Redesigning Pedagogy for Mathematics with the Help of Technology

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Abstract: Many teachers have experienced at one time or another the frustrations of trying to impart their knowledge to their students but the latter somehow seem not to grasp the full meaning of the content taught. This may be due to the constructivists' belief that knowledge cannot be transmitted from teachers to learners but is actively constructed by the learners themselves as they attempt to make sense of their experiences. So this paper attempts to look at how mathematics teachers can redesign their pedagogy by taking into account new teaching methods that are made possible by technology. The paper will also give a few examples of how to use various mathematical software to guide pupils to explore mathematical concepts so that they can construct their own knowledge.

1. INTRODUCTION

The traditional chalk-and-talk teacher-centred type of teaching assumes that students are passive objects waiting to be filled with knowledge. But more often than not, what students actually learn can be very different from what the teacher has taught. This is because students do not just absorb knowledge wholesale but they actively construct their own knowledge (Noddings, 1990), assimilating or accommodating new knowledge into their existing schema (Piaget, 1970). The practical implication of this theory of constructivism is the need to facilitate students' construction of their own knowledge, rather than attempting to 'force' the teacher's knowledge into them. This calls for a student-centred mode of learning (Henson & Eller, 1999).

One way to enhance pupil learning is to guide students to discover various mathematical ideas or concepts so that they can construct their own knowledge. This pedagogy is rooted in Bruner's (1961; 1974) guided discovery approach. But there is a limit to how much the students can explore (see Section 2.1). However, with the advance of technology, teachers can actually harness it to help their students in their mathematical explorations.

This paper will look at these 'new' teaching methods that are made possible by technology (although the idea behind them is not new). Some concrete examples for secondary school mathematics will also be given.

2. REDESIGNING PEDAGOGY

In this section, two examples will be given to illustrate the 'new' teaching methods made possible by technology.

2.1 Example 1: How to Teach Circle Theorems

How do teachers normally teach this angle property of circles: Angle at Centre = 2 × Angle at Circumference?

Method 1: Deductive Proof

Traditionally teachers usually begin with a deductive proof which involves two cases. In the first case, the diameter CX cuts through the angle AOB (as shown in Fig. 1.1). In the second case, the diameter CX does not cut through the angle AOB and so the proof for this angle property has to be modified.

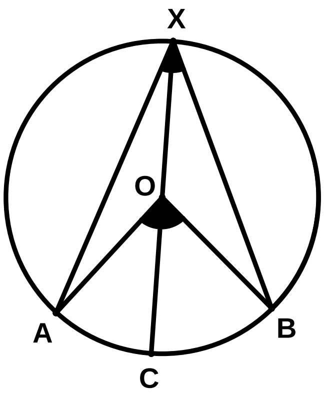

Figure 1.1

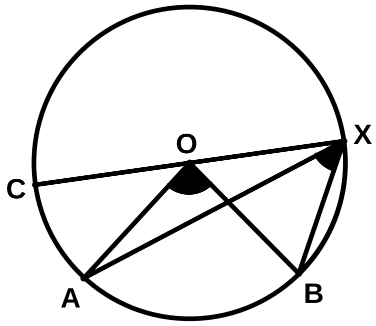

Figure 1.2

Many students have found this method rather difficult to understand. Why?

According to the Van Hiele's Theory (1986), students are at the *Recognition* Level[1] (or Level 1) where they have just begun to recognise what angles subtended at the centre or at the circumference of the circle are. The next level is *Analysis* (or Level 2) where students begin to recognise some angle properties of circles. But understanding a formal deductive proof for these angle properties is at the *Deduction* Level (or Level 4). Therefore it is a gigantic quantum leap to jump from Level 1 straight to Level 4. That is why many students find it difficult to understand all these deductive proofs.

Method 2: Induction by Drawing Circles

Some teachers, having learnt the Van Hiele's theory (1986), decide to let their students discover these angle properties without providing a formal proof right at the start. The teacher usually begins by asking every student to draw a circle, an angle subtended at the centre of the circle by an arc AB and an angle subtended at the circumference by the same arc AB (similar to Fig. 1.1). Then the teacher asks every student to measure both angles and find the relationship between them. Although each student only has one specific case to explore, the teacher can then collate the results from the whole class of about 40 students and all these specific examples will say the same thing. The teacher will then generalise the result using the thinking skill of inference by induction. It is important to note that this is not really a proof and induction does not tell a student *why* it happens. Pierre and Dina van Hiele are not saying that students do not need to learn deductive proofs but that they should learn them at a later stage.

