Chapter 9

Why Study Mathematics? Applications of Mathematics in Our Daily Life

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Most students would like to know why they have to study various mathematical concepts. Teachers usually cannot think of a real-life application for most topics or the examples that they have are beyond the level of most students. In this chapter, I will first discuss the purposes of mathematics, the aims of mathematics education and the rationales for a broad-based school curriculum. Then I will provide some examples of applications of mathematics in the workplace that secondary school and junior college students can understand. But students may not be interested in these professions, so I will examine how mathematics can be useful outside the workplace in our daily life. Lastly, I will look at how mathematical processes, such as problem solving, investigation, and analytical and critical thinking, are important in and outside the workplace.

1 Introduction

When teachers try to convince their students that mathematics is useful in many professions, such as engineering and medical sciences, many of their students may not be interested in these occupations. For example, when I was a teacher, some of my students wanted to be computer game designers instead, but they wrongly believed that this profession did not require much mathematics: only when I demonstrated to them that computer programming required some mathematics did they show any interest in studying mathematics. Other students of mine aspired to be soccer players but they did not realise that the sport could involve some mathematics: they erroneously thought that they would have to kick the ball high enough to clear it as far away as possible; obviously, it could not be at an angle of 90° from the ground but they believed it to be about 60° when in fact it should be 45°. Although this is a concept in physics, kinematics is also a branch of mathematics, not to mention that "mathematics is the queen of the sciences" (Reimer & Reimer, 1992, p. 83) — a famous quotation by the great mathematician Carl Gauss (1777-1855). Mathematics can also help soccer players to make a more informed decision if they know which position on the soccer field will give them the widest angle to shoot the ball between the goalposts (for more information, see Goos, Stillman & Vale, 2007, pp. 50-58).

However, there are still many jobs that may not require much mathematics, except perhaps for simple arithmetic like counting money and telling time, e.g., actors and actresses, taxi drivers, administrators, historians and language teachers. How often do they use, for example, algebra, in their workplace? Why study so much mathematics when many adults do not even use it in their professions? Isn't it a waste of time and resources?

I will address these issues by first examining the purposes of mathematics and the aims of mathematics education. Then I will discuss the rationales for a broad-based school curriculum which pertains not only to the learning of mathematics specifically but also the study of other subjects in general. After that, I will focus on real-life applications of mathematics in and outside the workplace by giving suitable examples that secondary school and junior college students can understand. But mathematics involves not just concepts and procedural skills but also thinking processes. Therefore, I will discuss how mathematical cognitive and metacognitive processes, such as investigation proficiency, problem-solving strategies, communication skills, and critical and creative thinking, are important in and outside the workplace in our daily life.

2 Purposes of Mathematics and Aims of Mathematics Education

In ancient times, mathematical practices were divided into scientific mathematics and subscientific mathematics (Høyrup, 1994): the former being mathematical knowledge pursued for its own sake without any intentions of applications; and the latter being specialists' knowledge that was applied in jobs such as architects, surveyors and accountants. Subscientific mathematics was often transmitted on the job and not found in books, but in modern days where mathematics is divided into pure and applied mathematical knowledge, the latter is usually taught at higher levels in schools because of the utilitarian ideology of the industrial trainers and the technological pragmatists who argue that schools should prepare students for the workforce (Ernest, 1991). However, groups with a purist ideology believe in studying mathematics for its own sake. For example, the old humanists advocate the transmission of mathematical knowledge because of their belief that mathematics is a body of structured pure knowledge, while the progressive educators prefer to use activities and play to engage students so as to facilitate self-realisation through mathematics because of their belief in the process view of mathematics and child-centreness. There is a third type of ideology called the social change ideology of the public educators who wish to create critical awareness and democratic citizenship via mathematics through the use of questioning, discussion, negotiation and decision making.

The different ideologies of these diverse social groups have affected and will continue to affect the aims of mathematics education (Cooper, 1985). For example, the New Math movement in the United States of America in the 1950s was an attempt by mathematicians, who could be viewed as old humanists with a purist ideology, to integrate the contents of academic mathematical practice into the school curriculum by teaching students more advanced mathematical concepts, such as commutative, associative and distributive laws, sets, functions and matrices, in order to understand the underlying algebraic structures and unifying concepts (Howson, 1983; Howson, Keitel & Kilpatrick, 1981). But many students were unable to cope with this type of high-level mathematics and they ended up not even skilled in basic mathematical concepts that were needed in the workforce. As a result, the industrial trainers and the technological pragmatists (groups with utilitarian ideology) won and it was 'Back to Basics' for school mathematics in the 1970s (Askey, 1999; Goldin, 2002).

Then, in the 1980s, the Standards of the National Council of Teachers of Mathematics (NCTM) (NCTM, 1980, 1989) sought to bring the *processes* of academic mathematics (as opposed to the *contents* of academic mathematics in the New Math movement) into the mathematics classrooms (Lampert, 1990; Schoenfeld, 1992). This model views students as mathematicians and it suggests that students should focus on activities that parallel what mathematicians do, for example, posing problems to solve, formulating and testing conjectures, constructing arguments and generalising (Moschkovich, 2002). This is more in line with the purist ideology of the progressive educators and perhaps even the social change ideology of the public educators who believe in the use of questioning and negotiation. It will be argued in Section 6 that these thinking processes have become important in the modern workplace where the requirements of the workforce are ever changing and technical skills are no longer enough (Wagner, 2008).

