

Using *LIVEMATH*TM to bring mathematics alive

Joseph B. W. Yeo describes an interactive algebra computer system to help students explore algebra and calculus

Many mathematics teachers are familiar with the use of software, such as the *Geometer's Sketchpad* with students to explore geometry and graphs (e.g. Fallstrom & Walter, 2011), and *Excel* or *Tinkerplots* to analyse statistics (e.g. Kazak, Wegerif & Fujita, 2013). But what about using a software package to explore algebraic and calculus concepts, such as factorisation and differentiation? Some teachers have used a computer algebra system (CAS) such as *Maple* (e.g. Dwyer, Moskal & Duke, 2007), which mathematicians, scientists, and engineers usually use to perform algebraic manipulations to solve complex problems. But, in general, students have no need for a CAS just to carry out symbolic manipulations because most formal assessments require them to do these with paper and pencil.

In this article, I will explain how teachers can use an interactive CAS called *LiveMath* (MathMonkeys, 2015) with their students to explore algebra and calculus, including online learning using a Web-based version of *LiveMath*. I will also outline briefly a research study that seems to support the benefits of using such software.

The Software: LiveMath

In the 1990s, James Kaput (1942-2005), the former Chancellor Professor of Mathematics at the University of Massachusetts, and an 'imagineer' and futurologist of mathematics education, was already impressed with an earlier version of *LiveMath* called *Theorist*. He commented that the software provides 'a unique user interface that allows one to perform 'natural' algebraic manoeuvres even more 'naturally' than one can achieve them on paper' (Kaput, 1992, p. 534). What this means is that when you change the value of a parameter or variable in the software, all the algebraic expressions, equations, and graphs linked to this parameter or variable will also change instantaneously. This interactive feature allows students to investigate many specific cases easily, instead of re-keying the algebra and re-plotting the graph for every case, which can be time-consuming. The software also allows teachers to do the programming to design the template beforehand, so that students can focus on exploring the mathematics without having to learn a new piece of software.

Figure 1 shows an example of a pre-designed *LiveMath* template from Yeo (2001) which can be used by students to explore what happens to the gradient of a curve in the vicinity of a stationary point, culminating in the discovery of the first derivative test.

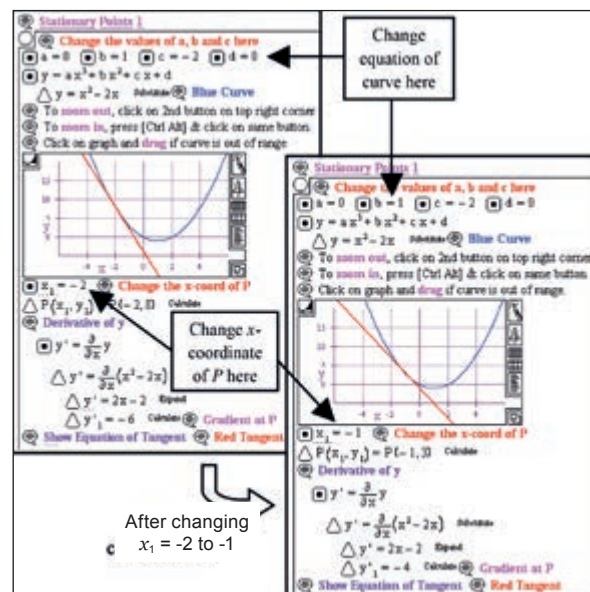
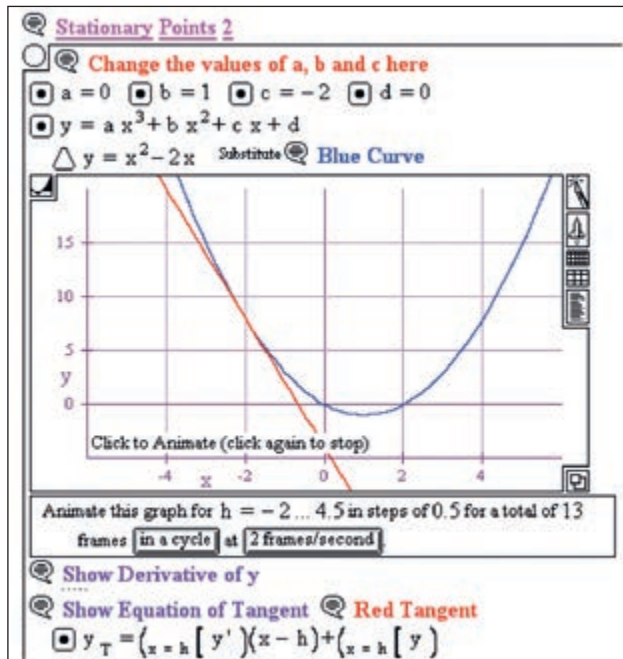


