Unpacking Big Idea about Proportionality

Connecting Ratio, Rate, Proportion and Variation

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Secondary Maths Syllabus for 2020

- **Eight Clusters of Big Ideas:** Invariance, Functions, Diagrams, Notations, Proportionality, Measures, Models, Equivalence
- "Big ideas highlighted in the syllabus are ***not*** meant to be authoritative or comprehensive."
- **Four Recurring Themes of Nature of Maths:**
  - Properties and Relationships
  - Operations and Algorithms
  - Representations and Communications
  - Abstractions and Applications

Nature of Mathematics

"Mathematics is the study of the ***properties, relationships***, ***operations, algorithms***, and ***applications*** of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. ***Abstractions*** are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for ***representing*** and ***communicating*** the ideas and results of the discipline."

Big Ideas

- Big ideas could be about one or more themes, e.g. Proportionality is about Properties and Relationships; while Equivalence is about Properties and Relationships, and Operations and Algorithms.
- "Big ideas express ideas that are central to mathematics. They bring **coherence** and **connect** ideas from different strands and levels."

Proportionality

- Unpacking big idea about proportionality: connecting ratio, rate, proportion and variation
- For a teacher, content knowledge is as important as pedagogical content knowledge (Ball, Thames & Phelps, 2008; Shulman 1986)
  - So that we do not teach wrong things (difficult for students to un-learn)
- As a teacher, we should also know more than what our students need to learn so that we can address their questions that may be beyond syllabus
  - But we have to decide when and what to tell our students

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Why proportionality?

- Essential basis of math knowledge (Lesh, Post & Behr, 1988)
- One of the most commonly applied mathematics in the real world (Lanius & Williams, 2003)
- "Proportional reasoning is at the core of so many important concepts in mathematics and science, including similarity, relative growth and size, dilations, scaling, pi, constant rate of change, slope, rates, percent, trig ratios, probability, relative frequency, density, and direct and inverse variations." (Heinz & Sterba-Boatwright, 2008, p. 528)

Heinz, K., & Sterba-Boatwright, B. (2008). The when and why of using proportions. *Mathematics Teacher, 101*, 528-533.
Lanius, C. S., & Williams, S. E. (2003). Proportionality: A unifying theme for the middle grades. *Mathematics Teaching in the Middle School, 8*, 392-396.
Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93-118). Hillsdale, NJ: Erlbaum.

Why proportionality?

- Research has found that many students and even adults have difficulties in reasoning proportionally (Behr, Harel, Post & Lesh, 1992; Staples & Truxaw, 2012)
- Comparative situation: 30 boys and 10 girls in a class
  - "How many more boys are there than girls?" **(absolute / additive)**
  - "How many times is the number of boys more than the number of girls?" **(relative / multiplicative)**
- Students tend to see comparative situations in absolute (additive) rather than relative (multiplicative) terms, partly because it is easier to see '20 more boys' than '3 times as many boys as girls'

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio, and proportion. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296–333). New York: McMillan.
Staples, M. E., & Truxaw, M. P. (2012). An initial framework for the language of higher-order thinking mathematics practices. *Mathematics Education Research Journal, 24*, 257–281.

Why proportionality?

- Another gap: many students are unable to distinguish "between and among concepts of decimal, percent, ratio, proportion, and proportional reasoning" (Ojose, 2015, p. 111)
- May be because of the way topics are sequenced by teachers during instruction without presenting the opportunity for students to see the **big picture** and the **interconnectedness** among the various mathematical concepts (ibid.)
- So the question is how are ratio, rate and proportion related? (survey)

Ratio (Primary 5)

- All 3 P5 textbooks give some examples of ratios (without any definition)

Ratio (Secondary 1)

- Both Sec 1 textbooks give a more formal definition of ratio

A Ratios Involving Rational Numbers

At the primary level, we have learnt to use a ratio to compare two similar quantities a and b.

> The ratio of a to b is denoted by $a : b$ which can also be represented by $\frac{a}{b}$, where $b \neq 0$.