However, the needs of the society may differ from the desires of individual students: the latter may not be interested to learn mathematics partly because they do not see the use of it, especially when most applications in the workplace are beyond their level, and partly because they may not be interested in professions that require a lot of mathematics. Therefore, there is a need to convince these students why they still need to study mathematics (and other subjects) which they may not need in their future working life — the rationales for a broad-based school curriculum, before I proceed to give examples of real-life mathematical applications in our daily life.

3 Rationales for a Broad-Based School Curriculum

Many students have ambitions but how many of them will end up fulfilling them? Just because a student wants to be a doctor does not mean he or she has the ability or the opportunity to become one. In Singapore, there is a limited vacancy for studying medicine in the university because the government does not want an oversupply of doctors (which is a popular choice of career here) and an undersupply in other jobs since the only asset that Singapore has is its people as there are scarce natural resources. So, even if a student has the aptitude to become a doctor, he or she may still lose out to the more academically-inclined students, and unless the student's parents have the means to send him or her overseas to pursue his or her ambition, the student will most likely end up with another job. This is the first thing that teachers can impress upon their students.

The second thing is for teachers to ask their students whether the ambition that they have now is their first ambition. Many students would have changed their ambitions at least once. Even if this is still their first ambition, there is no guarantee they will not change their minds in future. So, what will happen if students are allowed to specialise early? If their interests are to change later, it will be too late to switch to another subject. Moreover, some students cannot decide at this moment what job they want to do in future, so it is impossible to force them to choose now. Therefore, this is the basis of a broad-based school curriculum, to teach students the basics of different subjects so that when they need it in their area of specialisation later on, they will have some foundations to build upon.

Another reason for a broad-based curriculum is to expose students to different subjects so that they can find out where their interests lie. For example, if a student has never studied geography, how will the student know whether he or she will like to be a geologist or not? But what happens if a student studies a subject and then decides this is not for him or her in his or her future working life? Does that mean that the student can give up that subject? Although teachers can reason with their students that they may end up with a job involving a subject that they dislike in school, it may be more worthwhile to show them how they can still use the subject outside the workplace or in a different area of work, e.g., mathematical thinking processes can be applied to solve unfamiliar problems in other fields.

In the rest of this chapter, I will provide specific examples of how mathematics is applicable in our daily life. I distinguish between mathematical knowledge and processes, and between mathematics in and outside the workplace. Some educators (e.g., Moschkovich, 2002) have classified workplace mathematics as a subset of everyday mathematics.

4 Applications of Mathematical Knowledge in the Workplace

What are in the Singapore secondary syllabus and textbooks are mostly arithmetical applications such as profit and loss, discount, commission, interest rates, hire purchase, money exchange and taxation. But what about workplace uses of algebra, geometry, trigonometry and calculus? Usually, many of these applications are beyond the level of most students. However, this section will illustrate some suitable real-life applications which teachers can discuss with their students.

4.1 *Use of Similar Triangles in Radiation Oncology*

Geometry plays a very important role in radiation oncology (the study and treatment of tumours) when determining safe level of radiation to be administered to spinal cords of cancer patients (WGBH Educational Foundation, 2002). Figure 1 shows how far apart two beams of radiation must be placed so that they will not overlap at the spinal cord, or else a double dose of radiation will endanger the patient.

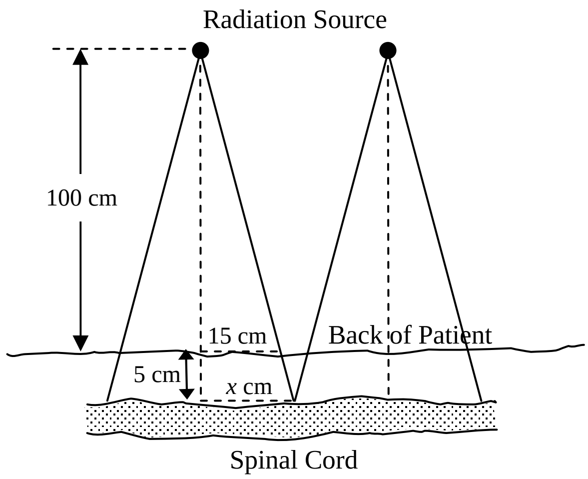

Figure 1. Radiation therapy for cancer patients

In this case, the distance of the radiation source from the back of the patient is 100 cm and 15 cm is only the length on the back. The actual length on the spinal cord, x cm, can be determined using a property of similar triangles:

$$\frac{100}{105} = \frac{15}{x}$$
$$x = \frac{15 \times 105}{100}$$
$$= 15.75$$

Therefore, the actual length is longer by 0.75 cm. If the two beams of radiation are placed only 2 × 15 cm = 30 cm (instead of 2 × 15.75 cm = 31.5 cm) apart, then 2 × 0.75 cm = 1.5 cm of the spinal cord will be exposed to a double dose of radiation.

4.2 *Optimisation Problems using Calculus*

Another real-life application is to minimise the total surface area S of a cylindrical can food for a *fixed* volume V in order to cut cost for the metal used. Since $V = \pi r^2 h$, where r is the radius and h is the height of the cylinder, then $h = \frac{V}{\pi r^2}$. Therefore, $S = 2\pi rh + 2\pi r^2 = \frac{2V}{r} + 2\pi r^2$.

