hypothesis 6

There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics between pupils in the experimental group and the control group after the experiment. (This questionnaire was based on a 5-point Likert scale.)

The table below shows the Mann-Whitney Test on Questionnaire on Affect towards Mathematics (ATM) for the CBL and GDA groups after the experiment.

Table 4.6 Mann-Whitney Test on Affect towards Mathematics after Treatment

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
CBL	32	3.65	0.621	–1.812	0.070	0.270
GDA	33	3.49	0.547			

The Z-value of –1.812 is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of ATM between pupils in the experimental group and the control group after the experiment. The effect size of 0.270 also suggests that the experimental approach has only a small effect on test performance.

Conclusion of Affect towards Mathematics between the Two Groups

In conclusion, the Mann-Whitney tests suggest that exploratory computer-based learning has no significant effect on pupils' affect towards mathematics.

4.1.4 *Affect towards Mathematics before and after Treatment*

In this section, the Wilcoxon Signed-Rank test is used to find out whether there is any significant difference in the pupils' affect towards mathematics in the experimental class (CBL) before and after the treatment. However the paired-samples t-test is used for the control class (GDA) because both sets of data for the control class are normally distributed (see Section 4.1). The effect size is also calculated. This will answer Research Question 3 from another angle.

Null Hypothesis 7

There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics (ATM) between pupils in the experimental group before and after the experiment. (This questionnaire was based on a 5-point Likert scale.)

The table below shows the Wilcoxon Signed-Rank Test on ATM for the CBL group before and after the experiment.

Table 4.7 Wilcoxon Signed-Rank Test on Affect towards Mathematics for CBL Group

Expt	n	Mean	Std Dev	Z-value	p-value	Effect Size
Before	32	3.48	0.721	–1.132	0.258	0.248
After		3.65	0.621			

The Z-value of –1.132 is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of ATM between pupils in the experimental group before and after the experiment. The effect size of 0.248 also suggests that the experimental approach has only a small effect on test performance.

Null Hypothesis 8

There is no significant difference in the mean scores of the Questionnaire on Affect towards Mathematics (ATM) between pupils in the control group before and after the experiment. (This questionnaire was based on a 5-point Likert scale.)

The table below shows the paired-samples t-test on ATM for the GDA group before and after the experiment. The degree of freedom is calculated using $df = n - 1$.

Table 4.8 Paired-Samples t-test on Affect towards Mathematics for GDA Group

Expt	n	Mean	Std Dev	t-value	df	p-value	Effect Size
Before	33	3.62	0.527	0.992	32	0.328	0.173
After		3.49	0.547				

The t-value of 0.992 with 32 degrees of freedom is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of ATM between pupils in the control group before and after the experiment. The effect size of 0.173 also suggests that the control approach has only a small effect on test performance.

Conclusion of Affect towards Mathematics before and after Treatment

In conclusion, the Wilcoxon Signed-Rank Test and the paired-samples t-test suggest that both exploratory computer-based learning and guided discovery approach have no significant effect on pupils' affect towards mathematics.

4.1.5 *Affect towards Exponential and Logarithmic Curves*

In this section, the Mann-Whitney test is used to find out whether there is any significant difference in the pupils' affect towards graphs of exponential and logarithmic functions between the experimental class (CBL) and the control class (GDA) after the treatment. The effect size is also calculated. This will answer Research Question 4.

Null Hypothesis 9

There is no significant difference in the mean scores of the Questionnaire on Affect towards Graphs of Exponential and Logarithmic Functions (ATG) between pupils in the experimental group and the control group after the experiment. (This questionnaire was based on a 5-point Likert scale.)

The table below shows the Mann-Whitney Test on ATG for the CBL and GDA groups after the experiment.

Table 4.9 Mann-Whitney Test on Affect towards Exponential and Logarithmic Curves

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
CBL	32	3.25	0.681	–1.754	0.079	0.283
GDA	33	3.06	0.725			

The Z-value of –1.754 is not significant at the 0.05 level. Hence the null hypothesis cannot be rejected. This suggests that there is no significant difference in the mean scores of ATG between pupils in the experimental group and the control group after the experiment. The effect size of 0.283 also suggests that the experimental approach has only a small effect on test performance.

Conclusion of Affect towards Exponential and Logarithmic Curves

In conclusion, the Mann-Whitney test suggests that exploratory computer-based learning has no significant effect on pupils' affect towards exponential and logarithmic curves.

4.2 Descriptive Statistics

Descriptive statistics are used to find out whether pupils in the experimental class (CBL) show any positive or negative affect towards the computer-based learning (see Section 1.6). This will answer Research Question 5.

4.2.1 *Affect towards Computer-Based Learning*

The table below shows the descriptive statistics on ATC for the CBL group after the experiment. (This questionnaire was based on a 5-point Likert scale.)

Table 4.10 Descriptive Statistics for Affect towards Computer-Based Learning

Group	n	Mean	Std Dev	Min	Max
CBL	32	3.49	0.475	2.28	4.33

The mean value of 3.49 with a standard deviation of 0.475 suggests that pupils in the experimental group have a moderately positive affect towards computer-based learning.

Conclusion of Affect towards Computer-Based Learning

In conclusion, the descriptive statistics suggests that pupils in the experiment group have a moderately positive affect towards the use of computers in learning exponential and logarithmic curves.

4.3 Summary of Data Analysis

Research Question 1

What is the effect of exploratory computer-based learning on pupils' understanding of exponential and logarithmic curves?

> Data analysis suggests that there is a significant difference in pupils' conceptual knowledge of exponential and logarithmic curves between the CBL and GDA classes. The effect size is also very large.

Research Question 2

What is the effect of exploratory computer-based learning on developing pupils' procedural skills?

> Data analysis suggests that there is a significant difference in pupils' procedural knowledge of exponential and logarithmic curves between the CBL and GDA classes. The effect size is also large.

Research Question 3

What is the effect of exploratory computer-based learning on pupils' affect towards mathematics?

Data analysis suggests that there is no significant difference in pupils' affect towards mathematics between the CBL and GDA classes. The effect size is also small.

Research Question 4

What is the effect of exploratory computer-based learning on pupils' affect towards exponential and logarithmic curves?

Data analysis suggests that there is no significant difference in pupils' affect towards exponential and logarithmic curves between the CBL and GDA classes. The effect size is also small.

Research Question 5

What is the affect of pupils in the experiment group towards the use of computers in learning exponential and logarithmic curves?

Data analysis suggests that pupils in the CBL class have a moderately positive affect towards the use of computers in learning exponential and logarithmic curves.

5. CONCLUSION

This chapter concludes the study by summarising the key findings, delineating the scope and limitations of this study, and discussing the implications for teaching and learning, and for further research.

5.1 Summary of Key Findings

The study suggests that pupils who used a piece of interactive software to explore mathematics had significantly higher scores in both the conceptual and procedural knowledge tests, as compared to pupils who explored mathematics without the use of computers. But there was no significant difference in their affect towards mathematics in general and towards the topic in particular, although pupils in the experimental class show a moderately positive affect towards computer-based learning.

The finding on procedural knowledge is consistent with most local dissertations which also reported a better score in the achievement test (which are biased towards procedural skills) for the experimental class (see Section 2.4 & Appendix A). But Barton’s (2000) meta-analysis of more than 60 studies in other countries since 1995 indicated no significant difference in procedural knowledge tests between pupils in the experimental and control groups in most of the studies. What is important is that the pupils in the experimental class did not perform significantly worse than those from the control class.

The finding on conceptual knowledge is in sharp contrast to the only local dissertation which dealt with mathematical understanding. Ingham (2001) reported that there was no significant difference in most categories of understanding that he investigated, although there were very few questions in his achievement test that actually tested conceptual understanding. However Barton's (2000) meta-analysis of more than 60 studies in other countries since 1995 indicated a significant difference in conceptual knowledge tests between pupils in the experimental and control groups in most of the studies.

The finding on affect towards mathematics in general and the topic in particular is not conclusive since the findings of most local dissertations are mixed (see Appendix A). The trend is that a short treatment like this may or may not produce a significant change in affect towards mathematics in general but it is still likely to produce a significant difference in affect towards the particular topic although this study finds otherwise. The finding on affect towards computer-based learning is consistent with most local dissertations which reported that the pupils in the experimental class enjoyed the computer lessons.

5.2 Scope and Limitation of the Study

It is beyond the scope of this dissertation to examine certain issues in width. For example, the study of the effect of computer-based learning of graphs is restricted only to logarithmic and exponential curves. It is beyond the scope of this dissertation to examine whether the effect is the same for other types of graphs.

It is also beyond the scope of this dissertation to examine certain issues in depth. For example, in the study of the effect of computer-based learning on pupils' conceptual and procedural knowledge, it is beyond the scope to examine each of these types of knowledge by breaking them into smaller components and classifying them according to some taxonomies (see Section 3.7.1). Similarly, it is beyond the scope of this dissertation to investigate the affective variables in depth. The study did not look at each of the affective variables in depth but it investigated the effect of computer-based learning on the overall affect of pupils (see Sections 3.7.6 to 3.7.10).

Being a small-scale study with a small non-representative sample (see Section 3.2), it is also not wise to generalize its findings. Moreover some of the data collected were not normally distributed and therefore non-parametric tests had to be used to analyse the data (see Section 4.1). But these tests are less powerful than parametric tests in that there is an increased chance of a Type II error, meaning that there is an increased chance of accepting that there is no difference between the groups when, in reality, a difference exists (Field, 2000). This will only affect Research Question 3 (affect towards mathematics) and Research Question 4 (affect towards exponential and logarithmic curves) where the results are negative. It will not affect Research Question 1 (conceptual knowledge) and Research Question 2 (procedural knowledge) because the results are positive.

5.3 Implications for Teaching and Learning

Although this study differs from most local dissertations in that the control class in this study also used the same pedagogical approach rather than the traditional teacher-directed teaching, the significant difference in the scores of both the conceptual and procedural knowledge tests suggests that there may be something inherent in the use of computers to explore mathematics that enables the users to not only master the procedural skills but also to understand the topic better. The use of the computer as an interactive tool to explore mathematical concepts seems to be better than the mere non-interactive graphs printed on paper for pupils to explore, or traditional teacher-directed teaching.

It is also worth noting that the control class had more time for drill and practice as the exploratory computer-based lessons definitely took up more curriculum time. But the pupils still performed significantly worse than those in the experimental class in the procedural knowledge test.

The findings of this study, together with those from most local dissertations and studies from other countries (as reported by Barton, 2000), have very important implications for teaching and learning. They suggest that the use of computers to explore mathematics is a better pedagogical approach than the mere teacher-directed teaching or even guided-discovery learning without the use of computers. Computer-based learning not only enhances pupils' understanding of mathematical concepts but it also does not affect their procedural knowledge, even though the pupils have less time for drill and practice.

Therefore teachers may like to make full use of the interactive features of software like *LiveMath* and *Geometer's Sketchpad* to guide their pupils to explore mathematical concepts so that the latter can construct their own mathematical understanding. However it is important to note that this does not mean every lesson must be computer-based or teachers must not engage in any direct teaching.

5.4 Implications for Further Research

A short treatment may not improve pupils' affect towards mathematics in general. But does an extensive use of exploratory computer-based instruction improve pupils' affect towards the subject? Since research has suggested that there is a direct correlation between interest and achievement (McLeod, 1992), there is room for further research to establish whether pupils become more interested in mathematics if they engage in a long period of exploratory computer-based learning.

Further research is also needed to investigate whether exploratory computer-based learning enhances pupils' understanding because of the use of computers itself or because of a change in pedagogy. Although this study suggests that it may be due to the use of computers itself because both experimental and control classes used the same pedagogy, most studies actually did not control this extraneous variable (Oppenheimer, 1997). Barton (2000) also defined the control group in her meta-analysis of the 60 odd studies as the group that was typically taught in a traditional manner. So more research is needed to attribute the increase in mathematical understanding to the use of the computers to explore mathematics, rather than to the guided discovery approach inherent in the exploratory use of computers. If more research studies can establish this correlation, then teachers will have more bases to engage in exploratory computer-based learning.