Recap (Concept of Ratio)

In primary school, we have learnt how to solve problems involving ratios. We shall have a quick revision. A **ratio** is used to compare two or more quantities of the *same kind* which are measured in the same unit.

> The **ratio** of a is to b, where a and b represent two quantities of the *same kind*, and $b \neq 0$, is written as $a : b$.

ATTENTION

A ratio has *no* units.

Ratio: Same kind?

- **Both Sec 1 textbook definitions of ratio:** Ratio is used to compare two or more quantities of the **same kind** (or similar quantities)
- No. of boys : no. of girls = 3 : 2
- This is a ratio because we are comparing numbers (same kind), not because boys and girls are human beings
- x boys and y girls: x and y are numbers
- If quantities of same kind are just **numbers**, they have no units

Ratio: Same kind?

- If quantities of same kind are **measurements**, must they have the same units? Can we write a map scale as 1 cm : 100 m?
- Ratio notation $a : b$ has no units, so map scale is 1 : 10 000
- Does this mean if two measurements have different units (e.g. 1 cm and 100 m), it is not a ratio?
- May be helpful to distinguish between **concept of ratio**, and **ratio notation *a* : *b***
- If two measurements have same type of units (that can be converted from one to the other), we can still use **ratio** to compare, but if we write it using **ratio notation**, it must not have units

Ratio: Same kind (Summary)

- Ratio is used to compare two or more quantities of the **same kind**, which refer to either
  - (i) **numbers** or
  - (ii) **measurements** with same type of units (that can be converted from one to the other, e.g. cm and metres; not cm and grams)
- Ratio **notation** $a : b$ has **no units**

Ratio Notation

- **Usual textbook definition of ratio notation:** $a : b$, where $b \neq 0$
- Why $b \neq 0$ is because $a : b$ can be written as $\frac{a}{b}$
- Can $a = 0$?
  - E.g. no. of boys : no. of girls = 0 : 40
  - No. of girls : no. of boys = 40 : 0?
  - Is 0 : 40 the same as 0 : 20 or 0 : 1?
- If a quantity is 0, **not meaningful** to use ratio as a basis of comparison with another quantity: we just say that the quantity is 0

Ratio Notation

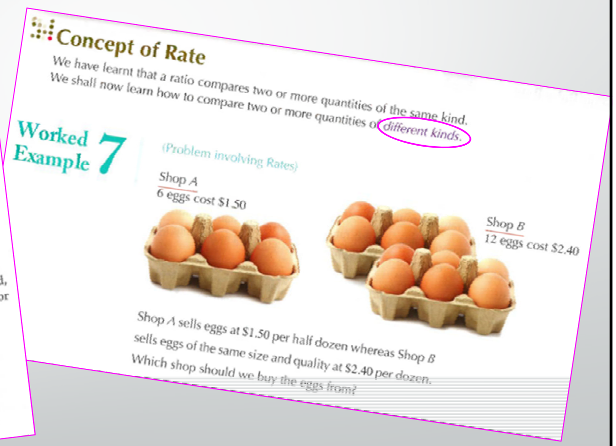
- **Usual textbook definition of ratio notation:** $a : b$, where $b \neq 0$
- Can a or b be negative?
  - Most (if not all) **scalar** quantities in real world are non-negative
  - Ideal gas law: $PV = nRT$, where T is absolute temperature (measured in Kelvin), so $T \geq 0$
  - Any textbook questions: $a, b > 0$
- **Proposed definition of ratio notation:** $a : b$, where $\boldsymbol{a, b > 0}$

Rate (Primary 5)

- Unlike ratio, all 3 P5 textbooks give a definition of rate after an example

Rate (Secondary 1)

- Both Sec 1 textbooks also define rate but they do it differently



Rate: Per unit of another quantity?