Using the first derivative test, we can find the value of r that gives the least value of S. What is interesting is the relationship between h and r when S is least: $h = 2r$, i.e., the height of the can is equal to its diameter. But many cylindrical can foods are not of these dimensions. Is it because the manufacturer does not know calculus or are there other factors? This is what happens in real life: there is more to it than mathematics. Teachers can ask their students why this is so. Below are some reasons:

- Hotdogs and sardines are long, so their cans are usually longer in shape, i.e., their height is greater than their diameter.
- Can drinks are also longer in shape because it is harder for our hands to hold them if their diameter is equal to their height.

- A can, whose diameter is equal to their height, may be aesthetically less appealing to the consumers.
- The metal sheets may come in a particular size such that fewer cans with equal diameter and height can be made from one metal sheet than if the cans are of a different dimension.

An article on John Handley High School (2006) Website reveals some more reasons why manufacturers do not make cylindrical cans which have equal height and diameter:

- Most can manufacturing lines run a variety of can sizes. To change both the height and diameter for cans with a different volume will require more time than just changing the height and using the same ends.
- Tall thin cans or short wide cans require less thermal processing time to sterilise than cans with equal height and diameter.
- Internal pressure that develops during thermal processing can distort the ends of the can, so the ends are made of thicker metal which is more costly. But if the ends are smaller in diameter, thinner metal can be used for both the ends and curved surface.
- Tall thin cans make better use of packaging and shipping space.

The last three reasons reveal that there are other types of mathematics involved, e.g., in thermal processing, and in packaging and shipping space.

4.3 *Encoding of Computer Data using Large Prime Numbers*

Before computer data are sent through the Internet, especially sensitive data like online banking, they are encoded so that hackers cannot make use of the data unless they know how to decode. Ronald **R**ivest, Adi **S**hamir and Leonard **A**dleman have invented the very secure RSA cryptosystem which makes use of very large primes and a complicated number theory. To break the code, hackers need to decompose a very large composite number into its two very big prime factors using trial division. Even with the fastest computers, this will take years because trial division for very large numbers is an extremely tedious process. Although the actual encryption (refer to Wolfram MathWorld Website, Weisstein, 2009a, for more information) is beyond the level of the students, there are three other things that teachers can do with their students.

The first thing to discuss is how big these primes are. Teachers can introduce to their students the search for very large Mersenne primes. Mersenne (1588-1648) used the formula $M_p = 2^p - 1$, where p is prime, to generate big primes, but M_p is only prime for certain values of p, e.g., M_2, M_3, M_5 and M_7 are primes but M_{11} is not a prime. The largest known prime, $M_{43,112,609}$, found by Edson Smith on 23 Aug 2008, contains 12 978 189 digits! If one newspaper page can contain 30 000 digits, then 433 newspaper pages will be needed to print the largest known prime! For more information, refer to the Great Internet Mersenne Prime Search or GIMPS Website (Mersenne Research, 2009).

Another interesting issue to discuss is the complicated number theory that is the basis of the RSA cryptosystem. For centuries, mathematicians had studied prime numbers and number theory for interest sake since there were no known real-life applications at that time. But because of their study, with the invention of computers, there is now a very secure way of protecting sensitive computer data. Teachers can let their students debate the tension and merit between education for interest sake and education to prepare students for the workplace — the purist ideology versus the utilitarian ideology discussed in Section 2.

The third thing is to let students have a sense of encryption by using some other simpler but not-so-secure systems since they will not be able to understand the RSA cryptosystem. An example of such a system is the use of a matrix to encode and its inverse to decode the data. The reader can refer to Teh, Loh, Yeo and Chow (2007a) pp. 44-51 for more details and a ready-to-use student worksheet.

4.4 *Use of Differentiation in Economics*

Economics, a subject which some junior college students take, uses differentiation extensively. For example, retailers are interested to find out whether an increase in the price of a product will cause the demand to drop so drastically that the total revenue will fall? Or will the demand drop only slightly such that the total revenue will still increase? In economics, this is called the price elasticity of demand and it measures the degree of responsiveness in the quantity demanded of a commodity as a result of a change in its price (Wikipedia, 2009a). Suppose a retailer does a survey and finds out that the daily demand curve of a particular product is given by $Q = 20 - 0.5P$, where Q is the quantity demanded of the commodity and P is its price in dollars. The price elasticity of demand E_d is calculated as follows:

$$E_d = \frac{dQ}{dP} \times \frac{P}{Q} = -0.5 \times \frac{P}{Q}.$$

What does this mean? If $|E_d| = 1$ (unit elastic), the percentage change in quantity demanded is equal to the percentage change in price. If $|E_d| > 1$ (elastic), the percentage change in quantity demanded is bigger than that in price, and so if the price is raised, the quantity demand will drop enough to cause the total revenue to fall, and if the price is lowered, the quantity demand will rise enough to cause the total revenue to increase. If $|E_d| < 1$ (inelastic), the percentage change in quantity demanded is smaller than that in price, and so if the price is increased, the total revenue will still rise despite the drop in demand, and vice versa.