However it is well established that teachers are resistant to the use of computers in teaching (Noble, 1998). If there are more research studies to show that exploratory computer-based learning is more effective than other approaches and teachers are still resistant to it, then the successful implementation of the IT initiative in Singapore depends on dealing with these other factors, for example, teachers' belief in the nature of mathematics and mathematics learning (Ang, 1999; Thompson & Thompson, 1996). Teachers who believe that mathematics is just "practice makes perfect" or procedural skills have no place for guided-discovery learning. They need to be convinced about the fallibilistic nature of mathematics (Ernest, 1991) and the social constructivistic approach in the teaching and learning of mathematics (Ernest, 1994). Otherwise they will still not engage in the use of computers to explore mathematics, regardless of what research says.

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APPENDIX A: SUMMARY OF LOCAL RESEARCH ON IT

Study	Research Design	Treatment	Subjects	Topics	Software	Variables	Findings
Lee-Leck (1985)	Quasi-experimental	Experimental (expt) class – IT lessons; Control (ctrl) class – Normal lecture	Preservice Primary School Teachers (n=72)	Straight Line Graphs and Quadratic Curves	*BASIC*	Achievement	Significant difference
						Attitudes	Significance difference in attitudes towards topic but not math in general
Tan (1987)	Quasi-experimental	Expt class – IT lessons; Ctrl class – Traditional expository instruction	Secondary Three Express Pupils (n=54)	Straight Line Graphs	*BASIC*	Achievement	Significant difference
						Attitudes	No significant difference in attitudes towards math in general
Woo-Tan (1989)	Quasi-experimental	Expt class – IT lessons; Ctrl class – Teacher-directed approach	Secondary Four Express Pupils (n=70)	Transformation Geometry	*BASIC*	Achievement	Significant difference
						Attitudes	Significant difference in attitudes towards both topic and math in general

Study	Research Design	Treatment	Subjects	Topics	Software	Variables	Findings
Yeo (1995)	Quasi-experimental	Three expt classes (one high, one medium & one low ability) – IT lessons; Three ctrl classes (one high, one medium & one low ability) – Traditional approach	Secondary Three Express Pupils (n=192)	Quadratic Curves	*BASIC*	Achievement	Significant difference for medium ability classes but not for high and low ability classes
						Attitudes	Significant difference in attitudes towards both topic and math in general
Ho (1997)	Quasi-experimental	Expt class – IT lessons; Ctrl class – Traditional expository approach	Secondary Four Normal Academic Pupils (n=70)	Transformation Geometry	*LOGO*	Achievement	Significant difference
						Attitudes	Significant difference in attitudes towards both topic and math in general
Ingham (2001)	Quasi-experimental	Two expt classes – IT lessons; No ctrl class	Secondary Three Express Pupils (n=58)	Quadratic Curves	*Graphmatica*	Achievement	Significant difference
						Understanding	No significant difference (in most categories)
						Attitudes	No significant difference (in most categories)

Study	Research Design	Treatment	Subjects	Topics	Software	Variables	Findings
Leong (2001)	Quasi-experimental	Expt Class A – Hands-on guided-inquiry IT lessons; Expt Class B – Teacher-demo guided-inquiry IT lessons; Expt Class C – Teacher-demo expository IT lessons	Secondary Two Express Pupils (n=121)	Transformation Geometry	*Geometer's Sketchpad* (*GSP*)	Spatial Ability	No significant difference
						Achievement	Significant difference between Classes A & C, and Classes B & C, but not Classes A & B
Lee C. M. (2002)	Quasi-experimental	Expt Class – IT lessons; No ctrl class	Primary Four Pupils (n=40)	Angles	*LOGO*	Achievement	Significant difference between pretest and posttest
Ong (2002)	Quasi-experimental	Expt class – IT lessons; Ctrl class – Teacher-centred approach (her definition is traditional expository instruction)	Secondary Three Express Pupils (n=60)	Angle Properties of Circles	*GSP*	Achievement	Significant difference
						Attitudes	Significance difference in attitudes towards topic but not math in general

APPENDIX B: COMPUTER-BASED LESSONS FOR EXPONENTIAL CURVES

The first section shows what the CD-ROM contains: pre-designed *LiveMath* templates for exploring the characteristics of the curves. The pupils just changed the values of the parameters, and the corresponding equations and graphs would change automatically. They could also perform pre-designed animations with a click of the mouse. The second section shows the worksheet. The last section shows the transparency masters for the answers to the worksheet.

Section B1: Pre-designed *LiveMath* Templates

IT4AMExpGraph1.thp

Graphs of Exponential Functions 1

$y_1 = a^x$ **Blue Curve**

$a = 4$ **Change a here (a>0)**

$y_2 = b^x$ **Red Curve**

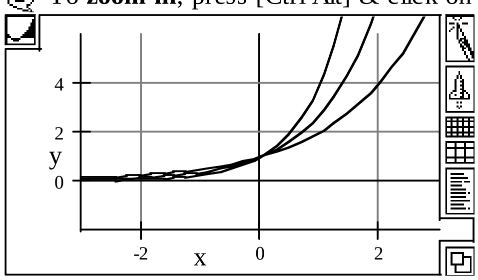
$b = e$ **Change b here (b>0)**

$y_3 = c^x$ **Green Curve**

$c = 2$ **Change c here (c>0)**

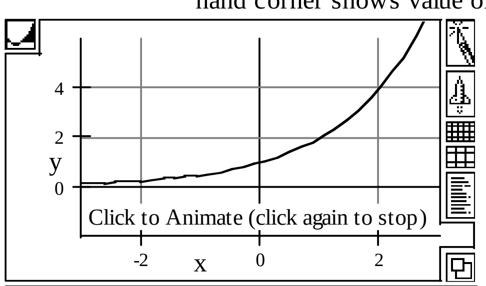
To **zoom out**, click on 2nd button on top right corner.

To **zoom in**, press [Ctrl Alt] & click on same button.



Graphs of Exponential Functions (with base a from 1/2 to 4)

$y = a^x$ During animation, number at top right hand corner shows value of a.



Animate this graph for $a = 0.5 \ldots 4$ in steps of 0.5 for a total of 7 frames [in a cycle] at [2 frames/second].

IT4AMExpGraph2.thp

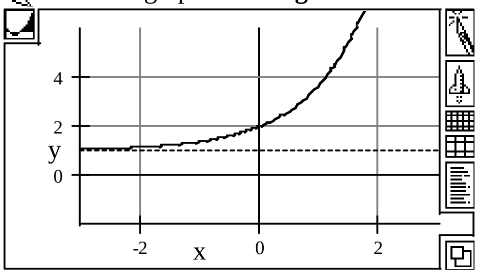
Graphs of Exponential Functions 2

$y = e^{x} + c$ **Blue Curve**

$y_1 = c$ **Red Dotted Line (Asymptote)**

$c = 1$ **Change c here**

Click on graph and **drag** if curve is out of range

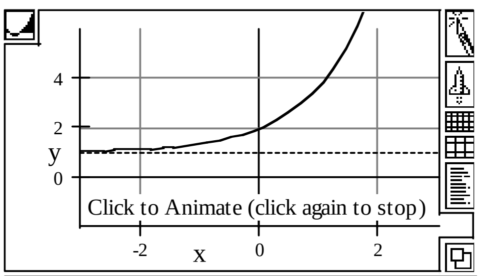


Graphs of Exponential Functions (with c from -2 to 4)

$y = e^{x} + c$

$y_1 = c$ **Red Dotted Line (Asymptote)**

During animation, number at top right hand corner shows value of c.



Animate this graph for $c = -2 \dots 4$ in steps of 1 for a total of 6 frames [in a cycle] at [2 frames/second].

IT4AMExpGraph3.thp

<u>Graphs of Exponential Functions 3</u>

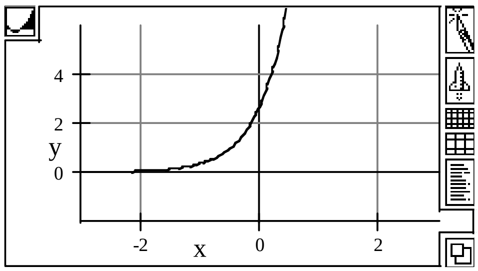
$y = e^{a x + b}$ **Blue Curve**

$a = 2$ **Change a here**

$b = 1$ **Change b here**

Click on graph and **drag** if curve is out of range

To find y-intercept to 3 sig. fig., use knife button (1st button on top right hand corner) to draw a region enclosing y-intercept and then release. This will zoom in on the selected region.



<u>Graphs of Exponential Functions (with a from -2 to 4)</u>

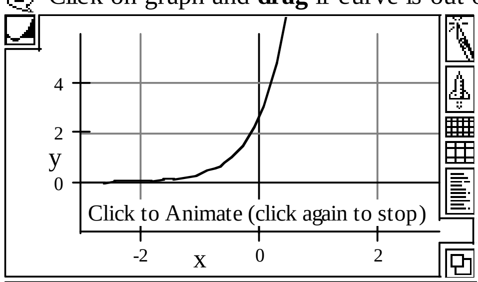
$y = e^{a x + b}$

$b = 1$ **Change b here**

$y = e^{a x + 1}$ Substitute **Blue Curve**

During animation, number at top right hand corner shows value of a.

Click on graph and **drag** if curve is out of range



Animate this graph for $a = -2 \ldots 4$ in steps of 1 for a total of 6 frames [in a cycle] at [2 frames/second].

Section B2: Worksheet for Experimental Class

EXPONENTIAL AND LOGARITHMIC FUNCTIONS IT WORKSHEET 1 – GRAPHS OF EXPONENTIAL FUNCTIONS

Software: ***LiveMath*** (previously known as ***MathView***)	Time Frame: 3 periods
Thinking Skills: **Induction** and **Deduction**	
At the end of the lesson, the students should be able to:	
(1) sketch the graphs of exponential functions,	
(2) describe the characteristics of exponential functions.	

A. GRAPHS OF EXPONENTIAL FUNCTIONS 1: $y = a^x$

1. The first document **IT4AMExpGraph1.thp** shows the graphs of the **exponential functions** $y_1 = a^x$, $y_2 = b^x$ and $y_3 = c^x$. Change the value of **a to 4**, the value of **b to 3** and the value of **c to 2**.

Question 1: What is the y-intercept for all the 3 curves? _____________

Question 2: What is the asymptote for all the 3 curves? _____________

Question 3: When x>0, which curve has the largest y-values (i.e. which curve is on top)?

y = ______ has the largest y-values, i.e. 4^x > _____ > _____ .

Question 4: When x<0, which curve has the largest y-values (i.e. which curve is on top)?

y = ______ has the largest y-values, i.e. 4^x < _____ < _____ .

Question 5: $y = e^x$ is called the **natural exponential function** where $e \approx 2.71828$. Where do you expect the curve $y = e^x$ to lie?

$y = e^x$ will lie between y = ______ and y = _______ .

Now change the value of **a to e** to check your answer in Q5.

2. Set **c = 1**.

Question 6: What is the shape of the curve $y = 1^x$? Is this an exponential function?

____

3. Set **a = 2** and **b = 1/2**.

Question 7: What do you observe between the curves $y = (1/2)^x$ and $y = 2^x$?

$y = (1/2)^x$ is the __________________ of $y = 2^x$ in the ________________ .

Question 8: Convert the equation of the curve $y = (1/2)^x$ to the form $y = d^{-x}$.

Therefore y = $\_\_\_\_^{-x}$ is the _______________ of $y = 2^x$ in the __________ .

4. Click on the second graph to see the animation of the curves $y = a^x$ **from a = 0.5 to a = 4** in steps of 0.5.
Note: The animation will stop at 3.5. It will not include the last value.

B. GRAPHS OF EXPONENTIAL FUNCTIONS 2: $y = e^x + c$

1. The second document **IT4AMExpGraph2.thp** shows the graph of $y = e^x + c$ and its asymptote. Change the value of **c to 0** so that the graph shows the curve $y = e^x$.