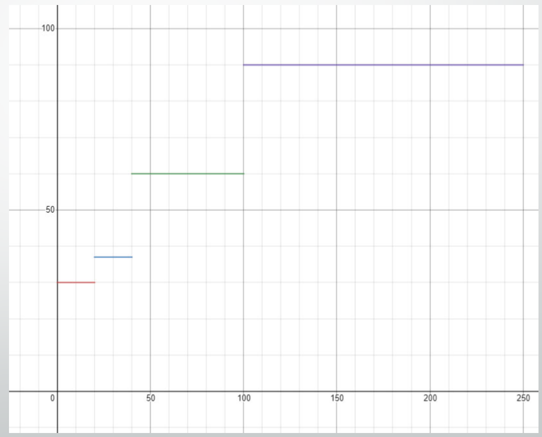
- **Most textbook definitions of rate:** Rate is **(usually)** expressed as one quantity **per unit of another quantity**
- The textbook that includes the word 'usually' does not give any exceptions. But can you think of some exceptions?
- E.g. an item costs $2 per 100 grams, or a smartphone plan costs $10 per 2 gigabytes of data
- Rate can be expressed as one quantity **per *n* units of another quantity**, where *n* is some 'nice' number
- Is this idea of rate comprehensive enough?

What kind of rates is this?

- 2016 O-level Exam Paper 2 on Problem in Real-World Contexts

LOCAL MAIL RATES (i

For Letters, Postcards, Printed Papers and Packets/Packages		
Weight Step Up To	Standard Mail	Non-Standard Mail
Regular		
20 g	$0.30	$0.60
40 g	$0.37	
Large		
100 g	$0.60	$0.90
250 g	$0.90	$1.15
500 g	$1.15	$1.70
1 kg	$2.55	
2 kg	$3.35	



Rate of Change

- **Rate of change** in Additional Mathematics under Calculus (Sec 4)
- E.g. rate of change of area A of circle w.r.t. radius r is $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r$
- $\frac{\mathrm{d}A}{\mathrm{d}r}$: expression of one quantity per unit of another quantity, e.g. $\frac{\mathrm{d}A}{\mathrm{d}r} = 10$ cm^2 **per cm** of change in radius when radius is $\frac{5}{\pi}$ cm
- **Proposed definition of rate:** Rate is a way of comparing how one quantity **changes** with another quantity. It is either a non-step rate (which can be expressed as one quantity per n units of another quantity) or it is a step rate.

Rate: Different Kinds?

- **One Sec 1 textbook definition of rate:** Rate is used to compare two quantities of **different kinds**
- The other textbook did not say different kinds but examples given all compare quantities of different kinds
- Can rates be used to compare two quantities of the same kind?
- Unemployment rate = no. of unemployed people divided by no. of working people, and then expressed as a percentage
- Singapore's unemployment rate in 2018 = 2.1%, i.e. 2.1 unemployed people per 100 working people, or 21 unemployed people per 1000 working people
- Rate of change of circumference of circle with respect to its radius
- Therefore, rate can be used to compare two quantities of the **same kind** or of **different kinds**

Is an exchange rate a constant rate?

- What is not constant is how an exchange rate changes with time: **rate of change of exchange rate with time** is not constant
- But an exchange rate is a **constant rate** at a given period time (e.g. at money changer)
  - US$1 = S$1.37
  - US$2 = S$2.74 (rate is still the same: US$1 = S$1.37)
  - US$0.50 = S$0.685 (rate is still the same: US$1 = S$1.37)
- "Is an exchange rate constant?" is vague because it can mean:
  - "Is an exchange rate constant with respect to time?"
  - "Is an exchange rate a constant rate?"

Ratio vs. Rate

- **Ratio** is a way of comparing two or more quantities of the same kind. The quantities can be numbers (of objects) or measurements. The ratio notation $a : b$, where $a, b > 0$, has no units.
- **Rate** is a way of comparing how one quantity changes with another quantity. The quantities can be of the same kind or of different kinds. It is either a non-step rate or a step rate.
- How are ratio and rate related?
- Some overseas textbooks (e.g. Cathcart, Pothier, Vance & Bezuk, 2016; Van de Walle, Karp & Bay-Williams, 2010): a rate is a ratio
- But they will not use ratio notation $a : b$ (no units) for rates
- Local textbooks: clear distinction between ratio and rate, but are they related?

