Making Connections using Mathematical Tasks

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Workshop S3
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Overview

- Understanding the **Why**
- Experiencing the **What**
- Thinking about the **How**
- The **beginning** of a journey

Understanding the WHY

Nature of Mathematics

Mathematics can be described as a study of the properties, relationships, operations, algorithms, and applications of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels.

[…]

Abstractions are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for representing and communicating the ideas and results of the discipline.

Recurring Themes

What are the properties of mathematical objects and how are they related?

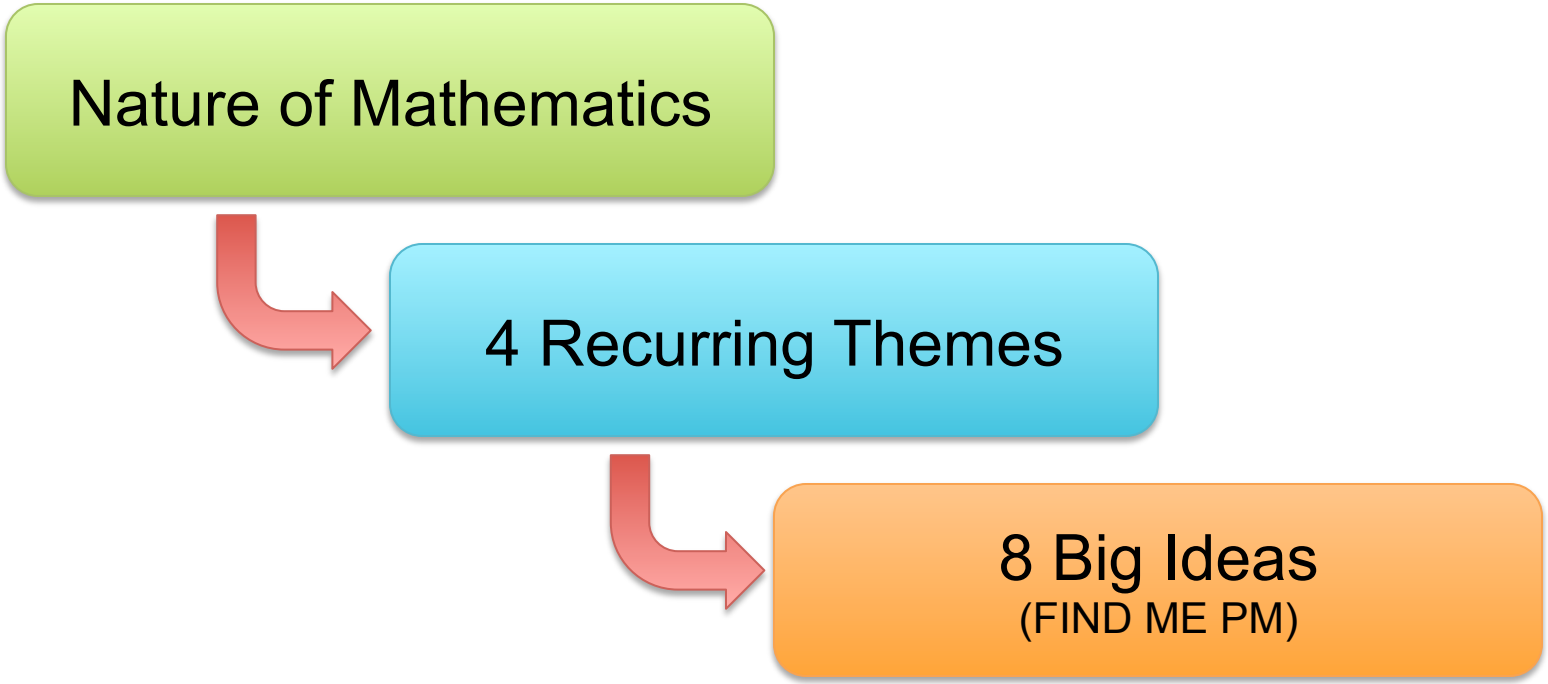
How can the mathematical objects and concepts be represented and communicated?

What meaningful actions can we perform on the objects and how do we carry them out?

How can the mathematical objects be further abstracted and where can they be applied?

Big Ideas

- Big ideas express ideas that are central to mathematics. They bring coherence and connect ideas from different strands and continuity across levels.