Question 9: For the curve $y = e^x$, the y-intercept is ____ and the asymptote is y = ____ .

2. Change the value of **c from –2 to 4** in steps of 1. Record the y-intercepts and the asymptotes in the table below.

Curve	y-intercept	Asymptote
$y = e^x - 1$		y =
$y = e^x$		
$y = e^x + 1$		
$y = e^x + 2$		
$y = e^x + 3$		
$y = e^x + 4$		

Question 10: With reference to the curve $y = e^x$, infer by **induction** the effect of c on the curve $y = e^x + c$.

c will shift the curve $y = e^x$ __________ if c > 0 and ___________ if c < 0.

3. Click on the second graph to see the animation of the curves $y = e^x + c$ from **c = –2 to c = 4** in steps of 1.
Note: The animation will stop at 3. It will not include the last value.

C. GRAPHS OF EXPONENTIAL FUNCTIONS 3: $y = e^{ax+b}$

1. The third document **IT4AMExpGraph3.thp** shows the graph of $y = e^{ax+b}$. Change the value of **a to 2** and the value of **b to 1**.

Question 11: For the curve $y = e^{2x+1}$, the y-intercept is _______ and the asymptote is y = _______ .

Hint: To find the y-intercept, put x = 0 and use a calculator to evaluate y to 3 significant figures. Or use the **knife button** (first button on top right hand corner of graph) to draw a region enclosing the y-intercept and then release. This will zoom in on the selected region. Repeat the process until you get the y-intercept to 3 significant figures.

2. Change the values of a and b to obtain the curves given in the table below. Record the y-intercepts and the asymptotes.

Curve	y-intercept (to 3 sig. fig.)	Asymptote
$y = e^{x+2}$	$e^2 =$	y =
$y = e^{2x-1}$		
$y = e^{-x+3}$		
$y = e^{3x-2}$		

Question 12: What do you notice about the asymptotes?

____

3. Set **b = 1**. Change the value of **a from –2 to 3** in steps of 1 except for a = 0 (why?). Record the y-intercepts and the asymptotes in the table below.

Curve	y-intercept (to 3 sig. fig.)	Asymptote
$y = e^{-2x+1}$	$e^1 =$	$y =$
$y = e^{-x+1}$		
$y = e^{x+1}$		
$y = e^{2x+1}$		
$y = e^{3x+1}$		

Question 13: What do you notice about the y-intercepts and the asymptotes?

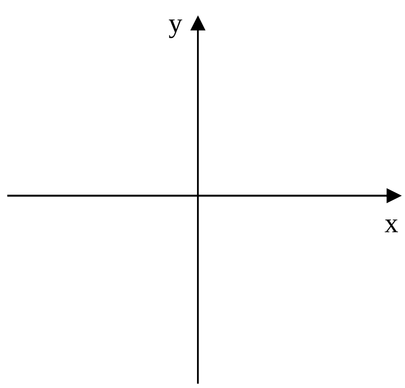
____

4. Click on the second graph to see the animation of the curves $y = e^{ax+b}$ from **a = –2 to a = 4** in steps of 1 (when b = 1).
Note: The animation will stop at 3. It will not include the last value.

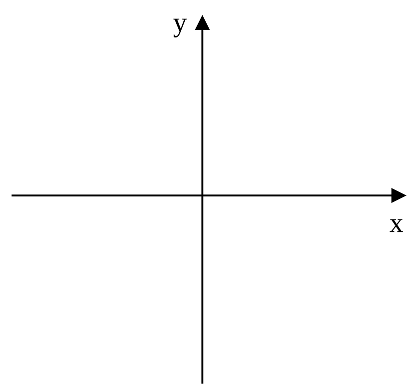
D. EXERCISE

1. **Do NOT use your computer to do the exercise.**
2. Apply what you have learnt to **deduce** the shapes of the following curves and sketch them, showing the **y-intercept** and the **asymptote** clearly.

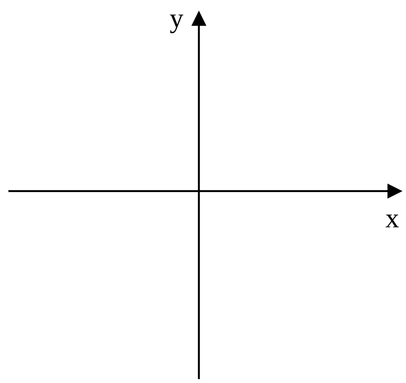
(a) $y = e^x$



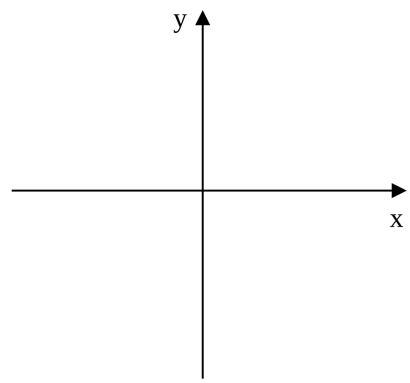
(b) $y = e^x - 2$



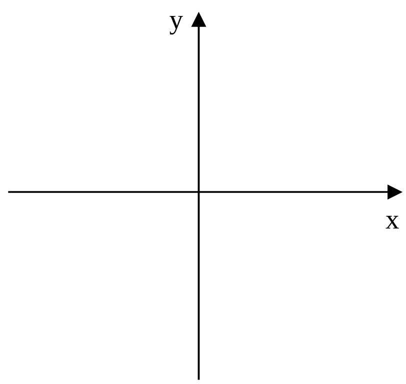
(c) $y = e^{-x}$



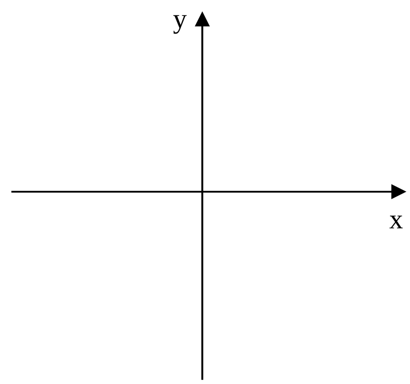
(d) $y = e^{3x+2}$



(e) $y = e^{-x} + 1$



(f) $y = e^{-2x+1}$



ANSWERS (OHT MASTER)

A. GRAPHS OF EXPONENTIAL FUNCTIONS 1: $y = a^x$

1. y-intercept = 1
2. asymptote: x-axis (or y=0)
3. For $x > 0$, $y = 4^x$ has the largest y-values, i.e. $4^x > 3^x > 2^x$.
4. For $x < 0$, $y = 2^x$ has the largest y-values, i.e. $4^x < 3^x < 2^x$.
5. $y = e^x$ will lie between $y = 2^x$ and $y = 3^x$.
6. $y = 1^x$ is a straight line. It is **not** an exponential function.
7. $y = (1/2)^x$ is the reflection of $y = 2^x$ in the y-axis.
8. $y = 2^{-x}$ is the **reflection** of $y = 2^x$ in the y-axis.

B. GRAPHS OF EXPONENTIAL FUNCTIONS 2: $y = e^x + c$

9. For $y = e^x$, y-intercept = 1; asymptote is $y = 0$ (x-axis).

Curve	y-intercept	Asymptote
$y = e^x – 1$	0	$y = –1$
$y = e^x$	1	$y = 0$
$y = e^x + 1$	2	$y = 1$
$y = e^x + 2$	3	$y = 2$
$y = e^x + 3$	4	$y = 3$
$y = e^x + 4$	5	$y = 4$

10. c will shift the curve $y = e^x$ **up if $c > 0$** and down if $c < 0$.

C. GRAPHS OF EXPONENTIAL FUNCTIONS 3: $y = e^{ax+b}$

11. For $y = e^{2x+1}$, y-intercept = $e^1 = 2.72$; asymptote is $y = 0$ (x-axis)

Curve	y-intercept	Asymptote
$y = e^{x+2}$	$e^2 = 7.39$	$y = 0$
$y = e^{2x-1}$	$e^{-1} = 0.368$	$y = 0$
$y = e^{-x+3}$	$e^3 = 20.1$	$y = 0$
$y = e^{3x-2}$	$e^{-2} = 0.135$	$y = 0$

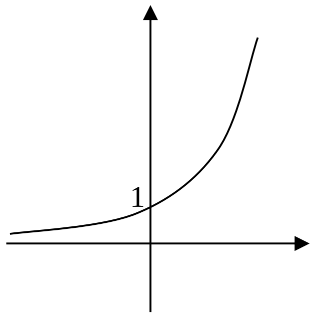
12. The asymptotes are the same (x-axis).

Curve	y-intercept	Asymptote
$y = e^{-2x+1}$	$e^1 = 2.72$	$y = 0$
$y = e^{-x+1}$	$e^1 = 2.72$	$y = 0$
$y = e^{x+1}$	$e^1 = 2.72$	$y = 0$
$y = e^{2x+1}$	$e^1 = 2.72$	$y = 0$
$y = e^{3x+1}$	$e^1 = 2.72$	$y = 0$

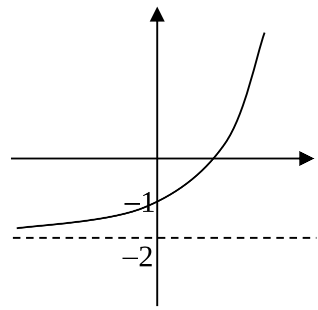
13. The y-intercepts and the asymptotes are the same.

D. EXERCISE

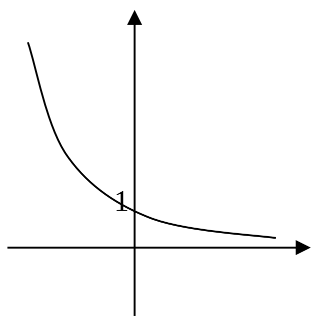
2. (a) $y = e^x$



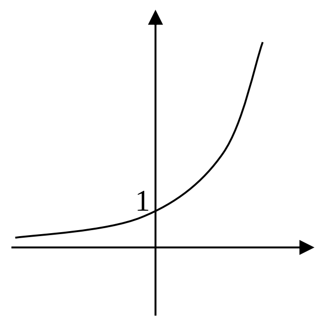
(b) $y = e^x - 2$



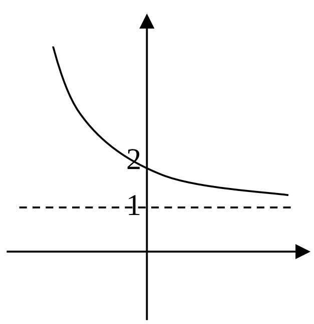
(c) $y = e^{-x}$



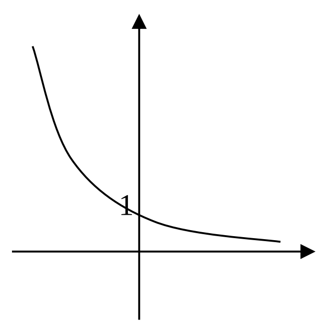
(d) $y = e^{3x+2}$



(e) $y = e^{-x} + 1$



(f) $y = e^{-2x+1}$



APPENDIX C: COMPUTER-BASED LESSONS FOR LOGARITHMIC CURVES

The first section shows what the CD-ROM contains: pre-designed *LiveMath* templates for exploring the characteristics of the curves. The pupils just changed the values of the parameters, and the corresponding equations and graphs would change automatically. They could also perform pre-designed animations with a click of the mouse. The second section shows the worksheet. The last section shows the transparency masters for the answers to the worksheet.