The problem with induction is how many cases the students can explore. In this example, the students collectively explore about 40 cases, which is a good number compared to what individual students can explore on their own. But it cannot hide the fact that each student *actually* explores only one case on his or her own. The other cases are what others have explored and so they are not within each student's own realm of experiences. Moreover what is important is whether these are representative cases, i.e. whether these cases represent all possible variations. In all likelihood, no student will draw the angles according to Figure 1.2 unless the teacher specifically tells some of them to do so. Moreover what happens if the angle subtended at the centre of the circle is a reflex angle?

Method 3: Induction by Using the Geometer's Sketchpad

With the advance of technology, exploring these angle properties has become somewhat easier. The students just use a pre-designed *Geometer's Sketchpad* (or GSP) template that looks like the following in Figure 2.1. The *Geometer's Sketchpad* is a dynamic geometry software: when the user moves a mathematical object such as a point, a line or a circle, all the other objects that are linked to it will also move automatically in order to preserve the geometrical properties among them. Therefore all the students need to do is to drag the various points (namely points A, B, X and R) around and the measurements of the angles subtended at the centre and at the circumference of the circle will also change instantaneously.

[1] Some name the Van Hiele's Theory Level 0 to 4 instead of Level 1 to 5. In this paper, we will use Level 1 to 5.

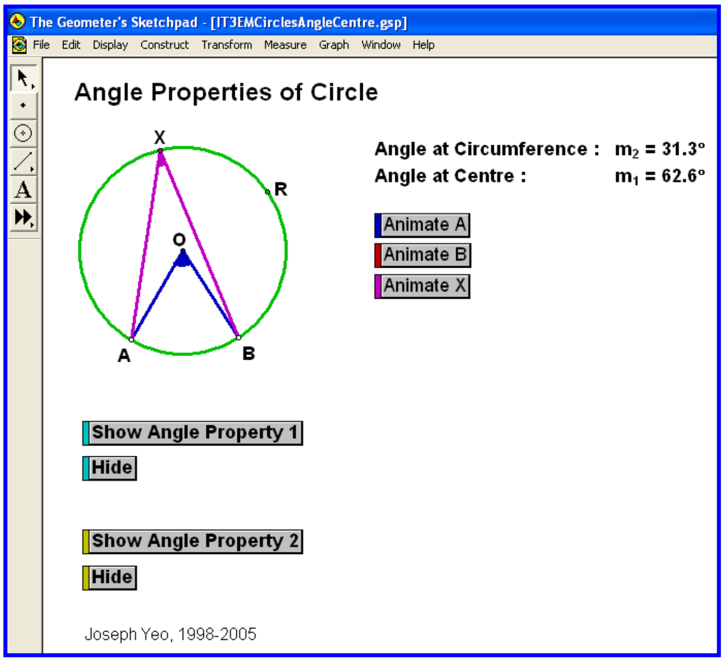

Figure 2.1

In this way, individual students can explore many cases all by themselves. It is also likely that their template may look like Figure 1.2 occasionally and some higher-ability students may also discover that the angle subtended at the circumference remains the same even if you move the point X around (unless the latter goes in between points A and B in which case the angle will now be subtended by the major arc AB instead of by the minor arc AB).

However care must still be taken to ensure that students explore the case where the angle subtended at the centre is reflex. Another area for students to explore is the effect on the angles if the size of the circle is changed by dragging the point R. Again it must be emphasised that the purpose of this template is to help students achieve Van Hiele's Theory Level 2 before going into deductive proofs at a later stage.

If we compare the use of this virtual manipulative with the concrete circles that students have to draw without the help of technology, we will see that the template is much more interactive.

2.2 Example 2: How to Teach Nature of Roots of Quadratic Equations

How do teachers normally teach the relationship between the nature of the roots of a quadratic equation $ax^2 + bx + c = 0$ and its discriminant D = $b^2 - 4ac$?

Method 1: Deductive Reasoning

Traditionally teachers usually begin by examining the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and telling their students that:

(a) If $b^2 - 4ac > 0$, then the quadratic formula will give rise to two unequal roots since $\pm\sqrt{b^2 - 4ac}$ will have two different values.

(b) If $b^2 - 4ac = 0$, then the quadratic formula will give rise to two equal roots since $\pm\sqrt{b^2 - 4ac}$ will have only one value, namely 0.

(c) If $b^2 - 4ac < 0$, then the quadratic formula will give rise to two complex roots since $\pm\sqrt{b^2 - 4ac}$ will not be defined because you cannot take the square roots of a negative number.

Some teachers may further illustrate these three cases by sketching a graph for each case. But some students just could not understand why this is so and they end up memorising these three cases by hard. This suggests that deductive reasoning may still be too difficult for these students at the beginning.