Big Ideas – the 4 C's

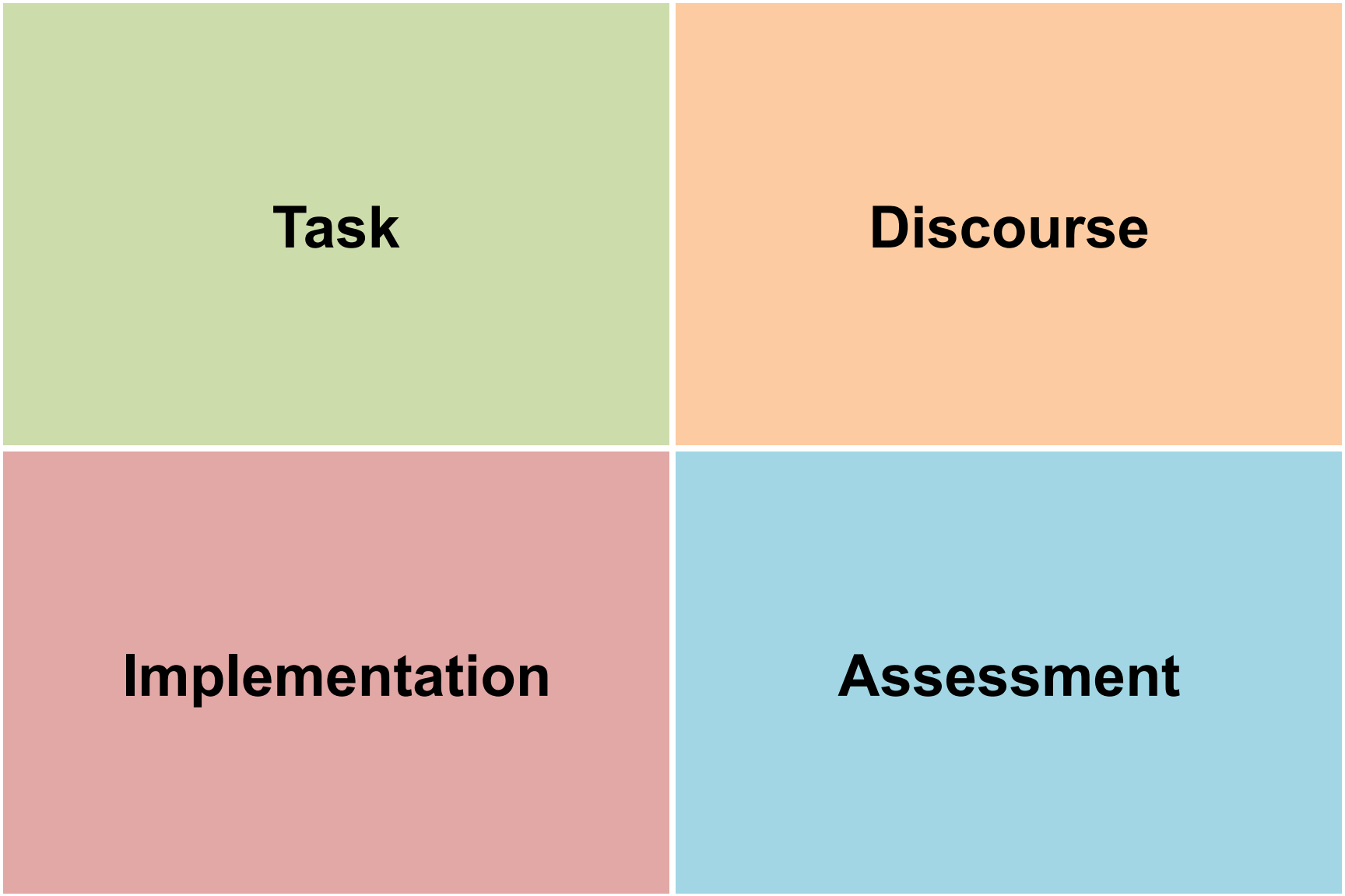
Coherence

Connections

Continuity

Centrality

The WHATs & HOWs



Experiencing the WHAT

The WHATs of…

Task

- Concepts/Skills?
- Big ideas?