Cathcart, W. G., Pothier, Y. M., Vance, J. H., & Bezuk, N. S. (2006). *Learning mathematics in elementary and middle schools* (4th ed.). Upper Saddle River, NJ: Pearson.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2010). *Elementary & middle school mathematics* (7th ed.). Boston, MA: Pearson.

Ratio vs. Rate

- Singapore's unemployment rate in 2018 = 2.1%, i.e. 2.1 unemployed people per 100 working people, or 21 unemployed people per 1000 working people
- Rate (comparing quantities of same kind) can be expressed as a ratio 21 : 1000
- Math Snacks: Bad Date (Video): https://www.youtube.com/watch?v=BZ1M01YBKhk
- Ratio of woman's no. of words to man's no. of words was 25 : 175 or 1 : 7
- The woman made this statement, "For every word I spoke, he spoke 7 words."
- Rate: "He spoke 7 words per word that I spoke."
- Ratio (of 2 quantities) can be converted to a rate
- Can you always do that? How about no. of boys to no. of girls in a class is 2 : 1? Can you say that there are 2 boys per girl? Does that make sense to you?

List of countries by sex ratio

From Wikipedia, the free encyclopedia

The human sex ratio is the number of males for each female in a population. This is a list of sex ratios by country or region.

Country/region	at birth (CIA estimate, 2016)[1]	0–14 years	15–24 years	25–54 years	55–64 years	over 65	total	at birth (WDB estimate, 2012)[5]
– *World*	1.09	1.07	1.07	1.02	0.95	0.805	1.015	1.07
Afghanistan	1.05	1.03	1.04	1.04	0.97	0.86	1.03	1.07
Albania	1.10	1.12	1.07	0.91	0.98	0.89	0.98	1.07
Singapore	1.07	1.05	0.97	0.95	1.00	0.83	0.96	1.07

- Singapore: 1.07 males per female
- Writing ratio as rate makes it easier to compare across different groups (or countries)

Ratio vs. Rate

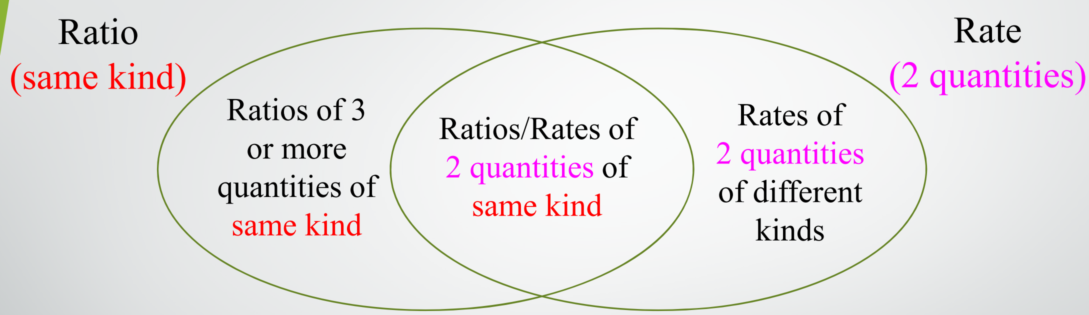
- Which class has a **higher proportion** of boys? (***not*** which class has more boys because the total no. of students in each class may be different)

Class	A	B	C
Boy to Girl Ratio	3 : 2	5 : 2	3 : 1

- Can easily tell Class B has a higher proportion of boys than Class A (because both ratios have same base 2)
- For Class A and Class C, have to reason a bit
- For Class B and Class C, even harder to tell

Class	A	B	C
Boy per girl (Rate)	1.5	2.5	3

Ratio vs. Rate (Summary)



- For **2 quantities of the same kind**, we can use ratio or rate (or both) to compare
- But sometimes it may be more **meaningful or helpful** to use one over the other