Section C1: Pre-designed *LiveMath* Templates

IT4AMLogGraph1.thp

Graphs of Logarithmic Functions 1

$y_1 = \log_a(x)$ **Blue Curve**

$a = 4$ **Change a here (a>0)**

$y_2 = \log_b(x)$ **Red Curve**

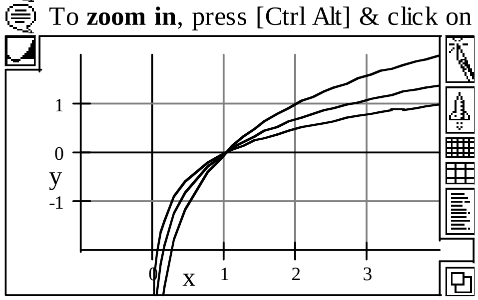
$b = e$ **Change b here (b>0)**

$y_3 = \log_c(x)$ **Green Curve**

$c = 2$ **Change c here (c>0)**

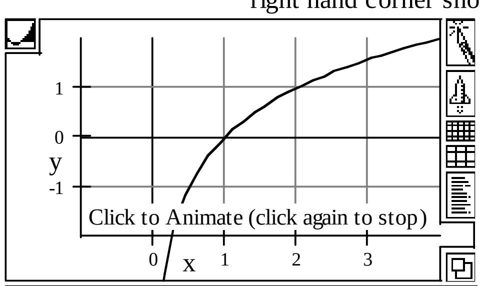
To **zoom out**, click on 2nd button on top right corner.

To **zoom in**, press [Ctrl Alt] & click on same button.



Graphs of Logarithmic Functions (with base a from 1/2 to 4)

$y = \log_a(x)$ During animation, number at top right hand corner shows value of a.



Animate this graph for $a = 2 \ldots 4.5$ in steps of 0.5 for a total of 5 frames [in a cycle] at [2 frames/second].

IT4AMLogGraph2.thp

Graphs of Logarithmic Functions 2

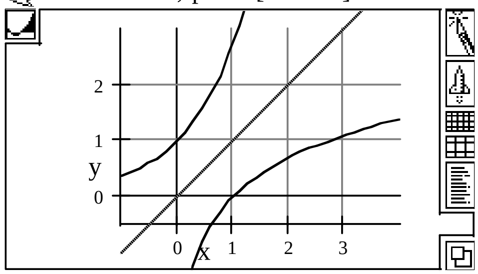
$a = e$ **Change a here (a>0)**

$y_1 = \log_a(x)$ **Blue Curve**

$y_2 = a^x$ **Red Curve**

To **zoom out**, click on 2nd button on top right corner.

To **zoom in**, press [Ctrl Alt] & click on same button.



Yellow Dotted Line

IT4AMLogGraph3.thp

Graphs of Logarithmic Functions 3

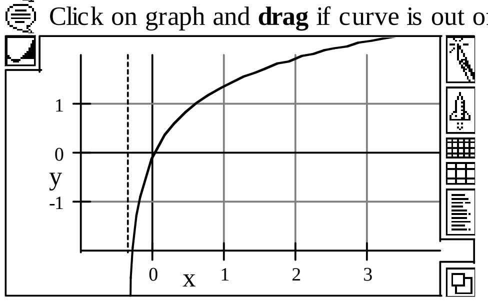
$y = \ln(a x + b)$

$a = 3$ $b = 1$ **Change a and b here**

To **zoom out**, click on 2nd button on top right corner.

To **zoom in**, press [Ctrl Alt] & click on same button.

Click on graph and **drag** if curve is out of range.



Red Asymptote

Section C2: Worksheet for Control Class

EXPONENTIAL AND LOGARITHMIC FUNCTIONS IT WORKSHEET 2 – GRAPHS OF LOGARITHMIC FUNCTIONS

Software: ***LiveMath*** (previously known as ***MathView***) Thinking Skills: **Induction** and **Deduction** At the end of the lesson, the students should be able to: (3) sketch the graphs of logarithmic functions, (4) describe the characteristics of logarithmic functions.	Time Frame: 2 periods

A. GRAPHS OF LOGARITHMIC FUNCTIONS 1: $y = \log_a x$

1. The first document **IT4AMLogGraph1.thp** shows the graphs of **logarithmic functions** $y_1 = \log_a x$, $y_2 = \log_b x$ and $y_3 = \log_c x$. Change the value of **a to 4**, the value of **b to 3** and the value of **c to 2**.

Question 1: What is the x-intercept for all the 3 curves? ____________

Question 2: What is the asymptote for all the 3 curves? ____________

Question 3: When x>1, which curve has the largest y-values (i.e. which curve is on top)?

$y =$ ________ has the largest y-values, i.e. $\log_2 x >$ ________ $>$ ________ .

Question 4: When x<1, which curve has the largest y-values (i.e. which curve is on top)?

$y =$ ________ has the largest y-values, i.e. $\log_2 x <$ ________ $<$ ________ .

Question 5: $y = \ln x$ (or $\log_e x$) is called the **natural logarithmic function** where $e \approx 2.71828$. Where do you expect the curve $y = \ln x$ to lie?

$y = \ln x$ will lie between $y =$ _________ and $y =$ __________ .

Now change the value of **a to e** to check your answer in Q5.

2. Click on the second graph to see the animation of the curves $y = \log_a x$ **from a = 2 to a = 4** in steps of 0.5.
Note: The animation will stop at 4. It will not include the last value 4.5.

B. GRAPHS OF LOGARITHMIC FUNCTIONS 2: $y = \ln x$

1. The second document **IT4AMLogGraph2.thp** shows the graphs of $y = \ln x$ and $y = e^x$.

Question 6: Describe **fully** the transformation that maps the natural logarithmic curve $y = \ln x$ to the natural exponential curve $y = e^x$.

____

2. Double click the icon beside the “Yellow Dotted Line” to check its equation for Q6.
3. Change the value of **a to 2**, then **to 3** and then **to 4**.

Question 7: Infer by **induction** the transformation that maps the logarithmic curve $y = \log_a x$ to the exponential curve $y = a^x$.

____

C. GRAPHS OF LOGARITHMIC FUNCTIONS 3: y = ln (ax+b)

1. The third document **IT4AMLogGraph3.thp** shows the graph of $y = \ln(ax+b)$. Change the value of **a to 2** and the value of **b to 1**.

Question 8: For the curve $y = \ln(2x+1)$, the x-intercept is _________ and the asymptote is x = _________ .

2. Double click on the icon beside the "Red Asymptote" to check its equation for Q8.

Question 9: Explain how you find the equation of the asymptote in terms of a and b.

____

Question 10: Explain how you find the x-intercept. You may want to express it in terms of a and b.

____

3. Change the values of a and b to obtain the curves given in the table below. Record the x-intercepts and the asymptotes.

No.	Curve	x-intercept	Asymptote
1.	$y = \ln(x+2)$		x =
2.	$y = \ln(2x-1)$		
3.	$y = \ln(-2x+1)$		
4.	$y = \ln(3x-2)$		

Question 11: How is the third curve different from the rest?

____

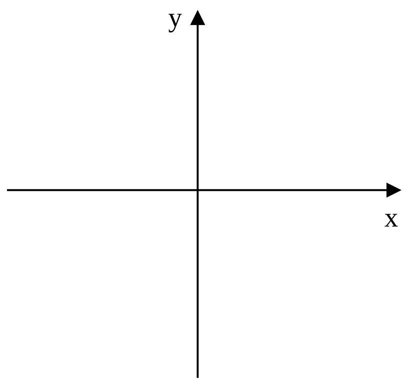
Question 12: What is the condition for the third curve to be different from the rest?

____

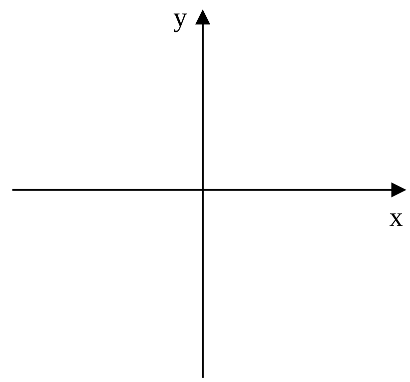
D. EXERCISE

1. **Do NOT use your computer to do the exercise.**
2. Apply what you have learnt (esp. Q9–10) to **deduce** the shapes of the following curves and sketch them, showing the **x-intercept** and the **asymptote** clearly.

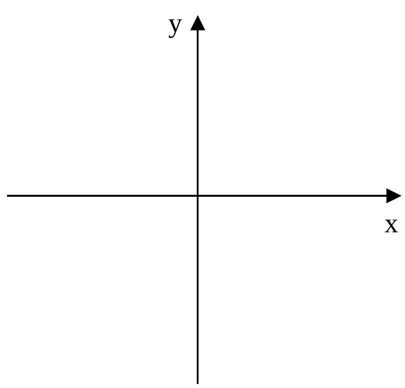
(a) $y = \ln(x+1)$



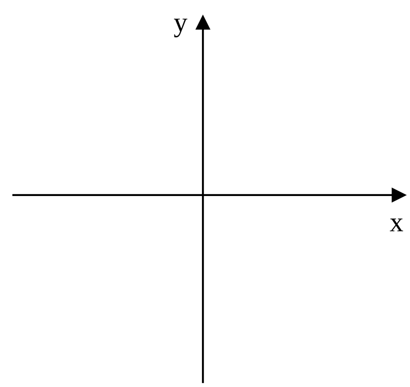
(b) $y = \ln(x-1)$



(c) $y = \ln(3x-1)$



(d) $y = \ln(-3x+1)$



ANSWERS (OHT MASTER)

A. GRAPHS OF LOGARITHMIC FUNCTIONS 1: $y = \log_a x$

1. x-intercept = 1
2. asymptote: y-axis (or x=0)
3. For $x > 1$, $y = \log_2 x$ has the largest y-values, i.e. $\log_2 x > \log_3 x > \log_4 x$.
4. For $x < 1$, $y = \log_4 x$ has the largest y-values, i.e. $\log_2 x < \log_3 x < \log_4 x$.
5. $y = \ln x$ will lie between $y = \log_2 x$ and $y = \log_3 x$.

B. GRAPHS OF LOGARITHMIC FUNCTIONS 2: $y = \ln x$

6. $y = \ln x$ is mapped onto $y = e^x$ by a **reflection in the line $y = x$**.
7. $y = \log_a x$ is mapped onto $y = a^x$ by a reflection in the line $y = x$.

C. GRAPHS OF LOGARITHMIC FUNCTIONS 3: $y = \ln(ax+b)$

8. For $y = \ln(2x+1)$, x-intercept = 0; asymptote is $x = -1/2$
9. Put $ax+b = 0$. Then $x = -b/a$ is the equation of the asymptote.
10. $\ln(ax+b) = 0 \Rightarrow ax+b = e^0$
$\Rightarrow ax+b = 1$
$\Rightarrow x = (1-b)/a$

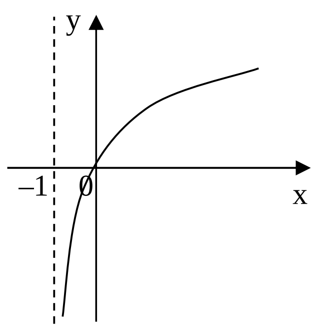
No.	Curve	x-intercept	Asymptote
1.	$y = \ln(x+2)$	-1	$x = -2$
2.	$y = \ln(2x-1)$	1	$x = 1/2$
3.	$y = \ln(-2x+1)$	0	$x = 1/2$
4.	$y = \ln(3x-2)$	1	$x = 2/3$

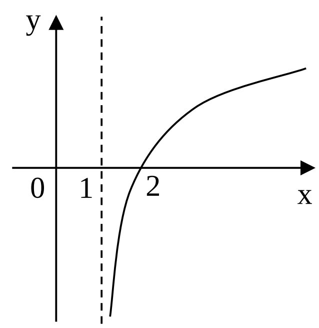
11. The third curve lies on the left of the asymptote whilst the other curves lie on the right.
12. For the third curve, $a < 0$.