Discourse

Implementation

Assessment

Finding the Height of the Flag Pole

10.1 ways to find it!

Method 1 [I post, I shoot, I calculate]

Secondary One. I pose, I shoot, I calculate

1. Write down your height.
2. Now, take a full-length photo of you standing next to the flagpole as shown.

3. Describe how you can use this photo to get an estimate of the height of the flagpole. You may want to draw a suitable diagram to make your explanation clearer. State any assumption(s) made.
4. State the mathematical concepts/skills you use to complete this task.

Method 1 [I post, I shoot, I calculate]

The Task

- Topic(s): Ratio (Sec 1); Scale (Sec 2)

The Discourse

- $$\frac{\text{Man's height in pic}}{\text{Actual man's height}} = \frac{\text{Pole's height in pic}}{\text{Actual pole's height}}$$

vs

$$\frac{\text{Man's height in pic}}{\text{Pole's height in pic}} = \frac{\text{Actual man's height}}{\text{Actual pole's height}}$$

- Assumptions? Accuracy? Alternatives?

Method 1b?

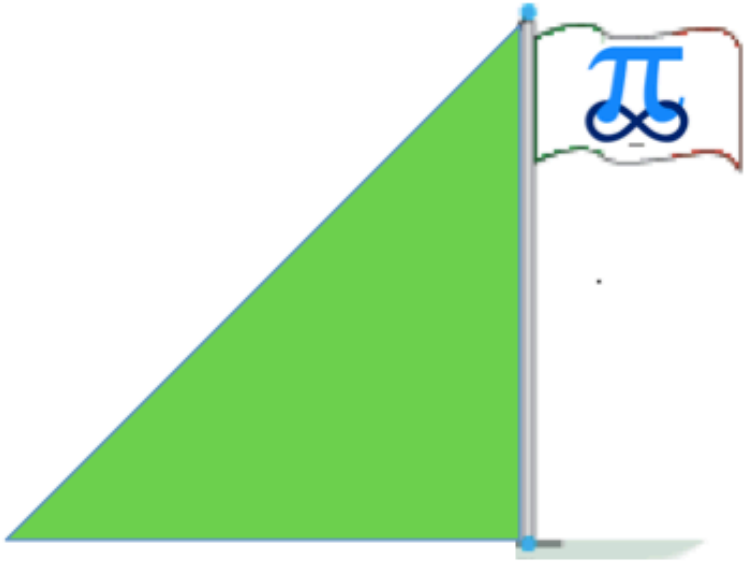
With a ruler?

By estimation?

Method 2 [A triangle affair]

Secondary One. A triangle affair

1. Have a friend to stand next to the flagpole.
2. Now, hold the isosceles right-angled triangle in front of you such that one of the short sides is horizontal and the other is vertical. Move nearer or further from the flagpole such that you can sight the very top of the flagpole at the vertical vertex of the triangle and the bottom of the flagpole at the right vertex of the triangle as shown.



3. Describe where you can ask your friend to move to and what distance to measure so as to get an estimate of the height of the flagpole. You may want to draw a suitable diagram to make your explanation clearer. State any assumption(s) made.
4. State the mathematical concepts/skills you use to complete this task.

Method 2 [A triangle affair]

The Task

- Topic(s): Polygons

The Discourse

- Geometrical properties
- Similar triangles?
- Assumptions? Accuracy? Alternatives?

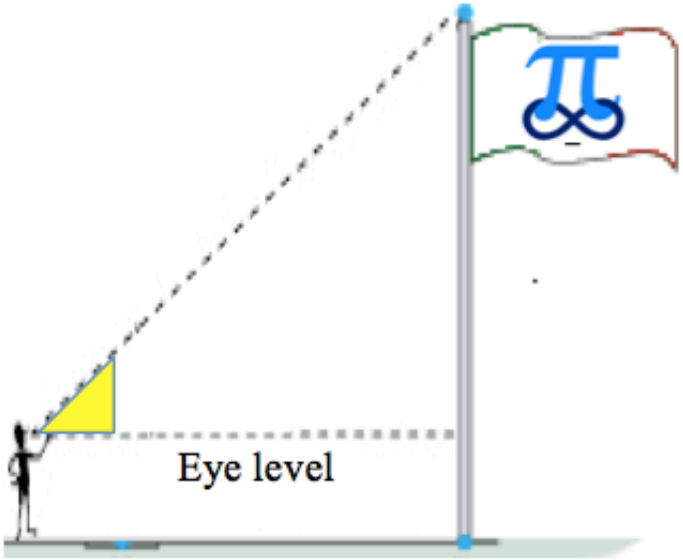
Method 2b?