Proportion: Different Meanings

proportion *noun* **(IMPORTANCE)**

[U] **used in a number of phrases to mean importance and seriousness:**

*You have to keep a **sense of** proportion (= the ability to understand what is important and what is not).*

*I think a certain amount of worry about work is very natural, but you've got got to **keep it in** proportion (= judge correctly its seriousness).*

proportion *noun* **(SIZE)**

C2 [C or U] **the correct or most attractive relationship between the size of different parts of the same thing or between one thing and another:**

*Your legs are very much **in** proportion **to** (= the right size for) the rest of your body.*

*His feet seem very small **in** proportion **to** his body.*

*My head was much nearer the camera than the rest of me so I'm all **out of** proportion.*

Proportion: Different Meanings

proportion *noun* **(AMOUNT)**

C1 [C, + sing/pl verb] **the number or amount of a group or part of something when compared to the whole:**

Children make up a large proportion ***of*** *the world's population.*

A higher proportion ***of*** *men are willing to share household responsibilities than used to be the case.*

The report shows that poor families spend a larger proportion ***of*** *their income on food.*

- **Proportion** can refer to a **part-whole ratio**

Proportion as Part-Whole Ratio

- Which class has a **higher proportion** of boys?

Class	A	B	C
Boy to Girl Ratio	3 : 2	5 : 2	3 : 1
Boy per girl (Rate)	1.5	2.5	3

- Previously, we have used rates to help us compare
- But actually, **proportion** of boys in Class A = $\frac{3}{5}$ or 60% **(part-whole ratio)**
- This more mathematical idea of proportion (part-whole ratio) is still different from direct (or inverse) proportion (relationship between two quantities)

Proportionality

- **Proportionality** refers to **direct and inverse proportions**
- Library fines for one book:

× 3   × 2   × 2   ÷ 2 or × 0.5

No. of days (x)	1	2	3	4	5	6	7	8
Fine (y cents)	15	30	45	60	75	90	105	120

× 2   × 3   × 2   ÷ 2 or × 0.5

- As x is doubled, y is doubled; as x is halved, y is halved; etc.
- **Concept of direct proportion:** as x is multiplied by n, y is also multiplied by n (n can be 0.5 etc.)

Direct Proportion

× 2   × 3

No. of days (x)	1	2	3	4	5	6	7	8
Fine (y cents)	15	30	45	60	75	90	105	120

× 2   × 3

- $\frac{x_2}{x_1} = 2$ and $\frac{y_2}{y_1} = 2$, so $\frac{y_2}{y_1} = \frac{x_2}{x_1} = 2$
- $\frac{x_2}{x_1} = 3$ and $\frac{y_2}{y_1} = 3$, so $\frac{y_2}{y_1} = \frac{x_2}{x_1} = 3$
- "For a mathematician, a **proportion** is a statement of **equality of two ratios**, i.e. $a/b = c/d$." (Tourniaire & Pulos, 1985, p. 181)
- But the **two ratios are not constant** (even for same scenario of library fines)

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics, 16*, 181-204.

Direct and Inverse Proportions (Summary)

- Two related ideas of direct proportion:
  - **(Direct) Proportional Reasoning:** As one quantity (or variable) is multiplied by n, the other quantity (or variable) is also multiplied by n
  - **Equality of two ratios:** $\frac{y_2}{y_1} = \frac{x_2}{x_1}$
- Two related ideas of inverse proportion:
  - **(Inverse) Proportional Reasoning:** As one quantity (or variable) is multiplied by n, the other quantity (or variable) is divided by n (or multiplied by $\frac{1}{n}$)
  - **Equality of two ratios:** $\frac{y_2}{y_1} = \frac{x_1}{x_2}$ (e.g. if $\frac{x_2}{x_1} = 2$, then $\frac{y_2}{y_1} = \frac{1}{2}$)

Direct and Inverse Proportions (Summary)