Note: For $y = \ln(ax+b)$,
$a>0$: curve lies on right of asymptote
$a<0$: curve lies on left of asymptote

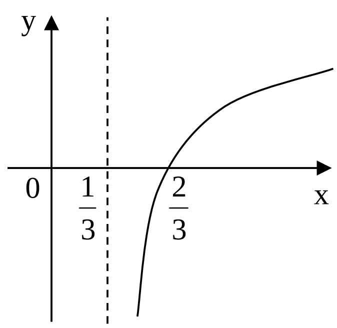
D. EXERCISE

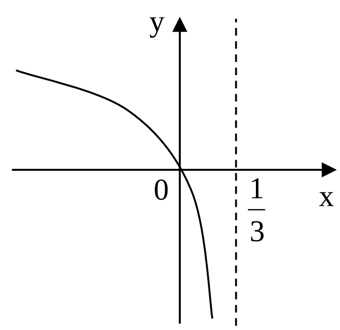
2. (a) $y = \ln(x+1)$ (b) $y = \ln(x-1)$





(c) $y = \ln(3x-1)$ (d) $y = \ln(-3x+1)$





Note: $y = \ln(-3x+1)$ is reflection of $y = \ln(3x-1)$ in the asymptote

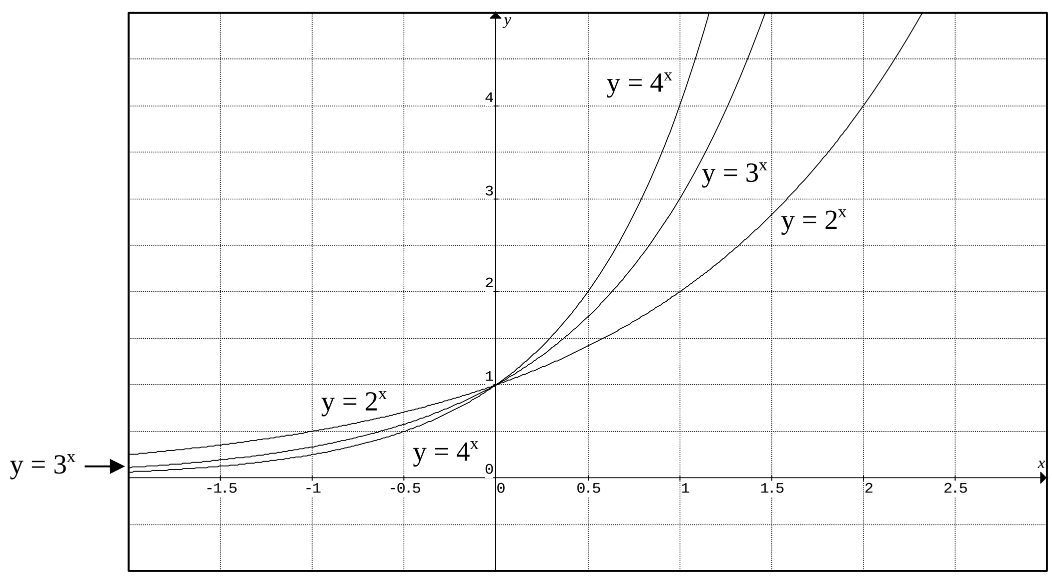
APPENDIX D: WORKSHEET ON EXPONENTIAL CURVES FOR CONTROL CLASS

EXPONENTIAL AND LOGARITHMIC FUNCTIONS WORKSHEET 1 – GRAPHS OF EXPONENTIAL FUNCTIONS

Thinking Skills: **Induction** and **Deduction**	Time Frame: 3 periods
At the end of the lesson, the students should be able to:	
(1) sketch the graphs of exponential functions,	
(2) describe the characteristics of exponential functions.	

A. GRAPHS OF EXPONENTIAL FUNCTIONS 1: $y = a^x$

1. The diagram below shows the graphs of the **exponential functions** $y = 4^x$, $y = 3^x$ and $y = 2^x$.



Question 1: What is the y-intercept for all the 3 curves? ____________

Question 2: What is the asymptote for all the 3 curves? ____________

Question 3: When x>0, which curve has the largest y-values (i.e. which curve is on top)?

y = ______ has the largest y-values, i.e. 4^x > _____ > _____ .

Question 4: When x<0, which curve has the largest y-values (i.e. which curve is on top)?

y = ______ has the largest y-values, i.e. 4^x < _____ < _____ .

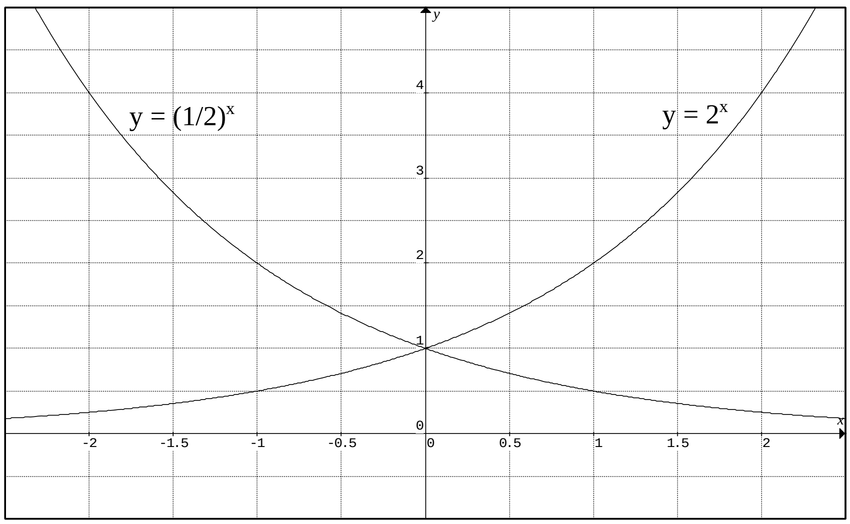
Question 5: $y = e^x$ is called the **natural exponential function** where $e \approx 2.71828$. Where do you expect the curve $y = e^x$ to lie?

$y = e^x$ will lie between y = ______ and y = _______ .

Question 6: What is the shape of the curve $y = 1^x$? Is this an exponential function?

____

2. The diagram below shows the graphs of the **exponential functions** $y = (1/2)^x$ and $y = 2^x$.



Question 7: What do you observe between the curves $y = (1/2)^x$ and $y = 2^x$?

$y = (1/2)^x$ is the __________________ of $y = 2^x$ in the ________________ .

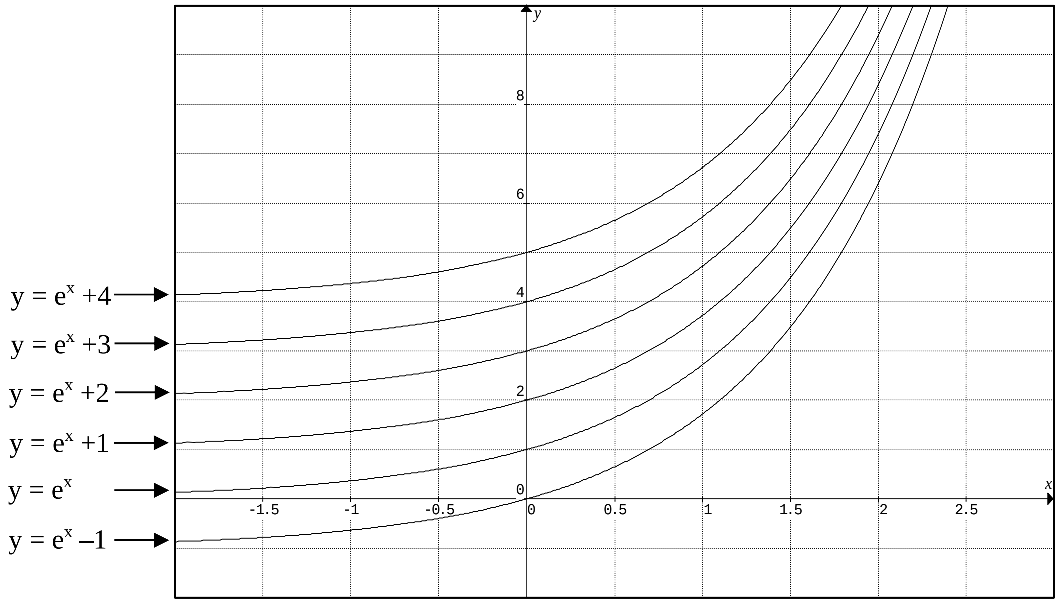
Question 8: Convert the equation of the curve $y = (1/2)^x$ to the form $y = d^{-x}$.

Therefore y = $\_\_\_\_^{-x}$ is the _______________ of $y = 2^x$ in the __________ .

B. GRAPHS OF EXPONENTIAL FUNCTIONS 2: $y = e^x + c$

1. The diagram below shows the graph of $y = e^x + c$, for c = –1, 0, 1, 2, 3, 4.

Question 9: For the curve $y = e^x$, the y-intercept is ____ and the asymptote is y = ____ .



2. Record the y-intercepts and the asymptotes in the table below.

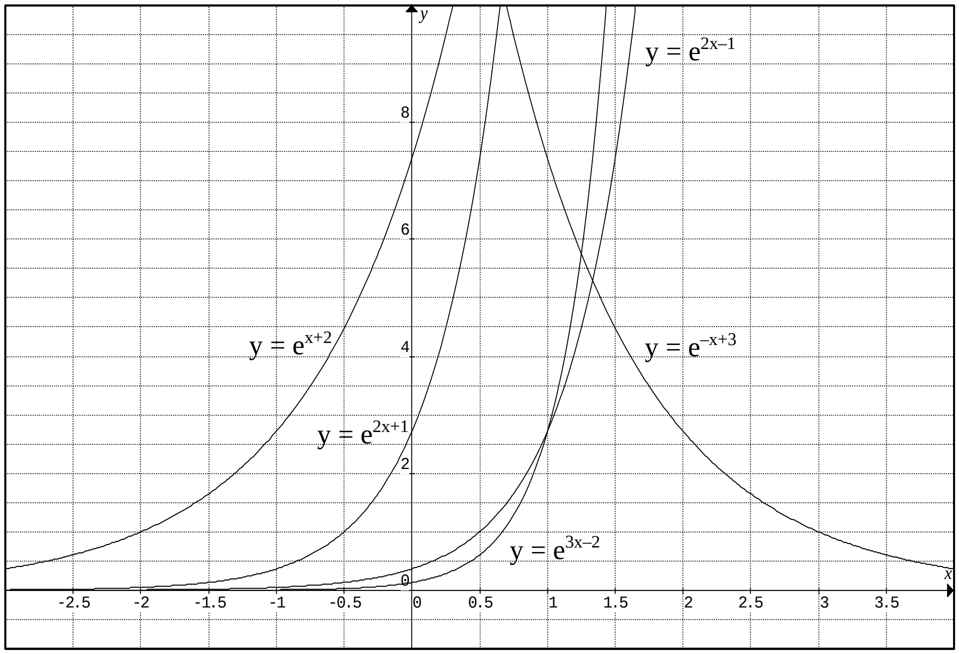
Curve	y-intercept	Asymptote
$y = e^x - 1$		y =
$y = e^x$		
$y = e^x + 1$		
$y = e^x + 2$		
$y = e^x + 3$		
$y = e^x + 4$		

Question 10: With reference to the curve $y = e^x$, infer by **induction** the effect of c on the curve $y = e^x + c$.

c will shift the curve $y = e^x$ __________ if c > 0 and ___________ if c < 0.

C. GRAPHS OF EXPONENTIAL FUNCTIONS 3: $y = e^{ax+b}$

1. The diagram below shows the graph of $y = e^{ax+b}$ for some values of a and b.



Question 11: For the curve $y = e^{2x+1}$, the y-intercept is ________ and the asymptote is y = ______ .

Hint: To find the y-intercept, put x = 0 and use a calculator to evaluate y to 3 significant figures.