With… a square? A rhombus?
Anything?

Method 3 [The right vision]

Secondary One. The right vision

1. Write down your eye's height.
2. Now, hold the isosceles right-angled triangle to your eye level as shown, such that one of the short sides is horizontal and the other is vertical. Look up along the longest side by raising your eyes/head. Move back from the flagpole until you can sight the top of the tree at the top tip of the triangle. Measure the horizontal distance between your current position and the flagpole.



3. Describe how you can use this distance to get an estimate of the height of the flagpole. You may want to draw a suitable diagram to make your explanation clearer. State any assumption(s) made.
4. State the mathematical concepts/skills you use to complete this task.

Method 3 [The right vision]

The Task

- Topic(s): Polygons

The Discourse

- Geometrical properties
- Similar triangles?
- Trigonometry?
- Assumptions? Accuracy? Alternatives?

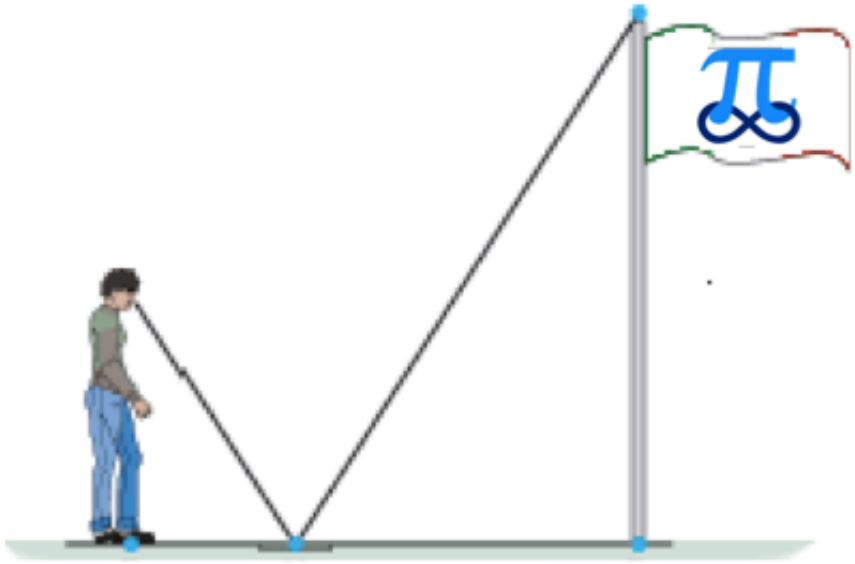
Method 3b?

Using a clinometer?

Method 4 [Mirror, mirror on the floor]

Secondary Two. Mirror, mirror on the floor

1. Write down your eye's height.
2. Now, place a mirror flat on the ground at _____ metres from the foot of the flagpole, such that the mirror is between you and the flagpole as shown. Looking at the mirror, walk backwards, in line with the flagpole and the mirror, till you see the tip of the flagpole in the mirror. Measure the horizontal distance between your current position and the mirror.



3. Describe how you can use this distance to get an estimate of the height of the flagpole. You may want to draw a suitable diagram to make your explanation clearer. State any assumption(s) made.
4. State the mathematical concepts/skills you use to complete this task.

Method 4 [Mirror, mirror on the floor]