Suppose the retailer is selling the product for \$30, i.e., $P = 30$. Then $Q = 20 - 0.5P = 5$. Since $|E_d| = |-0.5 \times 30 \div 5| = 3 > 1$, then the retailer should not increase but decrease the price. For example, if the retailer increases the price by \$2 from \$30 to \$32, then $Q = 4$ and the total revenue becomes $\$P \times Q = \$32 \times 4 = \$128$, which is a lot less than the original total revenue of $\$30 \times 5 = \$150$. But if the retailer decreases the price by \$2 to \$28, then $Q = 6$ and the total revenue becomes \$168, which is more than the original total revenue of \$150. Figure 2 shows the graph of Q against P and the graph of the total revenue TR against P.

The question is how low the price can drop before the total revenue starts to decrease. Unit elasticity, i.e., $|E_d| = 1$, occurs when $P = 20$ (and $Q = 10$), so the retailer should lower the price to \$20 for the total revenue to reach its peak of $\$20 \times 10 = \$200$ (refer to the maximum point of the bottom graph in Fig. 2). If the price is decreased further to, say, \$18, then $Q = 11$ and the total revenue is only $\$18 \times 11 = \$198 < \$200$.

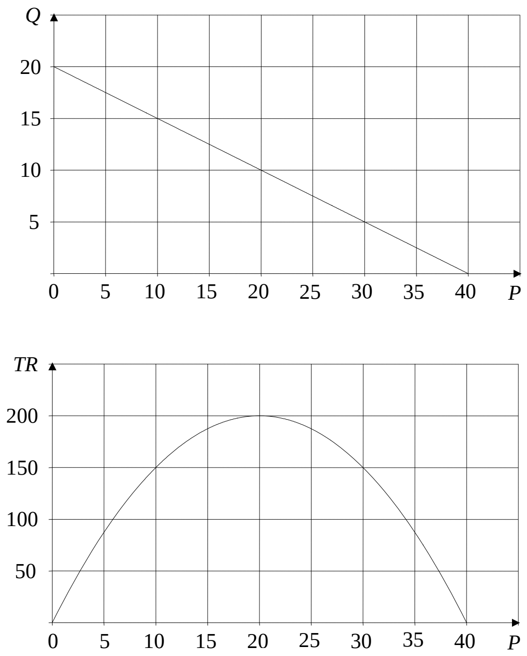

Figure 2. Graphs of *Q* against *P* and *TR* against *P*

In fact, when $P = 18$ and $|E_d| \approx 0.818 < 1$, i.e., the price elasticity of demand is inelastic, the retailer can increase or decrease the price without affecting the demand so much. This means the retailer should raise the price so that the total revenue will increase. But how high can the price rise before the total revenue starts to decrease? The answer is still the same: when unit elasticity is reached, i.e., when the price is \$20 and the total revenue is at its maximum of \$200.

In real life, things are not that simple. For example, how does one conduct a survey to obtain a reliable demand curve? And if the retailer drops the price from \$30 to \$20, he or she will have to bring in more stock, which translates to more storage space, more capital if the supplier refuses to give the retailer a higher credit, more time and effort in bringing in and selling more goods, and a higher risk if the product does not sell according to the demand curve.

A question some students may ask is whether it is possible for the total revenue to fall but the profit will actually increase? For example, if the retailer makes a profit of $1 for each item, when P = 30 and Q = 5, he or she only earns $5. But if the price is increased by $2 to $32, then Q = 4 and the total revenue falls from $150 to $128, but the profit increases from $5 to $3 × 4 = $12. This scenario can only happen if the profit per item is lower than the small increase in price. However, in real life, no retailer will make a profit of only $1 if the cost price of the item is $29 because a 3.4% profit will not be enough to cover overheads such as rental and salaries of employees. Retailers usually make about 40% to 60% profit per item, so if the selling price of the item is $30, the retailer will make between $9 and $11. When P = 30 and Q = 5, the retailer will make a profit of $45 to $55, but when P = 32 and Q = 4, the retailer will earn about $44 to $52. Therefore, the higher the percentage profit per item, a small increase in price and a corresponding decrease in demand will not only result in a fall in the total revenue but also a drop in profit.

4.5 *Other Generic Examples*

It is usually not possible to find a real-life application for every topic that students can understand. But there are applications that teachers can still discuss with their students without going into specific details. For example, complex numbers are used extensively in electrical engineering to understand and analyse alternating signals (United States Naval Academy Website, 2001); GPS (global positioning system) makes use of complex vectors and geometric trilateration to determine the positions of the objects (Wikipedia, 2009b); and land surveying equipment uses trigonometry and triangulation (Wikipedia, 2009c). Teachers can also let their students have a sense of how the latter works by using a clinometer (a simple instrument that measures the angle of elevation), a measuring tape and trigonometry to find the height of a tree or a building (for more details and a ready-to-use worksheet, see Teh, Loh, Yeo & Chow, 2007a, pp. 108-111).

Another example is the use of the formulae for finding arc length and sector area, and the symmetric and angle properties of circles, in the design and building of road tunnels, bridges, buildings and any arc-shaped structures. Teachers can let their students have a sense of how this works by getting them to draw an arc-shaped balcony, given the following dimensions in Figure 3. To construct the arc, the students will have to find the centre and radius of the corresponding circle using some circle properties. In fact, there is more than one way to do it.