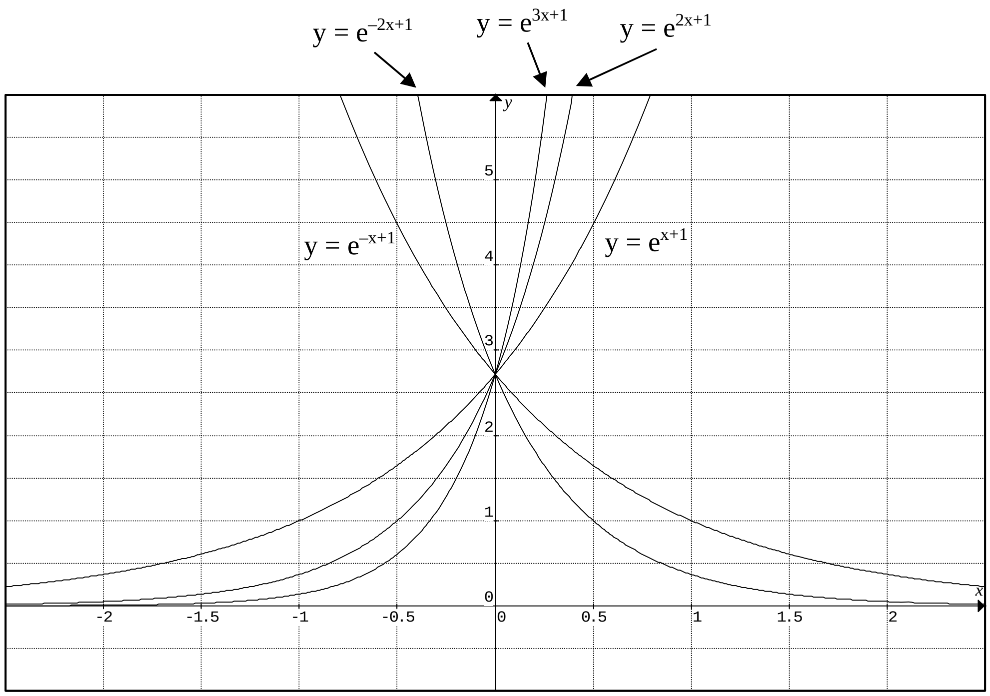
2. Record the y-intercepts and the asymptotes for the curves given in the table below.

Curve	y-intercept (to 3 sig. fig.)	Asymptote
$y = e^{x+2}$	$e^2 =$	y =
$y = e^{2x-1}$		
$y = e^{-x+3}$		
$y = e^{3x-2}$		

Question 12: What do you notice about the asymptotes?

____

3. The diagram below shows the graph of $y = e^{ax+b}$ for some other values of a and b. Record the y-intercepts and the asymptotes in the table below.



Curve	y-intercept (to 3 sig. fig.)	Asymptote
$y = e^{-2x+1}$	$e^1 =$	$y =$
$y = e^{-x+1}$		
$y = e^{x+1}$		
$y = e^{2x+1}$		
$y = e^{3x+1}$		

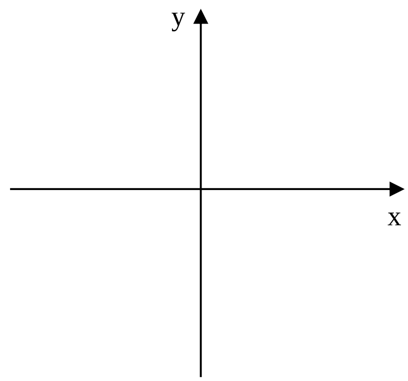
Question 13: What do you notice about the y-intercepts and the asymptotes?

____

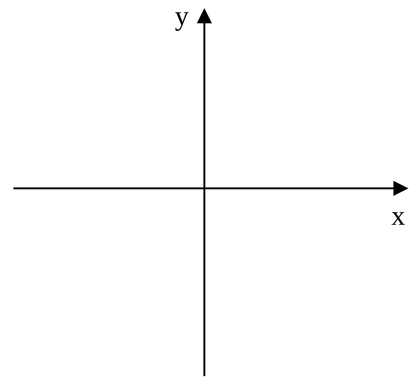
D. EXERCISE

1. Apply what you have learnt to **deduce** the shapes of the following curves and sketch them, showing the **y-intercept** and the **asymptote** clearly.

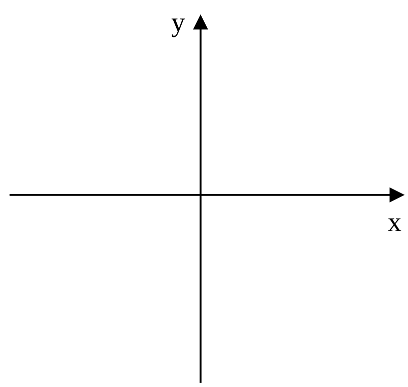
(a) $y = e^x$



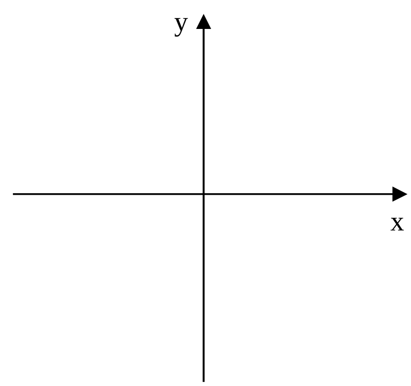
(b) $y = e^x - 2$



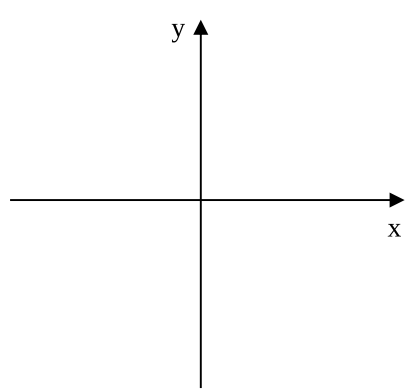
(c) $y = e^{-x}$



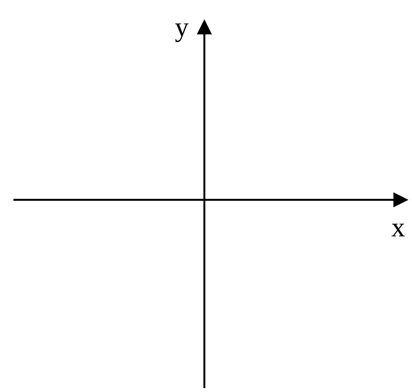
(d) $y = e^{3x+2}$



(e) $y = e^{-x} + 1$



(f) $y = e^{-2x+1}$



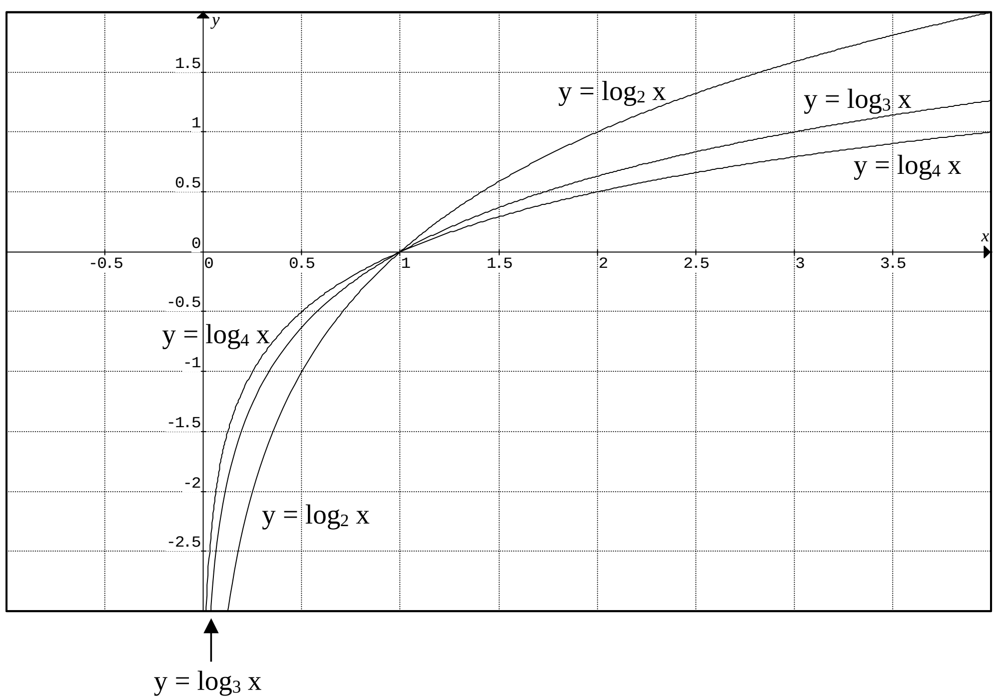
APPENDIX E: WORKSHEET ON LOGARITHMIC CURVES FOR CONTROL CLASS

EXPONENTIAL AND LOGARITHMIC FUNCTIONS WORKSHEET 2 – GRAPHS OF LOGARITHMIC FUNCTIONS

Thinking Skills: **Induction** and **Deduction**	Time Frame: 2 periods
At the end of the lesson, the students should be able to: (5) sketch the graphs of logarithmic functions, (6) describe the characteristics of logarithmic functions.	

A. GRAPHS OF LOGARITHMIC FUNCTIONS 1: $y = \log_a x$

1. The diagram below shows the graphs of the **logarithmic functions** $y = \log_4 x$, $y = \log_3 x$ and $y = \log_2 x$.



Question 1: What is the x-intercept for all the 3 curves? ____________

Question 2: What is the asymptote for all the 3 curves? ____________

Question 3: When x>1, which curve has the largest y-values (i.e. which curve is on top)?

y = ________ has the largest y-values, i.e. $\log_2 x >$ ________ > ________ .

Question 4: When x<1, which curve has the largest y-values (i.e. which curve is on top)?

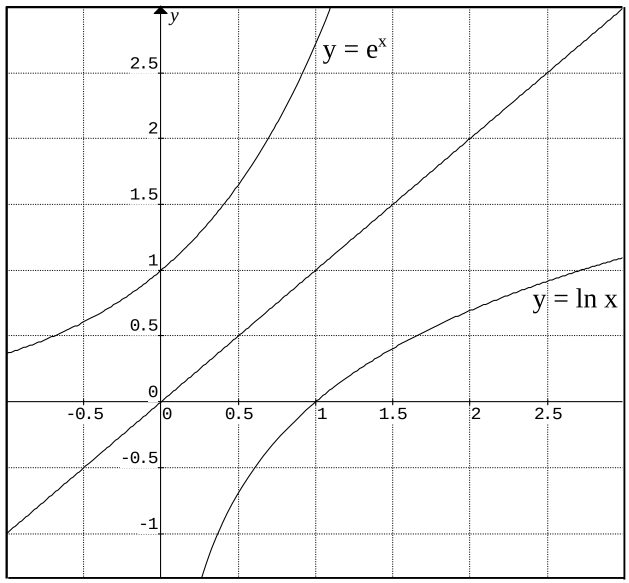
y = ________ has the largest y-values, i.e. $\log_2 x <$ ________ < ________ .

Question 5: $y = \ln x$ (or $\log_e x$) is called the **natural logarithmic function** where $e \approx 2.71828$. Where do you expect the curve $y = \ln x$ to lie?

$y = \ln x$ will lie between y = _________ and y = __________ .

B. GRAPHS OF LOGARITHMIC FUNCTIONS 2: $y = \ln x$

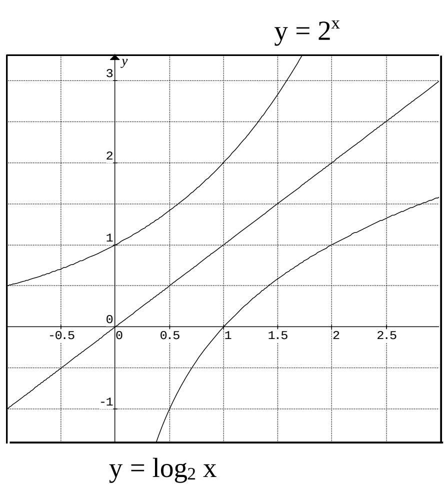
1. The diagram below shows the graphs of $y = \ln x$ and $y = e^x$.