Figure 1 Interactive *LiveMath* Template

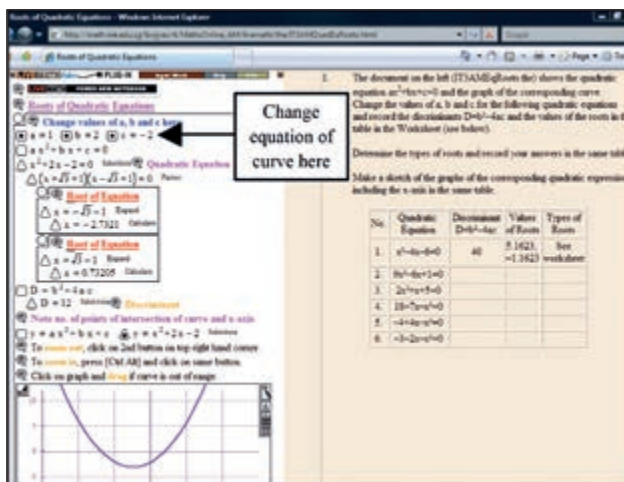
Without this software, students would have to differentiate to find the gradient at different points for each curve. If there is a need to visualise the graph, students would have to plot each curve, and the tangent to the curve for every point of contact. But, with this template, students just need to change the x-coordinate of the point of contact P between the curve and the tangent, and the gradient of the curve at P , the equation of the tangent and the graph will change automatically, see Figure 1. For another curve, students just need to change the equation of the curve and everything that is linked to the equation will change instantaneously. This facilitates the ease of investigation.

Furthermore, *LiveMath* is able to support animation. Figure 2 shows another template that animates the position of the tangent at different point of contact with the curve. This allows students to see how the gradient of the curve, which is equal to the gradient of the corresponding tangent, changes as x increases.

Figure 2 *LiveMath* Template Showing Animation

LiveMath Internet

The marvel of *LiveMath* is that it is still interactive on the *World Wide Web* and end users do not even need the software to interact with it. All they need to do is to install a free *LiveMath* plug-in and then they can view the template and change the values of the variables. Figure 3 shows an example of a Web-based *LiveMath* template from; <http://math.nie.edu.sg/bwjyee> which students can use to investigate the nature of the roots of a quadratic equation.

Figure 3 Web-based Interactive *LiveMath* Template

By changing the values of the parameters a , b , and c of the quadratic equation, the template will automatically find the roots of the new equation, re-calculate the discriminant, and re-plot the corresponding graph. Students will then be able to observe how the roots of a quadratic equation are related to the discriminant and the number of times

the curve cuts the x -axis. Imagine if a teacher wants students to discover these relationships without the use of an appropriate piece of software. Students would have to solve *each* quadratic equation, calculate its discriminant and plot its corresponding graph on a sheet of graph paper.

Alternatively, teachers can set the task as an *e-learning task* where students can use a computer at home to go online to interact with the template in order to learn, and understand the required mathematical concepts. Teachers can also design their own templates and put them up in their school website if they have the software and expertise.

Computerised Learning – some issues

One important issue is whether the use of computer technology in teaching really helps to enhance student learning. If it does not, then it will be a complete waste of time and money. Although many research studies show that students learn better with the help of computers, for example Barton's (2000) meta-analysis of more than 60 studies in various countries since 1995, Oppenheimer (1997) raised a valid objection: ... most of these studies did not control extraneous factors such as differences in pedagogies. The flaw with many of these studies is that they usually compare student-centred computerised learning with traditional direct teaching, and the better performance of students using computer technology may well be due to the employment of a different teaching method than the mere use of computers. If teachers were to change from a teacher-centred pedagogy to a student-centred approach without the help of computers, Oppenheimer argued that their students would also perform better.

However, some research studies compared the use of the same student-centred pedagogies in both the experimental and the control classes: ... the only difference was that the experimental classes learned through the use of computer technology while the control classes did not have access to computers. For example, Yeo's (2006) study indicates that students, who used *LiveMath* templates to explore exponential and logarithmic functions, performed significantly better in a test of both conceptual and procedural knowledge on the topic, than students who were taught using a student-centred approach without the help of computer technology. Therefore, it seems that the use of interactive *LiveMath* templates do help to enhance student learning, although more research needs to be done.

Conclusion and Implications

With the advance of new innovative technology such as *LiveMath*, teachers can use the interactive CAS as a cognitive tool for their students to

explore algebraic and calculus concepts more effectively, instead of the traditional way of using a CAS as a computing tool to perform symbolic manipulations. The instantaneous changing effect of varying any parameter in a template has made mathematics *come alive* for many students, thus the name *LiveMath*. Research seems to indicate that it is possible to enhance learning through the appropriate use of this software. The free provision of the *LiveMath Plug-in* can also help to propagate the use of the software in schools since the end-users, the students, do not need to acquire the software. The interactive feature of these *web-based* templates has also opened a new chapter of engaging online learning for students.



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Letter from a Reader

In MT 246, Janine Blinko poses the problem of finding two factors of 4095 between 60 and 70.

There is an even easier way of finding these factors than dividing 4095 by 65.

By similar reasoning to that which leads to 65 being one factor, the other must be 63. The sum of the digits of 4095 is 18. Therefore 9 must be a factor. The only multiple of 9 in the given range is 63.

I do enjoy reading MT!

Yours sincerely

Jenny Maxwell