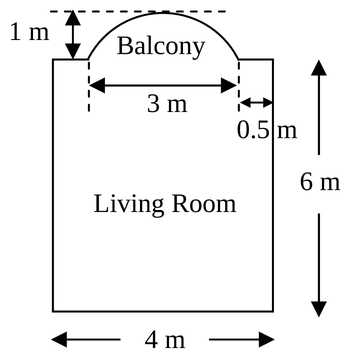

Figure 3. Arc-shaped balcony

An inspiring story to tell is the ingenious plug-in called Auto-Tune that can transform mediocre singing into note- and pitch-perfect singing (Tyrangiel, 2009). Many students are interested in songs and so using an example on singing may capture their attention. The inventor is Andy Hildebrand who has worked for many years in the oil industry by providing an accurate map of potential drilling sites using sound waves sent into the ground and recording their reflections. This technique, which uses a mathematical formula called autocorrelation (refer to Wolfram MathWorld Website, Weisstein, 2009b, for more information), has saved oil companies lots of money and allows Hildebrand to retire at 40! After retirement, at a challenge from a dinner party guest to invent something that would allow her to sing in tune, he invented Auto-Tune, which also uses autocorrelation. Even if students do not know how autocorrelation works, they may be impressed that a mathematical formula can allow a person to earn so much money that he can retire at 40! This shows the usefulness of mathematics in the workplace.

5 Applications of Mathematical Knowledge Outside the Workplace

Although mathematics is useful in many professions, what if students end up with occupations that do not require much mathematics? Does that mean the mathematics education that they have received is all gone to waste? Therefore, I will discuss in this section how mathematics is useful outside the workplace in everyday life, e.g., understanding everyday events and analysing newspaper reports, even when the students have become adults holding jobs that do not need mathematics.

5.1 *Earthquakes*

Logarithm is used in the Richter scale to measure the magnitude of an earthquake, e.g., the Indonesian earthquake, that set off giant tidal waves on 26 Dec 2004 and killed at least 159 000 people, measured 9.0. What does this magnitude mean and how does it compare with a 4.8-magnitude earthquake that struck Northern Italy on 5 Apr 2009 and killed almost 300 and left 60 000 homeless? Although some people may know that a 9.0-magnitude earthquake is ten times stronger than an 8.0 earthquake, complications arise when the difference in magnitude is not an integer.

The Richter scale is a measurement of earthquake magnitudes based on the formula $R = \lg\left(\frac{x}{0.001}\right)$ where x is the intensity of the earthquake as registered on a seismograph. Teachers can get their students to find x in terms of R by transforming the logarithmic form to the index form (this procedural skill is tested in GCE O-level exam!), i.e., $x = 0.001\times10^R$. To compare two intensities x_2 and x_1, we have:

$$\frac{x_2}{x_1} = \frac{0.001\times10^{R_2}}{0.001\times10^{R_1}} = 10^{R_2-R_1}.$$

Thus, to contrast the strength of two earthquakes, we just need to find the difference in magnitude $d = R_2 - R_1$ and then calculate 10^d. For example, the difference in magnitude between the Indonesian earthquake that set off the tsunami, and the recent earthquake in Northern Italy, is

d = 9.0 – 4.8 = 4.2, and so the Indonesian earthquake is $10^{4.2} \approx 15\,800$ times stronger than the Italian one. Although the difference of 4.2 in magnitude looks small, the difference in intensity is actually a lot bigger, which has surprised many students. For a ready-to-use student worksheet on earthquakes and logarithms, the reader can refer to Teh, Loh, Yeo and Chow (2007b) pp. 25-28.

5.2 *Lottery*

A 4D draw in Singapore has a total of 23 prizes: first, second and third prizes, ten starter prizes and ten consolation prizes. There are 10 000 possible four-digit numbers: 0000 to 9999. The first and second prizes for the 4D Draw on 27 Jun 2007 were both the same number 6904. The Sunday Times on 8 Jul 2007 quoted a spokesman from Singapore Pools (the company that organises the 4D Draw) as saying that the probability for a number to appear twice in the same draw is 1 in 10 000 times 10 000, or one in 100 millions (Mak, 2007). Surprisingly, many mathematics teachers, whom I have spoken to, agreed with the statement.

Let us begin with a simpler question: what is the probability for the same number to appear as first and second prizes in the same draw? Consider a related problem: what is the probability of obtaining the same number in a toss of two dice? The total number of possible outcomes for throwing two dice is 36. The favourable outcomes are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6). Thus the probability of obtaining the same number on the two dice is $\frac{6}{36} = \frac{1}{6}$. Similarly, the total number of possible outcomes for the first and second prizes is 10 000 × 10 000. The favourable outcomes are (0000, 0000), (0001, 0001), (0002, 0002), …, (9999, 9999) and so the probability of obtaining the same number for the first two prizes is $\frac{10\,000}{10\,000 \times 10\,000} = \frac{1}{10\,000}$.

Another method is as follows. Since it does not matter what the number on the first die is, the probability of obtaining any number on the die is 1. Then there is only one favourable outcome for the second die and so the probability of obtaining the same number on both dice is

$1 \times \frac{1}{6} = \frac{1}{6}$, and not $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Using the same argument, the probability of the same number appearing as the first and second prizes in the same draw is $1 \times \frac{1}{10\,000} = \frac{1}{10\,000}$, and not $\frac{1}{10\,000} \times \frac{1}{10\,000} = \frac{1}{100\,000\,000}$.