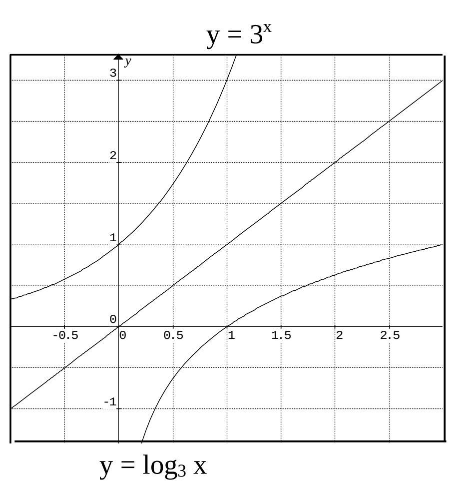


Question 6: Describe **fully** the transformation that maps the natural logarithmic curve $y = \ln x$ to the natural exponential curve $y = e^x$.

____

2. The diagrams below show the graphs of $y = \log_2 x$ and $y = 2^x$, and the graphs of $y = \log_3 x$ and $y = 3^x$.



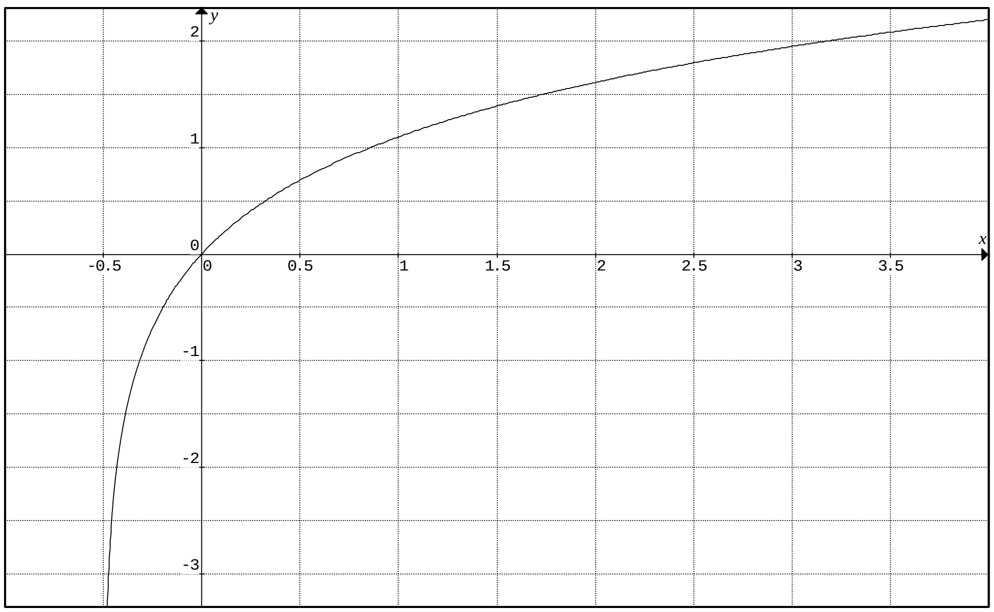


Question 7: Infer by **induction** the transformation that maps the logarithmic curve $y = \log_a x$ to the exponential curve $y = a^x$.

____

C. GRAPHS OF LOGARITHMIC FUNCTIONS 3: y = ln (ax+b)

1. The diagram below shows the graph of y = ln (2x+1).



Question 8: For the curve y = ln (2x+1), the x-intercept is ___________ and the asymptote is x = _________ .

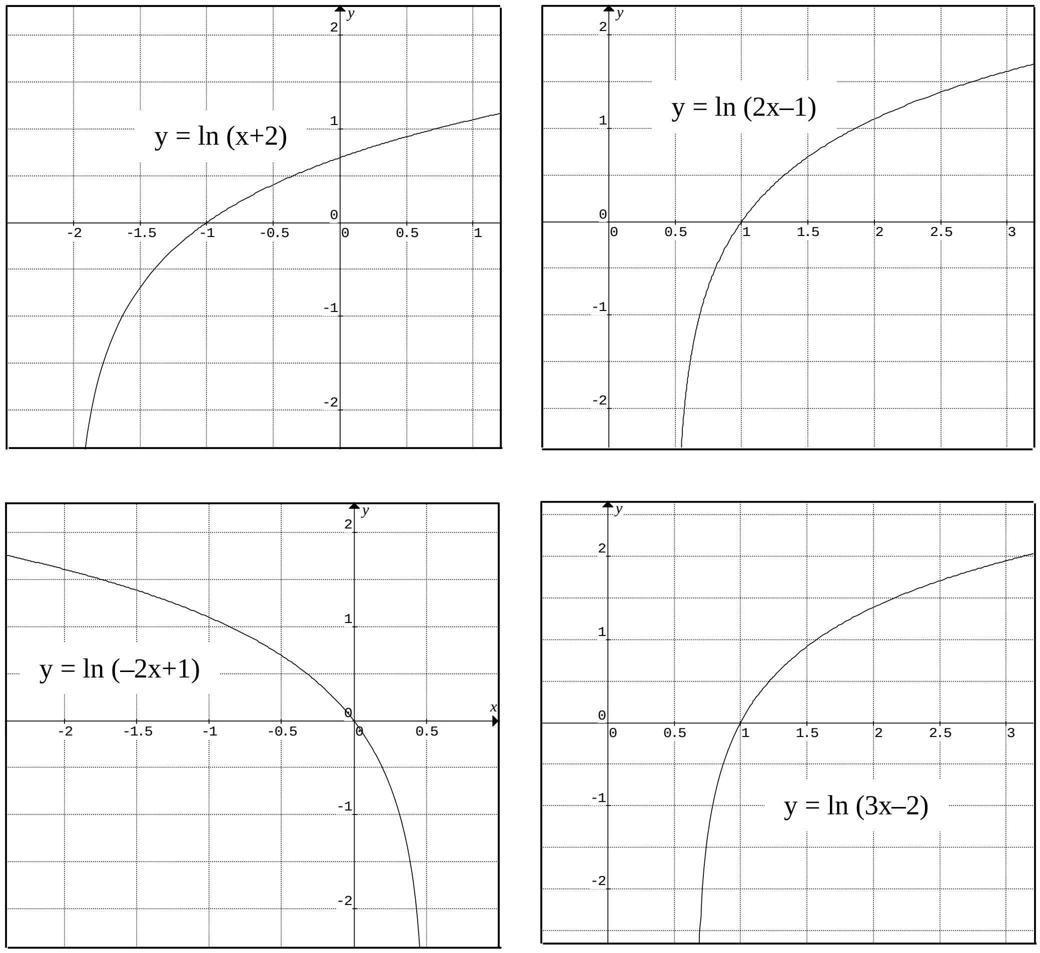
Question 9: Explain how you find the equation of the asymptote for the graph of y = ln (ax+b) in terms of a and b.

____

Question 10: Explain how you find the x-intercept. You may want to express it in terms of a and b.

____

2. The diagrams below show the graphs of y = ln (x+2), y = ln (2x–1), y = ln (–2x+1) and y = ln (3x–2). Write down their x-intercept and asymptote in the table below.



No.	Curve	x-intercept	Asymptote
1.	y = ln (x+2)		x =
2.	y = ln (2x–1)		
3.	y = ln (–2x+1)		
4.	y = ln (3x–2)		

Question 11: How is the third curve different from the rest?

____

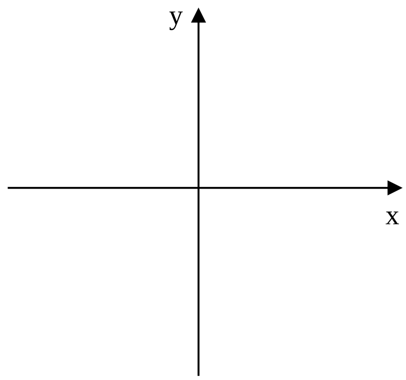
Question 12: What is the condition for the third curve to be different from the rest?

____

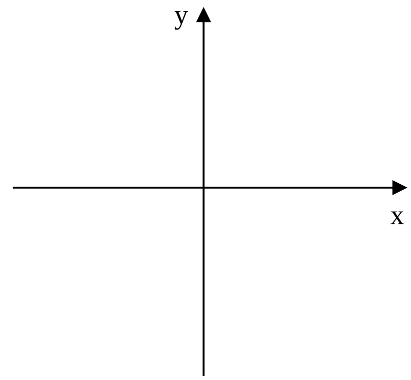
D. EXERCISE

1. Apply what you have learnt (esp. Q9–10) to **deduce** the shapes of the following curves and sketch them, showing the **x-intercept** and the **asymptote** clearly.

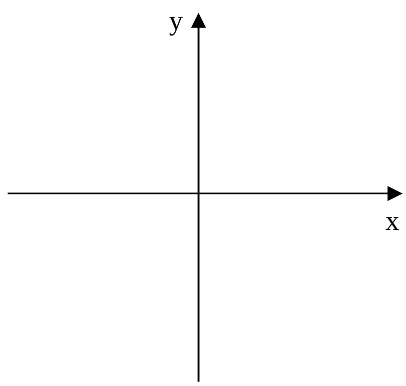
(a) $y = \ln(x+1)$



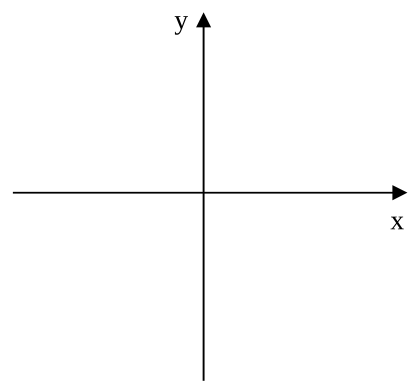
(b) $y = \ln(x-1)$



(c) $y = \ln(3x-1)$



(d) $y = \ln(-3x+1)$



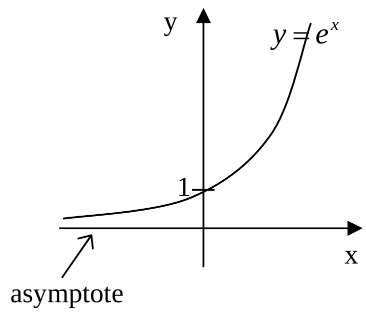
APPENDIX F: TEST ON PROCEDURAL KNOWLEDGE

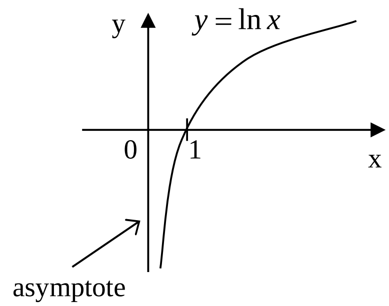
TEST ON EXPONENTIAL AND LOGARITHMIC FUNCTIONS (PART 1)

NAME: ____________________________ **CLASS:** _________________

Time Limit: 35 min Total Marks: 25

The graphs of $y = e^x$ and $y = \ln x$ are shown below.





For Questions 1-5, sketch the graph of the given equation in the box below. **Label clearly all the important points** (such as the intercepts of the axes) and the **asymptotes** (if any) in your graph. Show any necessary working on the right of the box.

1. $y = e^x + 1$ [4]

Working

Describe **fully** the transformation that maps $y = e^x$ to $y = e^x + 1$.

____

____

2. $y = e^{-x}$ [4]

Working

Describe **fully** the transformation that maps $y = e^{x}$ to $y = e^{-x}$.

3. $y = -\ln x$ [4]

Working

Describe **fully** the transformation that maps $y = \ln x$ to $y = -\ln x$.

4. $y = \ln x - 2$ [5]

Working

Describe **fully** the transformation that maps $y = \ln x$ to $y = \ln x - 2$.

5. $y = \ln(x - 2)$ [5]

Working

Describe **fully** the transformation that maps $y = \ln x$ to $y = \ln(x - 2)$.