The Task

- Topic(s): Similar triangles

The Discourse

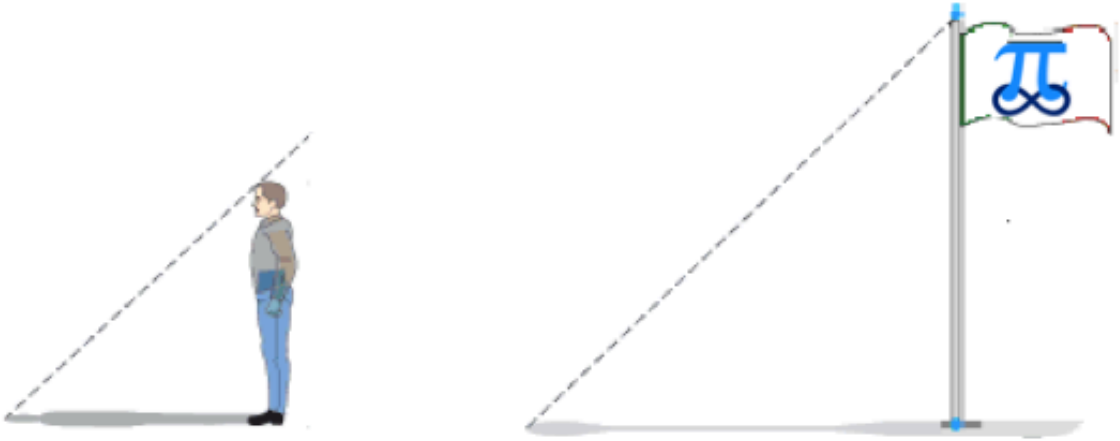
- Proof of similarity?
- Assumptions? Accuracy? Alternatives?

Method 4b?

Method 5 [Lights, shadow, action]

Secondary Two. Lights, shadow, action!

1. Write down your height.
2. Now stand next to the flagpole such that you can see your own shadow and the shadow casts by the flagpole under the sun as shown. Measure the length of each shadow.



3. Describe how you can use these lengths to get an estimate of the height of the flagpole. You may want to draw a suitable diagram to make your explanation clearer. State any assumption(s) made.
4. State the mathematical concepts/skills you use to complete this task.

Method 5 [Lights, shadow, action]

The Task

- Topic(s): Similar triangles

The Discourse

- Proof of similarity?
- Assumptions? Accuracy? Alternatives?

Method 10?

Rule of thumb?

Method 10.1?

Ask OM!

The WHATs of…

Task

Discourse

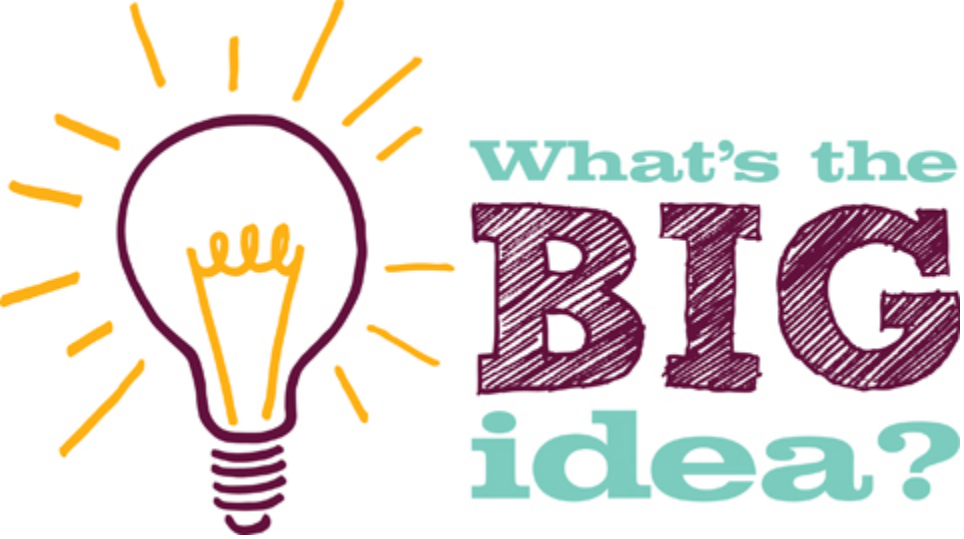
- Assumptions? Accuracy? Alternatives?
- What if?
- Connections?