Quite a number of mathematics teachers, whom I have spoken to, were still not convinced. The main confusion is that it now *seems* that the probability of a number appearing twice in a draw is equal to the probability of a draw having all different numbers, which the teachers have this intuitive feeling that it is not right. But the probability of a number appearing twice in a draw is *not* $\frac{1}{10\,000}$; rather, $\frac{1}{10\,000}$ is the probability of a number appearing in both the first and second prizes. Similarly, the probability of a draw having all different numbers is not $\frac{1}{10\,000}$, but it is equal to $\frac{{}^{10\,000}P_{23}}{10\,000^{23}} \approx 0.975$ since there are 23 prizes in a draw. Secondary school students, who have not learnt permutations, can find the probability as follows, although computing it is rather tedious:

$$1 \times \frac{9999}{10\,000} \times \frac{9998}{10\,000} \times \frac{9997}{10\,000} \times \ldots \times \frac{9978}{10\,000}.$$

Thus the probability of a draw having at least one number appearing more than once is about 0.025. As there are three draws every week, and so about 156 draws a year, we should expect about four draws to have at least one number appearing more than once every year. Although the latter is rarer than a draw having all different numbers, it is theoretically not so rare at all! However, in reality, most years have gone by without a single draw having at least one number appearing more than once.

5.3 *Water Supply*

An article from Streats (a free newspaper in Singapore that has since ceased publication) on 26 May 2001 quoted Mr Lim Swee Say, then

Acting Minister for the Environment of Singapore, as saying, "If we increase the supply of fresh water by 20 per cent and at the same time reclaim 30 per cent of the used water, we will be able to increase our total water capacity by as much as 70 per cent." ("Water: Add and Multiply", 2001) But 20% plus 30% of 1.2 is only 56%. Was Mr Lim wrong?

Let V be the volume of water that we have. If we increase the supply of fresh water by 20%, then the total volume of water will be $1.2V$. If we reclaim 30% of the used water (all the $1.2V$ of water will be used after some time), then the volume of reclaimed water will be $0.3 \times 1.2V$, i.e., the total volume of water will be $1.2V + 0.3 \times 1.2V = 1.56V$, an increase of 56%. But this is only after the first reclamation. The reclaimed water will be used after some time and then we can still reclaim 30% of it. So the total volume of water after the second reclamation will be $1.2V + 0.3 \times 1.2V + 0.3^2 \times 1.2V$. Continuing this process of reclamation, the total volume of water in the long run will be:

$$\begin{aligned} & 1.2V + 0.3 \times 1.2V + 0.3^2 \times 1.2V + 0.3^3 \times 1.2V + \ldots \\ &= 1.2V \times \left(1 + 0.3 + 0.3^2 + 0.3^3 + \ldots\right) \\ &= 1.2V \times \frac{1}{1 - 0.3} \\ &\approx 1.71V \end{aligned}$$

Therefore, Mr Lim was correct to say that it was possible to increase our water supply by as much as 70%. A question that teachers should invite their students to ask is, "Do we have to wait forever to get a 70% increase since this is the sum to infinity? Or after how many times of reclamation will we get near enough to a 70% increase?" Fortunately, we do not have to wait so long: after the third reclamation, the total volume of water will be $1.7004V$. Although the initial calculation involves summing to infinity for a geometric progression (GP), which is only taught at the junior college level, secondary school students can actually find the answer for three times of reclamation without using the GP formula.

5.4 *Quality of Life*

Some students may question the purpose of understanding everyday events and analysing newspaper reports as outlined above. The American philosopher and educational theorist John Dewey (1859–1952) believed that education was a process of enhancing the quality of life. “By *quality*, Dewey meant a life of meaningful activity, of thoughtful conduct, and of open communication and interaction with other people.” (Hansen, 2007, p. 9) If people are ignorant or cannot make sense of the events that happen during the daily course of their lives, then they may not be able to engage in meaningful discourse with other people, make well-informed decision and lead a life of accountable conduct.

However, some people think that life will still go on if they do not understand all these. But ignorance is not bliss. They may make bad ill-informed decisions because they do not understand, for example, the spread of the HIN1 flu virus as reported in the media. Some elderly people think that they are immune to the virus since it attacks mostly young adults with medical problems (personal communication), and so the former may not take any precautions; this not only affects themselves but they may end up spreading the virus to others. Hence, knowledge and the ability to reason logically are necessary not only for leading a meaningful life but also a responsible one.

6 Applications of Mathematical Processes in the Workplace

Mathematics is not only about conceptual knowledge and procedural skills, but it involves cognitive processes such as problem-solving heuristics and thinking skills; metacognitive processes such as the monitoring and regulation of one’s own thinking; and attitudes such as interest, confidence and perseverance in solving unfamiliar problems. Concepts, skills, processes, metacognition and attitudes are the five components of the Pentagon Model which is the framework for the Singapore mathematics curriculum (Ministry of Education, 2006). But are these thinking processes, metacognition and affective variables important in the workplace, and if yes, why?