6. By sketching a straight line in the graph in Q5, find the solution of the equation $x = e^{3-x} + 2$. Show your working clearly in the space below. [3]

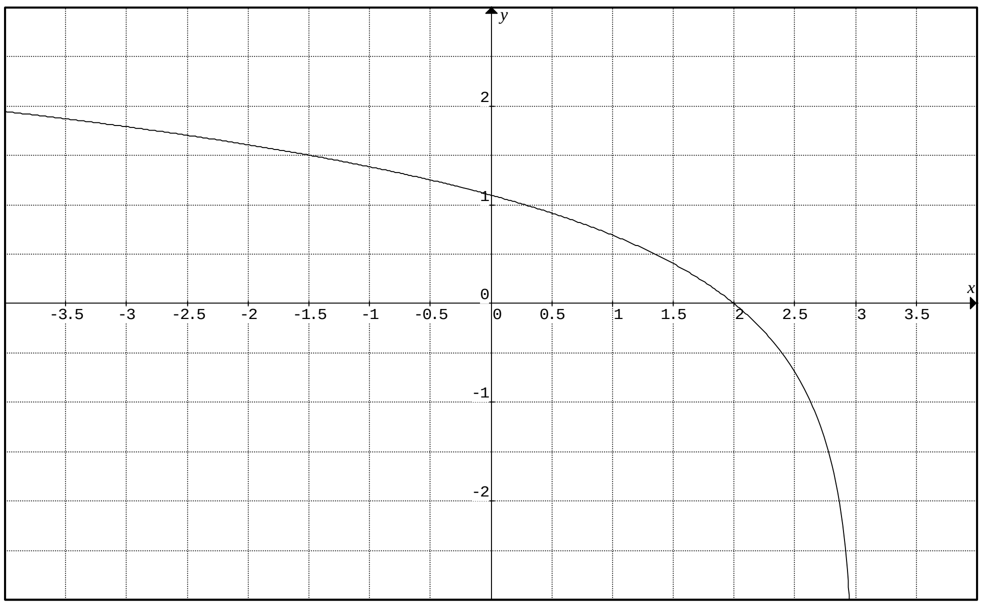
APPENDIX G: TEST ON CONCEPTUAL KNOWLEDGE

TEST ON EXPONENTIAL AND LOGARITHMIC FUNCTIONS (PART 2)

NAME: ______________________________ **CLASS:** __________________

Time Limit: 35 min Total Marks: 25

For Questions 7 and 8, refer to the graph below.



The diagram above shows the graph of $y = \ln(3 - x)$.

7. Is there any restriction on the values of x? If yes, state the restriction. [1]

____

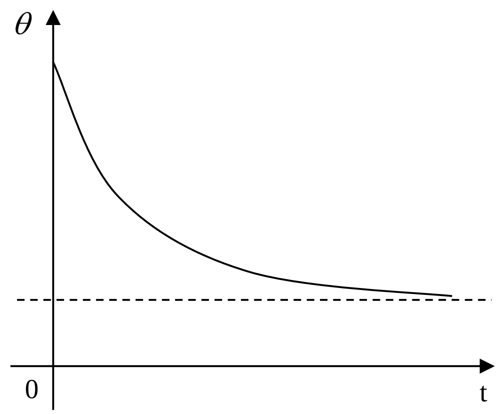
8. Is there any restriction on the values of y? If yes, state the restriction. [1]

____

Suppose you were doing your mathematics homework and you felt very sleepy. So you made yourself a cup of coffee. But you became too engrossed in your homework and forgot to drink your coffee. After some time, the coffee cooled down to a certain temperature. If you think this is unlikely, you can change the scenario to this: You became distracted by so many sms from your friends that you forgot both your homework and your coffee!

The graph below shows the cooling curve of the coffee. Its temperature θ °C, when it has been cooling for t minutes, is given by the equation $\theta = 30 + 50e^{-kt}$.

Answer Questions 9-14.



9. What was the initial temperature of the coffee? [1]

---

10. What was the room temperature? Show clearly how you derive your answer. [2]

---

11. If the temperature of the coffee was 32°C after cooling for 20 minutes, find the value of k, leaving your answer in terms of ln or correct to 4 significant figures. [2]

12. What was the temperature of the coffee after one hour, leaving your answer correct to 3 significant figures? [2]

13. If you consider the nature of the graph, the coffee will never reach the room temperature. Why? [2]

____

____

14. But we know from our daily experience that the coffee will eventually reach the room temperature. So how do you reconcile the differences? Hint: See Q12. [2]

____

____

The profit for Company A was \$90K (i.e. \$90 000) last year (t = 0). Let the profit be \$y K. Then when t = 0, y = 90.

It is projected that the profit for Company A for this year (t = 1) will be \$110K. From next year onwards, the profit is projected to be \$130K, \$150K, \$170K, …

A new company B only made a profit of \$1K last year (t = 0) but its profit is expected to double every year, i.e. \$2K, \$4K, \$8K, ...

Answer Questions 15-21.

15. Write down an equation relating the profit, y, of Company A, in terms of t. [1]

____

16. What is the name given to the type of increase for Company A? [1]

____

17. Write down an equation relating the profit, y, of Company B, in terms of t. [1]

____

18. What is the name given to the type of increase for Company B? [1]

____

19. Sketch, in the same diagram, the graphs of profit y against the time t for both companies. **Label clearly all the important points.** [3]

20. By using a calculator, **estimate** the value of t (to the nearest whole number) when both companies make about the same profit. Label this value clearly in the diagram in Q19. [2]

---

21. In the long run, which company will make more money? Explain your reason clearly. [3]

APPENDIX H: SPECIFIC INSTRUCTIONAL OBJECTIVES

The following are the specific instructional objectives (SIO) for the graphs of exponential and logarithmic functions (taken from the Scheme of Work from the school of the subjects):

At the end of the lessons, the students should be able to:

- sketch exponential functions of the form $y = ke^{ax+b} + c$.
- sketch logarithmic functions of the form $y = k\ln(ax+b) + c$.
- state the relationship and graphical significance between $y = e^x$ and $y = \ln x$.
- describe the geometrical transformation of $y = ke^x + c$ from $y = e^x$.
- describe the geometrical transformation of $y = k\ln x + c$ from $y = \ln x$.
- use the graphs of exponential functions to solve equations of the form

  $ke^{ax+b} = mx + c$.
- use the graphs of logarithmic functions to solve equations of the form

  $k\ln(ax+b) = mx + c$.

APPENDIX I: QUESTIONNAIRE ON AFFECT TOWARDS MATHEMATICS

CLASS: ____________________________

We are interested to find out what you think or feel about mathematics. Please read each statement carefully and circle the number that best represents how you feel.

You do not need to write your name and so your answers are completely anonymous. Feel free to express your honest thoughts and feelings.

SD: STRONGLY DISAGREE
D: DISAGREE
U: UNDECIDED
A: AGREE
SA: STRONGLY AGREE

NO.	ITEMS	SD	D	U	A	SA
1.	I enjoy discovering new things in mathematics.	1	2	3	4	5
2.	Mathematics is not important in daily life.	1	2	3	4	5
3.	I would like to read more books on mathematics.	1	2	3	4	5
4.	Mathematics helps to develop a person's analytical skills.	1	2	3	4	5
5.	No matter how hard I try, I'll never be good in mathematics.	1	2	3	4	5
6.	Working on mathematical problems is boring.	1	2	3	4	5
7.	Mathematics has contributed greatly to science, technology and other fields of knowledge.	1	2	3	4	5
8.	When I hear the word 'mathematics', I have a feeling of dislike.	1	2	3	4	5

NO.	ITEMS	SD	D	U	A	SA
9.	Mathematics is just memorizing formulae and practice.	1	2	3	4	5
10.	Mathematics is easy for me.	1	2	3	4	5
11.	Solving mathematical puzzles is fun.	1	2	3	4	5
12.	Mathematics is not important for the advance of civilization and society.	1	2	3	4	5
13.	I would like to spend less time in school doing mathematics.	1	2	3	4	5
14.	Mathematics is a worthwhile subject to study.	1	2	3	4	5
15.	I am able to solve mathematics problems.	1	2	3	4	5
16.	If mathematics is not a compulsory school subject, I will not take it.	1	2	3	4	5
17.	Mathematics helps a person to appreciate patterns in the world.	1	2	3	4	5
18.	I appreciate the beauty of mathematics.	1	2	3	4	5
19.	Mathematics is less important than art, literature, geography or music.	1	2	3	4	5
20.	I usually understand what we are talking about during mathematics class.	1	2	3	4	5

APPENDIX J: QUESTIONNAIRE ON AFFECT TOWARDS EXPONENTIAL AND LOGARITHMIC CURVES

CLASS: ____________________________

We are interested to find out what you think or feel about the topic on the graphs of exponential and logarithmic functions. Please read each statement carefully and circle the number that best represents how you feel.

You do not need to write your name and so your answers are completely anonymous. Feel free to express your honest thoughts and feelings.

SD: STRONGLY DISAGREE
D: DISAGREE
U: UNDECIDED
A: AGREE
SA: STRONGLY AGREE

NO.	ITEMS	SD	D	U	A	SA
1.	I enjoyed learning the graphs of exponential and logarithmic functions.	1	2	3	4	5
2.	I regard the graphs of exponential and logarithmic functions as some kind of curves without meaning.	1	2	3	4	5
3.	The topic on the graphs of exponential and logarithmic functions was easy to understand.	1	2	3	4	5
4.	It was boring to explore the properties of exponential and logarithmic curves.	1	2	3	4	5
5.	Exponential and logarithmic curves have applications in real life.	1	2	3	4	5
6.	I find it tedious to locate points on the exponential and logarithmic curves.	1	2	3	4	5
7.	I find it interesting to solve problems on exponential and logarithmic curves.	1	2	3	4	5

NO.	ITEMS	SD	D	U	A	SA
8.	Graphs of exponential and logarithmic functions are only useful for passing tests and exams.	1	2	3	4	5
9.	I am able to relate exponential and logarithmic curves to their equations and vice versa.	1	2	3	4	5
10.	I dislike this topic.	1	2	3	4	5
11.	This topic helps me to understand why I need to study mathematics.	1	2	3	4	5
12.	I am able to solve problems on exponential and logarithmic curves.	1	2	3	4	5
13.	Studying the graphs of exponential and logarithmic functions was a waste of time.	1	2	3	4	5

APPENDIX K: QUESTIONNAIRE ON AFFECT TOWARDS COMPUTER-BASED LEARNING

CLASS: ____________________________

We are interested to find out what you think or feel about the use of computers in learning the topic on the graphs of exponential and logarithmic functions. Please read each statement carefully and circle the number that best represents how you feel.

You do not need to write your name and so your answers are completely anonymous. Feel free to express your honest thoughts and feelings.

SD: STRONGLY DISAGREE
D: DISAGREE
U: UNDECIDED
A: AGREE
SA: STRONGLY AGREE

NO.	ITEMS	SD	D	U	A	SA
1.	I enjoyed using computers to learn the graphs of exponential and logarithmic functions.	1	2	3	4	5
2.	I can understand this topic in a shorter time without using computers.	1	2	3	4	5
3.	Using computers to learn this topic is more interesting than normal classroom teaching using the whiteboard.	1	2	3	4	5
4.	The use of computers has helped me to understand the concepts of the graphs better.	1	2	3	4	5
5.	I feel that the software is not interesting.	1	2	3	4	5
6.	The interacting feature of the software helps me to explore the properties of the curves better.	1	2	3	4	5
7.	I was more involved in using the computer than in understanding the lesson.	1	2	3	4	5

NO.	ITEMS	SD	D	U	A	SA
8.	The computer graphics make it easier for me to visualize the nature of the graphs.	1	2	3	4	5
9.	Time seems to pass very slowly in computer-based lessons.	1	2	3	4	5
10.	I feel that the use of computers can help me to be a more independent learner.	1	2	3	4	5
11.	I dislike using computers to learn mathematics.	1	2	3	4	5
12.	The animation in the software helps me to understand the lesson better.	1	2	3	4	5
13.	I did not know enough of the software to engage in any meaningful activity during computer lessons.	1	2	3	4	5
14.	Using computers gives me more control over what I learn.	1	2	3	4	5
15.	I find it interesting to use the software to explore the properties of the graphs.	1	2	3	4	5
16.	I think that it is not worthwhile spending so much curriculum time on computer lessons.	1	2	3	4	5
17.	I find computer-based lessons enjoyable.	1	2	3	4	5
18.	It makes no difference to me whether my teacher uses the computer or the whiteboard to teach this topic.	1	2	3	4	5