- Direct proportion:
  - Can we say, "For direct proportion, as x increases, y increases"?
  - But there are **other types of increase**, such as quadratic or exponential increase
  - Important to say, "As x increases, y increases **proportionally**…"
  - "… and by 'proportionally', we mean if x is multiplied by n, then y is also multiplied by n." (may be too formal for students)
  - Or "… and **by 'proportionally', we mean** as x is doubled, y is also doubled; as x is halved, y is also halved; etc." (may be easier for students to understand)
- Why is it important to emphasise that the increase is proportional, other than there are other types of increase? (later)

Direct Proportion

No. of days (x)	1	2	3	4	5	6	7	8
Fine (y cents)	15	30	45	60	75	90	105	120
Rate $\frac{y}{x}$ (cents/day)	15	15	15	15	15	15	15	15

- $\frac{y}{x} = k$, where k is a non-zero constant
- $y = kx$, where $k \neq 0$
- $\frac{y_2}{x_2} = \frac{y_1}{x_1}$ **(equality of 2 rates)** can be obtained from $\frac{y_2}{y_1} = \frac{x_2}{x_1}$ **(equality of 2 ratios)** by some manipulations
- But the **rate** is a **constant**: $\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{y_3}{x_3} = \ldots$ **(equality of rates)**
- Unlike **ratio**, which is **not a constant**, e.g. $\frac{y_2}{y_1} = \frac{x_2}{x_1} = 2$, or $\frac{y_2}{y_1} = \frac{x_2}{x_1} = 3$, etc.

Inverse Proportion

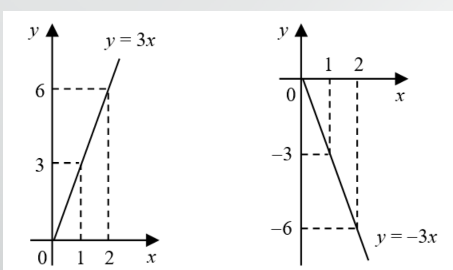
- $xy = k$, where k is a non-zero constant
- $y = \frac{k}{x}$, where $k \neq 0$
- $\frac{y_2}{x_1} = \frac{y_1}{x_2}$ **(equality of 2 rates)** can be obtained from $\frac{y_2}{y_1} = \frac{x_1}{x_2}$ **(equality of 2 ratios)** by some manipulations
- But unlike direct proportion, for **inverse proportion**, the **rate is not a constant**; what is **constant** is the **product** of the two quantities (or variables)
- Direct proportion:
  - **Equality of two ratios** $\frac{y_2}{y_1} = \frac{x_2}{x_1}$, or equality of (two or more) rates $\frac{y_2}{x_2} = \frac{y_1}{x_1}$, or **rate $\frac{y}{x}$ is constant**
- Inverse proportion:
  - **Equality of two ratios** $\frac{y_2}{y_1} = \frac{x_1}{x_2}$, or equality of two rates $\frac{y_2}{x_1} = \frac{y_1}{x_2}$, or **product xy is a constant**

Direct and Inverse Variations

- 2001 syllabus: If y varies directly as x (we write $y \propto x$), then $y = kx$, where $k \neq 0$.

  If y varies inversely as x (we write $y \propto \frac{1}{x}$), then $y = \frac{k}{x}$, where $k \neq 0$.
- 2007 syllabus: Variations were combined with proportions (equations remain but symbol $\propto$ removed)
- For proportions, textbooks always deal with positive quantities x and y (although they can be 0), so constant of proportionality $k > 0$
- For variations, textbooks also deal with positive quantities x and y, and so $k > 0$
- But definitions for variations remain: $k \neq 0$
- What if $k < 0$? What will it affect?