Tony Wagner, Co-director of the Change Leadership Group at the Harvard Graduate School of Education, interviewed CEOs of big companies to find out what key skills they valued most when hiring new staff (Wagner, 2008). To his surprise, many of them put critical thinking and problem solving as one of the top priorities and “the heart of critical thinking and problem solving is the ability to ask the right questions” (p. 21). For example, Clay Parker, President of the Chemical Management Division of BOC Edwards, a company that makes machines and supplies chemicals for the manufacture of microelectronics devices, replied that he would look for someone who asked good questions. He said, “We can teach them the technical stuff, but we can’t teach them how to ask good questions — how to think.” (p. 20) (Yet, teachers are expected to teach their students how to think!) As one senior executive from Dell revealed, “Yesterday’s answers won’t solve today’s problems.” (p. 21)

Other vital skills include leadership and collaboration, agility and adaptability, initiative and entrepreneurialism, effective oral and written communication, accessing and analysing information, and curiosity and imagination. Thus there is value in developing students’ mathematical problem-solving strategies, investigative and research skills, analytical, critical and creative thinking processes, teamwork and communication, and arousing their curiosity and interest in mathematics, because these are essential skills in the workplace. Although the domain may not be mathematics per se, hopefully, students are able to transfer their habits of minds to new and unfamiliar situations in other fields. If we believe that one of the purposes of education is to prepare students for the workforce (Cowen, 2007), then schools and teachers should heed Wagner’s (2008) advice to teach and test the skills that matter most.

7 Applications of Mathematical Processes Outside the Workplace

Mathematical processes are also important outside the workplace in everyday life, e.g., analysing statistical data reported in newspapers critically so as not to be deceived by biased reporting (in fact, this type of analytical skills is also important in the workplace when reading reports from clients). In this section, we will analyse how statistics, which is the science of collecting, organising, displaying and interpreting data (Yap, 2008), can be manipulated in each of the four components to tell different stories, whether intentionally or unintentionally.

7.1 *Collection of Data*

In a UEFA (Union of European Football Associations) Website poll involving 150 000 fans, Zinedine Zidane was named the best European footballer in the past 50 years (Reuters, 2004). He polled 123 582 votes, while second-placed Franz Beckenbauer received 122 569 votes, followed by Johan Cruyff with 119 332 votes. Teachers can ask their students this question: “Do most fans *really* think that Zidane is the best European soccer player of the last 50 years?” Most students, who are soccer fans, know who Zidane is, but few have heard of Beckenbauer or Cruyff. This is because Beckenbauer and Cruyff were famous in the 1970s while Zidane was at the peak of his career in the 1990s. Moreover, the poll was conducted online, meaning that the older fans who knew Beckenbauer and Cruyff were less likely to vote since they are usually less technological savvy than the younger fans. Despite all these, the polls show that Beckenbauer was only slightly behind Zidane — about 1000 votes or less than 1%. Therefore, if the polls were conducted in such a way that all the people who voted knew who Beckenbauer was, most likely Beckenbauer would dethrone Zidane. This is an example of how an inappropriate polling method can affect the voting outcome: the convenient sample is not representative of the population.

7.2 *Organisation of Data*

The first two paragraphs of a Straits Times article *Insurance firms, banks, top hate list* read, “Banks and insurance companies have made it to the top of the consumer hate list for the first time. They were the target of 1,915 complaints to the Consumers Association of Singapore (Case) between January and last month, edging out the usual suspects — timeshare companies (1,228 complaints), motor vehicle shops and companies (1,027), renovation companies (963), and electrical and electronics shops (710).” (Arshad, 2003, p. 1) Teachers can get their students to question the report critically. One of the main problems of this article was the organisation of the data: why were banks and insurance companies grouped together, while the others were single entities? Although banks do sell insurance, they are still different from insurance companies. If banks and insurance companies were considered separately, it might be possible that timeshare companies would now top the hate list, e.g., if the 1915 complaints were split equally between banks and insurance companies. This is an example of how the decision to group or not to group the data, and how the method of grouping, can distort the truth.

7.3 *Display of Data*

The headline of a survey reported on the front page of the Straits Times was SEX AND THE SINGAPORE GIRL (Mathi, 1998). It showed a big picture of two students kissing. The subheading 'Girls date — and kiss — before boys do' was supported by the first two bar graphs in the article, and the fourth bar graph showed that more girls had engaged in sex than boys. Figure 4 reproduces these three bar graphs in the article.

On page 26 of the same newspaper, a small report (with no picture) of the same survey stated, "Overall, the ST survey gave a reassuring picture of the Singapore teen. As Dr V. Atputharajah, Kandang Kerbau Women's and Children's Hospital's senior consultant with the department of reproductive medicine, said: 'Many teenagers have this impression that everyone is having sex. But in reality this is not true, *as the survey has shown* [italics mine].'" (Mathi & Wong, 1998, p. 26) But most readers, including my students at that time, had the impression that many girls were sexually active because of the front page news with such a telling photograph since a picture says a thousand words.

If the bar graphs were displayed differently as shown in Figure 5, then the picture presented will be entirely different: most of the girls did not date at 13 years or younger (85%), did not kiss by 14 years or younger (92%) and did not ever have sex (93%). This agrees with what Dr Atputharajah has said on page 26, "Many teenagers have this impression that everyone is having sex. But in reality this is not true, as the survey has shown." Hence, the display of data can affect how the readers perceive the survey results.