Implementation

Assessment

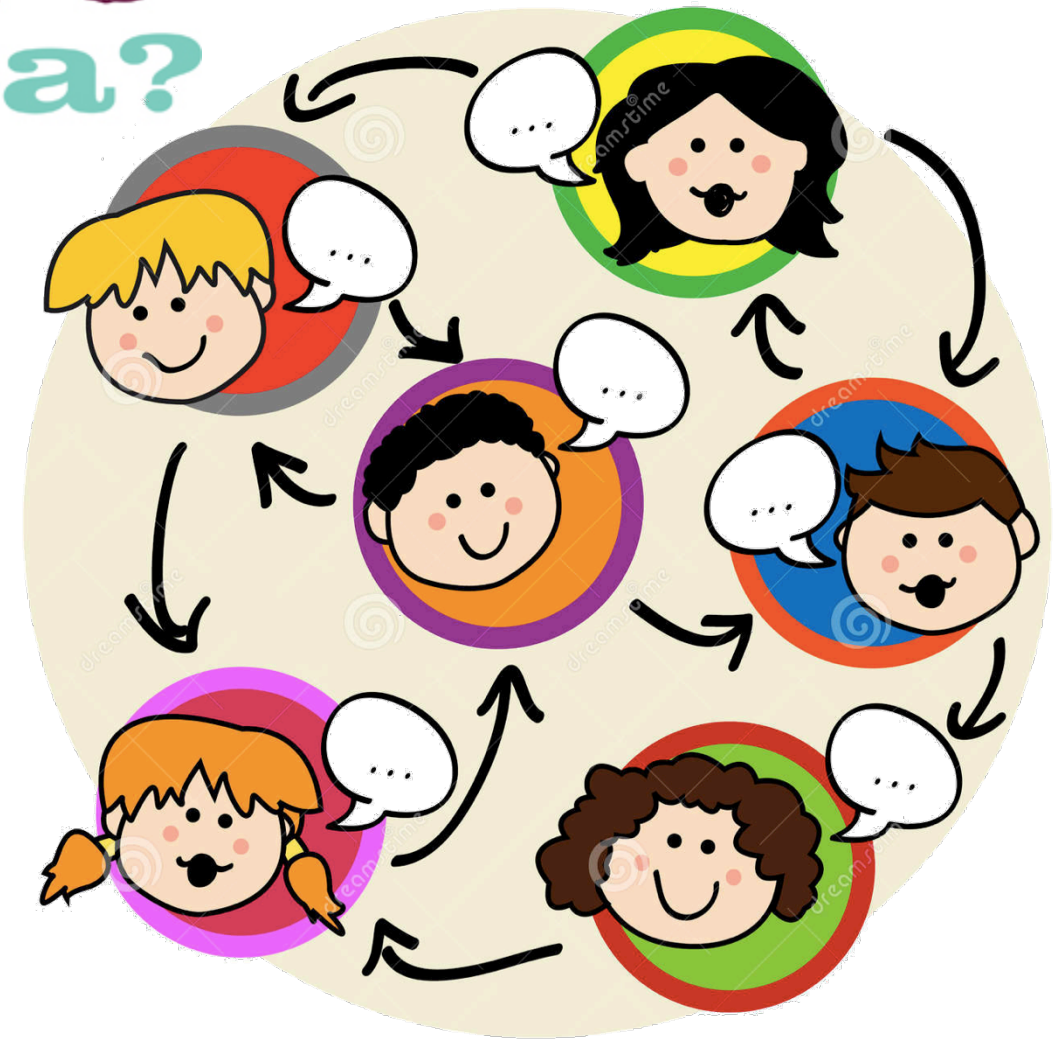
Generic Questions for Discourse

1. What is the problem about?
2. What information is given / do you need / is important?
3. What mathematics topics/concepts/skills can you use to solve this problem?
4. How do you know that your solution is correct / you have arrived at the correct answer?
5. Are there alternative methods to solve this problem?
6. Compare the different methods / the method(s) used this year with that used last year.
   - How are the methods similar or different?
   - Which do you prefer? Why?
   - Which is more efficient / accurate? Why?
   - What is the common topic/concept/skill behind these methods?



- Proportionality
- Diagrams
- Equivalence
- Models

What's the BIG idea?



Classroom Discourse

Thinking about the HOW

The HOWs of…

Task

Discourse

Implementation

- LE? AA? PEA?
- Instructions? Scaffolds?
- Orchestration of discourse?
- Other considerations? E.g. safety

Assessment

The HOWs of…

Task

Discourse

Implementation

Assessment

- Mathematics? Problem solving? 21CC?
- Rubrics?

