Engaged Learning in Mathematics

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Abstract: Engaged learning is an integral part of the ‘Teach Less, Learn More’ initiative. This paper will describe various strategies to engage secondary schools students in their learning of mathematics. It will provide concrete examples of how to use worksheets to guide students to discover certain mathematical concepts, and how to use real-life investigative tasks to engage the minds of the students and to develop in them an inquiry attitude towards mathematics. All these strategies were taught in an inservice course for secondary school teachers and the paper will present their views and attitudes towards engaged learning for their students.

1. INTRODUCTION

Teach Less, Learn More (TLLM) is about "teaching better, to engage our learners and prepare them for life, rather than teaching more, for tests and examinations" (MOE, 2005). It aims to touch the hearts and engage the minds of our learners. Instead of teaching more content using a teacher-centred pedagogy, TLLM is about student-centred learning; it is about how to engage the students' minds and hearts so that they can learn more.

This idea is not new in the educational circle. We can trace its roots to how students learn. Research has shown that students are not empty vessels waiting for teachers to transmit their knowledge but students actively construct their own knowledge by linking them to their existing schema (Ernest, 1994; Noddings, 1990). The implication is to provide a conducive environment for students to make sense of what they are learning.

As the scope of TLLM is very wide, this paper will only discuss one aspect: how to make use of effective pedagogies to engage learners in both their minds and hearts. It will first present examples of using worksheets to guide students to discover mathematical concepts. Then it will discuss how to use real-life investigative tasks to develop in students an inquiry attitude towards mathematics. Finally it will present the views of teachers who attended an inservice course where these strategies are taught.

2. GUIDED DISCOVERY APPROACH

To help students construct their own knowledge, one method is to guide them to explore certain mathematical concepts. This is based on Bruner's discovery approach (1961, 1974). We will illustrate with a simple example. How do teachers teach their students that a negative number multiplied by a negative number is positive? One method is to teach students the rule: "*negative times negative equals positive*". But this is teaching for instrumental understanding which is not understanding at all (Skemp, 1976). If students cannot make sense of what they are learning, the link between this piece of new knowledge and their existing schema is weak or it may not even exist at all. They may then easily forget what they have learnt or they may confuse this rule with other rules. For example, some students will write

(−2) + (−3) = 5 because “*negative and negative equals positive*” without realising that this rule applies for multiplication.

Another approach is to guide students to discover the rule for themselves by providing them with a worksheet. Figure 1 shows a simplified version of the worksheet which is half completed.

1^{st} Number × 2^{nd} Number =	Value	
4 × 3 =	12	–3
3 × 3 =	9	–3
2 × 3 =	6	–3
1 × 3 =	3	–3
0 × 3 =	0	–3
– 1 × 3 =		
– 2 × 3 =		
– 3 × 3 =		
– 4 × 3 =		

1^{st} Number × 2^{nd} Number =	Value	
4 × (–3) =	–12	+3
3 × (–3) =	–9	+3
2 × (–3) =	–6	+3
1 × (–3) =	–3	+3
0 × (–3) =	0	+3
– 1 × (–3) =		
– 2 × (–3) =		
– 3 × (–3) =		
– 4 × (–3) =		

Figure 1

In the first table, students are guided to observe the pattern that −1 × 3 = 0 – 3 = –3, etc. Then they apply what they have learnt to complete the first part of the second table and then observe the pattern that –1 × (–3) = 0 + 3 = +3, etc. In this way, students will have a sense of why a negative number times a positive number is negative, and why a negative number times a negative number is positive.

Even in this approach, the teacher can explain the pattern on the whiteboard to the entire class. But how effective is this teacher-centred method when most students just sit and listen, or maybe just sit and daydream? To engage the students in their minds, it may be more effective to let the students try the worksheet themselves so that they can discover the concept for themselves.

3. MATHEMATICAL INVESTIGATIONS

Another idea to engage the students is to make use of investigative tasks. True mathematical investigations are different from explorations of mathematical concepts in that the former has no specific goals (Frobisher, 1994; Skovsmose, 2002). In exploring mathematical concepts, students are guided to discover a particular concept. But in true mathematical investigations, there is plenty of scope for students to discover many new things that even the teacher may not have anticipated. We will illustrate this with the following example.