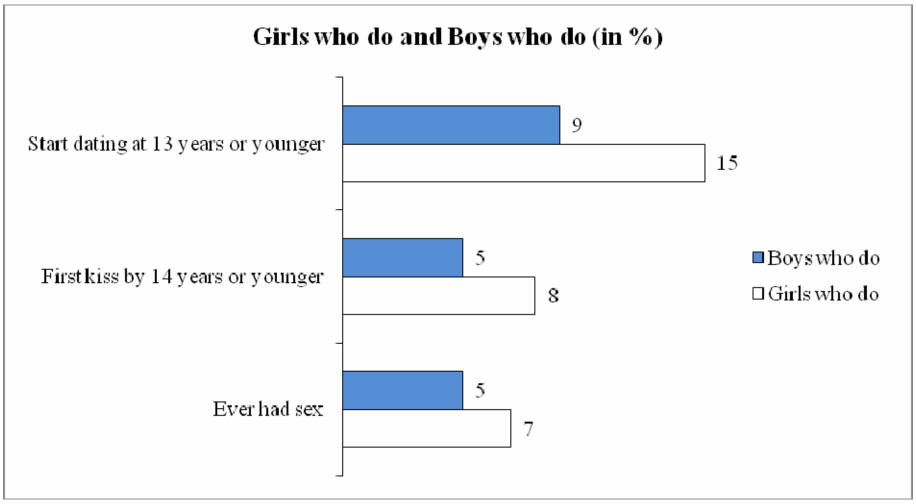

Figure 4. Bar graphs comparing girls who do and boys who do
(as depicted in the Strait Times article)

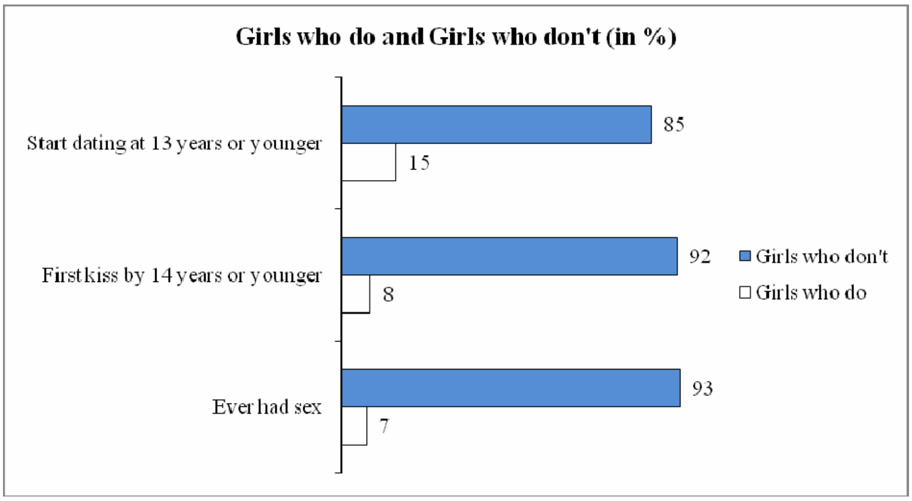

Figure 5. Bar graphs comparing girls who do and girls who don't
(which tell a different story)

7.4 *Interpretation of Data*

The headline of a survey reported in the Straits Times was ‘Survey finds discipline in schools not a big problem’ (Mathi, 1997). The survey interviewed 285 teachers over two years and three out of five teachers said that the discipline problem was not serious and so the researcher concluded, “This is the good news — that the state of discipline in schools is not as bad as it has been made out to be.” Of course, one can question about the sampling, but the main focus in this section is the interpretation of data. In order for discipline to be viewed as a serious problem, does one need more than 50% of teachers to agree that it is? Think about this: in a school with 100 teachers, if 40 teachers believe that the discipline problem in the school is bad enough, do *you* just treat it lightly and dismiss it? Most of the teachers, whom I have spoken to, agreed that it should be a concern if 40% of the teachers believed that the discipline in their school was serious.

The key issue here is that one does not always need a simple majority (i.e., more than 50%) to decide on a matter. For example, to pass a law in the Singapore Parliament that results in a constitutional amendment, at least two-thirds of the elected Members of Parliament must agree on it; a simple majority is not enough. On the other hand, a simple majority may not even be needed for an issue to become a problem in some cases. This is an example of how the basis or criterion for making a decision can affect the interpretation of the data.

7.5 *Can statistics lie?*

Evan Esar (1899-1995) believed that statistics was the only science that enabled different experts using the same figures to draw different conclusions (Moncur, 2007). But we go one step further: even the figures can lie! If teachers do not teach their students to analyse survey reports critically, then they may be misled by others. Although it is good to make informed decisions in everyday lives, it is even more important in the workplace because a mistake in judging the reliability of reports from clients can cause the company to suffer huge financial loss. However, the con of teaching students how to evaluate the validity of the collection, organisation, display and interpretation of data is that the students may end up using this knowledge to mislead others. That is why moral education in school is also very important: an intelligent crook is worse than a stupid one! Teachers should also teach their students to be responsible and useful citizens — the social change ideology of the public educators (Ernest, 1991) discussed in Section 2.

8 Conclusion

Mathematics is of practical value in many professions. It is not just the mathematical knowledge itself but the thinking processes acquired in genuine mathematical problem solving and investigation that can be applied to unfamiliar situations in other fields. Mathematical knowledge and processes are also useful outside the workplace in everyday life to understand and interpret certain events and news reports so as not to be deceived or swayed by others' opinions without any reasonable basis, thus improving one's own quality of life when one is able to lead a meaningful and responsible life. Teachers should impress upon their students the usefulness of mathematics in their daily life, and they should prepare their students for the future by focusing on the essential skills and processes that are required in the workplace.

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