The beginning of a journey…

- The Mathematical Task
  - Challenges students to make sense of both the contexts and the mathematics embedded in the task
  - Has multiple solution pathways such that students of different levels can do it
  - Teaches towards big ideas; encourages mathematical reasoning, metacognition & problem solving

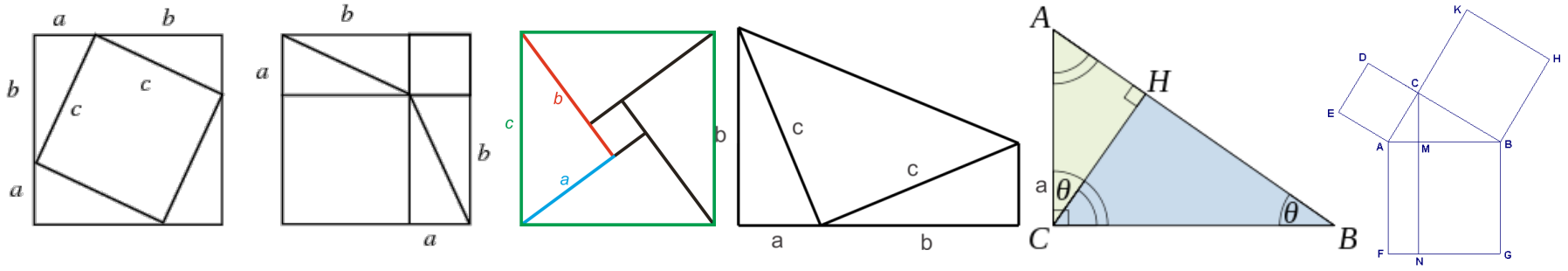
The beginning of a journey…

- The Teacher
  - Creates/recognises tasks
  - Plays an important role as a facilitator in the student learning as he/she orchestrates classroom discourse
  - Makes explicit the various connections both within and across the levels so as to deepen understanding

The beginning of a journey…

Possible Task 1: **Pythagoras Theorem**

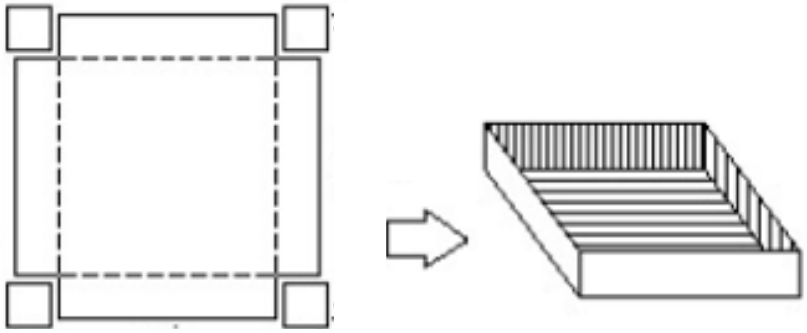
Level	Task	Topics
Sec 1	Pythagorean triplets	• Prime & square numbers • Number patterns
Sec 2	Proof of Pythagoras Theorem	• Algebra • Area of trapezium
Sec 3	Proof of Pythagoras Theorem	• Similar triangles • Triangles with common height



The beginning of a journey…

Possible Task 2: **Folding a box from cardboard**

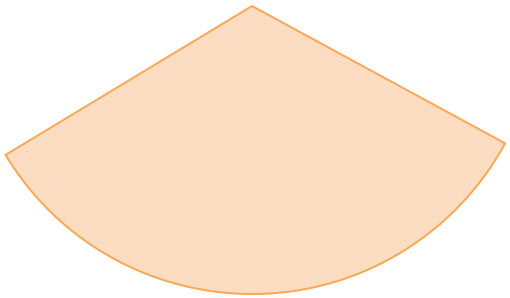
Level	Task	Topics
Sec 1	Tabulation	• Computational thinking
Sec 2	Graphical	• Functions & graphs
Sec 3	Solving equation	• Algebra / Polynomimals
Sec 4	Differentiation	• Calculus