> *Some numbers can be expressed as the sum of consecutive whole numbers. For example,*
>
> $$9 = 4 + 5 = 2 + 3 + 4$$
> $$14 = 2 + 3 + 4 + 5$$
>
> *Investigate.*

This investigative task is open-ended in that different students can pursue different trains of thoughts. For example, a student may want to look at which numbers can be expressed as a sum of consecutive whole numbers; another student may want to investigate which numbers can be expressed as a sum of consecutive whole numbers in more than one way; a third student may want to find out whether a prime number can be expressed as a sum of consecutive whole numbers.

A good investigative task will not end here. There is plenty of scope to extend the problem. What if we express the numbers as the sum of consecutive natural numbers or integers instead of whole numbers? What if we express the numbers as the product of consecutive whole numbers, natural numbers or integers? As you can see, a true mathematical investigation is very different from the exploration of specific mathematical concepts because the former has no fixed goals.

This investigative task was given to a group of teachers in an inservice course and most of them did not know how to begin. This is not surprising as most teachers have not been exposed to such tasks. Similarly, their students will not know what to investigate if they are given the task in this form. Therefore, scaffolding is needed to guide students at the initial stage. For example, the first part of the worksheet can be structured as follows:

> *The odd number 9 can be written as the sum of two consecutive whole numbers: 9 = 4 + 5. Express the following odd numbers as the sum of two consecutive whole numbers.*

$$\begin{aligned} 1 &= \quad + \\ 3 &= \quad + \\ 5 &= \quad + \\ 7 &= \quad + \\ 9 &= 4 + 5 \\ 11 &= \quad + \end{aligned}$$

> *What pattern(s) do you notice?*

Then the students will be guided to investigate which numbers can be expressed as the sum of three consecutive whole numbers. After that, the scaffold is removed and the students are asked:

> *Which numbers can be written as the sum of four consecutive whole numbers? Investigate.*

Hopefully the students will know how to investigate.

The purpose of mathematical investigations is to give students a taste of what mathematicians do, for example, formulating and testing conjectures, constructing arguments and generalising (Lampert, 1990). These are useful thinking skills that can stimulate the students' minds.

The example shown above in rather academic. To engage students in their hearts as well, we can use real-life investigative tasks to arouse their interest. For example, the students can investigate what mathematics has to do with the birth rates of rabbits and the number of petals of flowers, or how the golden ratio and the perfect rectangle affect the design of buildings in ancient times (Yeap, Yeo & Chow, 2006). There are also mathematically-rich games that are suitable for investigations. The context is very real for these students because the outcome, whether they win or lose, matters to them (Ainley, 1990).

4. RESEARCH STUDY

This was a small study where teachers were taught these pedagogies in an inservice course. Four sessions were conducted over a period of four weeks. Each session lasted three hours. At the beginning of the first session and at the end of the last session, the teachers were required to fill up a survey on their views of mathematics teaching and learning. They were also asked for their comments on these pedagogies at the end of the course.

Analysis of the pre and post survey showed that there was no significant difference in the teachers' views of mathematics teaching and learning. This might be due to the short duration of the course: the intervention was too short to influence the teachers' beliefs. Moreover, some teachers were more concerned about covering the syllabus and preparing their students for tests and examinations that they did not see the relevance of exploring mathematical concepts and doing mathematical investigations.

Nevertheless, a number of teachers found the course useful. Some positive comments were:

- Especially when students can generalise their findings after a series of investigations, the students will be able to draw their own conclusion and we would have achieved teach less learn more.
- Good worksheets to help pupils to discover and make meaning of patterns in mathematics. Multiplication of negative numbers using the worksheet helps pupils remember the signs better.

5. CONCLUSION

Guiding students to explore mathematical concepts and performing academic and real-life investigative tasks are just two ways to engage the minds and hearts of students. Some teachers have found these approaches useful while others do not see the relevance of such methods because they are more concerned about covering the syllabus and preparing their students for tests and examinations. For the ‘Teach Less Learn More’ initiative to work, teachers must first believe the value of it. Thus it is very important to change the mindsets of these teachers before they can teach less so that their students will learn more.

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