Method 2: Induction by Using a Worksheet

Some teachers decide to let their students explore the three cases on their own. A worksheet is designed to guide each student to solve various quadratic equations by factorisation or using the quadratic formula and to calculate the discriminant for each case. The students are then asked to observe whether they see any relationship between the nature of the roots and the value of the discriminant.

The advantage of such a hands-on activity is that the students spend considerable time pondering on the relationship and when they finally discover this relationship, they are more likely to remember it because they are the ones who construct this relationship themselves. But induction does not tell the students *why* the relationship happens. So it is still necessary at the end of the worksheet to ask the students to try to explain *why* the relationship happens by looking at the quadratic formula.

One more thing is still lacking. How does the graph look like? It may be useful to look at the graph because a picture says a thousand words. But the students have not learnt how to sketch a quadratic curve at this stage. The only thing that they can do is to plot all the quadratic curves in the worksheet but this is really time-consuming.

Method 3: Induction by Using LiveMath

With the advance of technology, the students can use the same worksheet together with a pre-designed *LiveMath* template that looks like the following in Figure 2.2. *LiveMath* is an interactive computer algebra system (CAS): when the user changes the value of a variable, all the expressions, equations and graphs that are linked to it will also change automatically. Therefore all the students need to do is to change the values of *a*, *b* and *c*, and the quadratic equation, the roots, the discriminant and the graph will also change instantaneously.

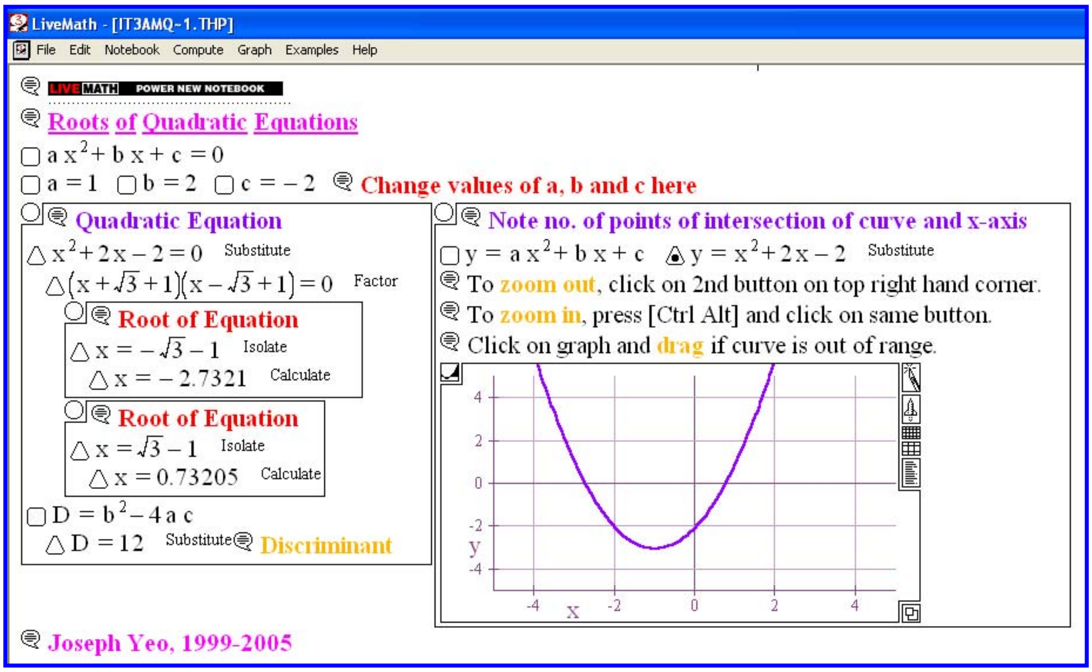

Figure 2.2

In this way, the students can save a lot of time solving the equations and plotting the graphs. The students can then have more time to explore more cases compared to what was previously possible without technology.

3. CONCLUSION

The Information Technology (or IT) Masterplan 2 (or mp2) was launched by the Ministry of Education (MOE) of Singapore in July 2002 to integrate IT into the design of a more flexible and dynamic curriculum by taking into account new teaching methods that are made possible by technology (Ministry of Education, 2003). This is different from the first IT Masterplan which used IT to support an existing curriculum. However the idea behind some of these ‘new’ teaching methods is not new. Some teachers have already used this idea to help their students construct their own knowledge by guiding them to discover mathematical concepts but this is often limited to a few cases for each student to explore. However, with the advance of technology, there are useful interactive software that facilitate the exploration of many cases with ease. Teachers can now harness the use of technology to enhance their pupils’ learning by changing their pedagogy from the traditional teacher-centred type of teaching to a student-centred mode of learning where the students can actively construct their own knowledge by making sense of their mathematical experiences.

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