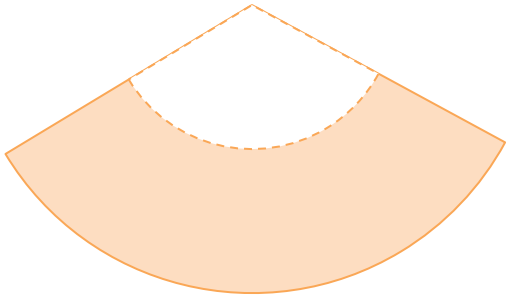


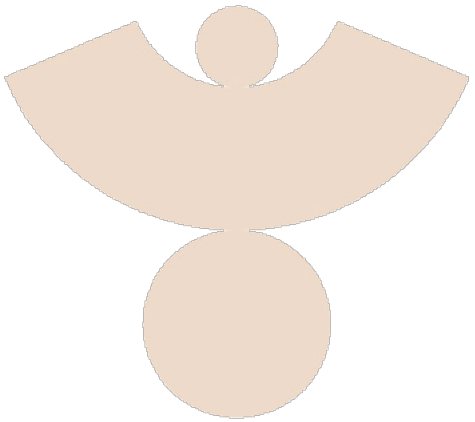
The beginning of a journey…

Possible Task 3: **Making a cup**

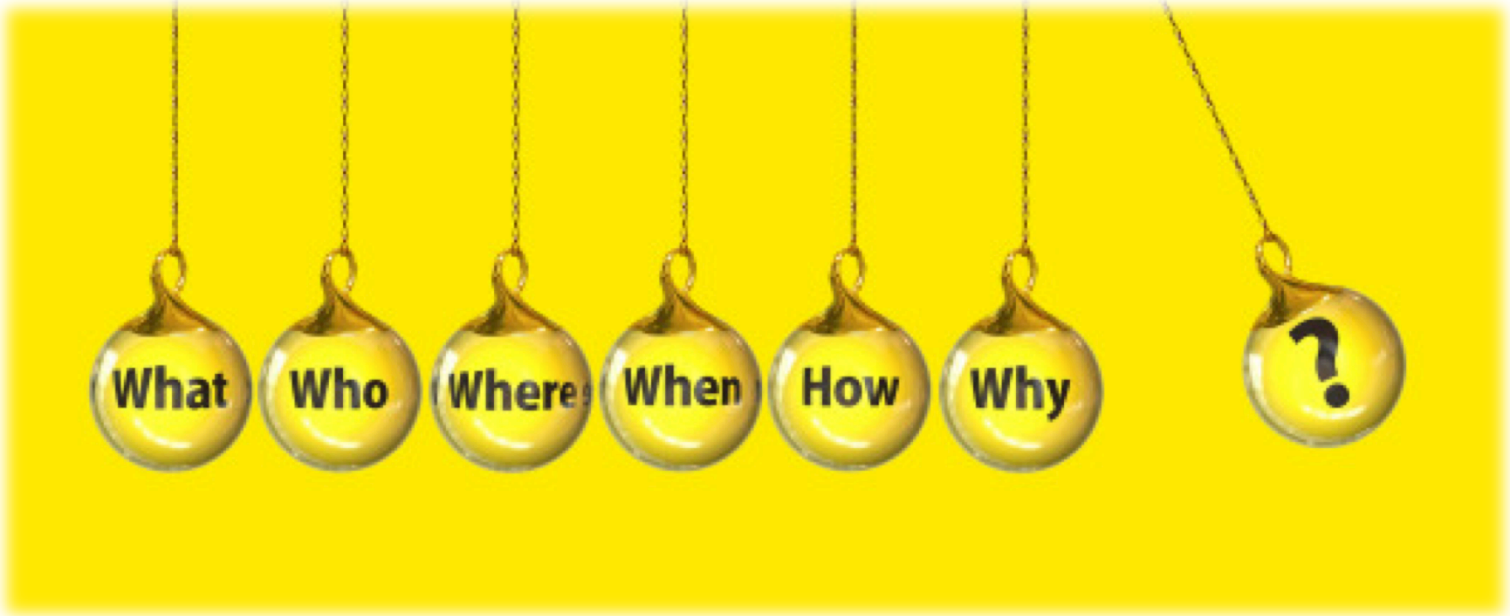
Level	Task	Topics
Sec 1	• Area of a sector & its truncated	• Mensuration
Sec 2	• Surface area & volume of a cone	• Mensuration
Sec 3	• Minimum/Maximum area/ volume given volume/area	• Similar volumes







The beginning of a journey…



with an idea & many questions

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