Direct Variation



- For $y = 3x$, as x is doubled from 1 to 2, y is doubled from 3 to 6
- For $y = -3x$, as x is **doubled** from 1 to 2, y is **doubled** from –3 to –6, but y **decreases!**
- Is this inverse proportion?
- But proportional decrease means as x is doubled, y is **halved**
- If $\boldsymbol{k < 0}$ for **direct variation**, as x increases, y **decreases but not proportionally** (unlike inverse proportion)
- For original concept of direct proportion (as x increases, y increases proportionally) to work, $k > 0$
- During **extension to $\boldsymbol{k < 0}$** in direct variation, something still works (e.g. as x is doubled, y is doubled), but others no longer work (as x increases, y decreases)

Extension of Concepts in Mathematics

- In mathematics, when a **concept is extended**, the original idea (or some parts of it) may no longer work
- E.g. original idea of a^n applies for positive integer n: a multiplied by itself n times
- What if $n = 0$ or -2? What does it mean for a to multiply by itself 0 or –2 times?
- a multiplied by itself 0 times is not 0, but $a^0 = 1$ (if $a \neq 0$)
- Similarly, in extending proportions (where $k > 0$) to variations (where $k \neq 0$), some parts of the original idea may not work
- Nevertheless, although current Sec 2 textbooks on direct and inverse proportions state that $k \neq 0$, all examples and questions only deal with $k > 0$ (and all quantities are positive also)

Proportionality in the real world

- "Although most people are probably unaware of the mathematical definition of proportions [i.e. the equality of two ratios], they do use them in familiar situations." (Tourniaire & Pulos, 1985, p. 181)
- How people use proportions is via the idea of **proportional reasoning**
- Mickey saves \$72 in 18 days. He saves the same amount each day. How many days will Mickey take to save \$160? (**Primary 5** Textbook, 2017, p. 74)

$$\$72 \rightarrow 18 \text{ days}$$

$$\$1 \rightarrow \frac{18}{72} \text{ day}$$

$$\$160 \rightarrow \frac{18}{72} \times 160 \text{ days}$$

- **Unitary method** makes use of **proportional reasoning**
- Not natural to use equality of two ratios or form equation $y = kx$ to solve

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics, 16*, 181-204.

Proportionality in the real world

- Proportion is in Sec 2 syllabus, so what can we do in Sec 1?
- A shopping centre charges 'per minute parking', i.e. a fixed amount is charged per minute of parking. Ivy pays \$6 for parking her car in the shopping centre for 2.5 hours. On another occasion, she parks her car for 1 hour 45 minutes. How much does she have to pay for the parking?
- Method 1: Find **rate** of parking = $\frac{\$6}{2.5\text{ h}}$ = \$2.40 per hour
- Method 2: 2.5 hours of parking cost \$6

  1 hour of parking costs $\frac{\$6}{2.5}$

  1.75 hours of parking cost $\frac{\$6}{2.5} \times 1.75 = \$4.20$
- **Direct proportional reasoning** can be used in **Sec 1 chapter on constant rate**

Proportionality in the real world

- Does that mean that when the relationship between two variables is a constant rate, then the two variables are directly proportional to each other?
- Counter example: $y = 2x + 3$
  - Rate = 2 (constant), but this is not direct proportion because, e.g. when x is doubled from $x = 1$ to $x = 2$, y is not doubled from $y = 5$ to $y = 7$
- Direct proportion: $y = kx$, i.e. y-intercept = 0 (straight line graph must pass through origin)
- May be too much to tell Sec 1 students this, but must we always form the equation or draw the graph to check that it passes through origin?
- Easier to just check that when $x = 0$, $y = 0$
- Situation with **constant rate** and **both quantities = 0 at same time**: this is **direct proportion**

Recognising Proportional Situations

- Many students fail to distinguish proportional situations from absolute comparison situations, so need to teach them how to recognise proportional situations (Behr, Harel, Post & Lesh,1992; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005), e.g. constant rate and both quantities = 0 at same time
- But after teaching them, some may go to the other extreme: is this a proportional situation?

If 30 musicians take 30 minutes to play Beethoven's Fifth Symphony, how long does it take 60 musicians to play the same symphony?

Behr, M., Harel, G., Post, T., & Lesh, R. (1992). Rational number, ratio, and proportion. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 296–333). New York: McMillan.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: effects of age and problem type on propensities for overgeneralisation. *Cognition and Instruction, 23*, 57–86.

Proportionality in the real world

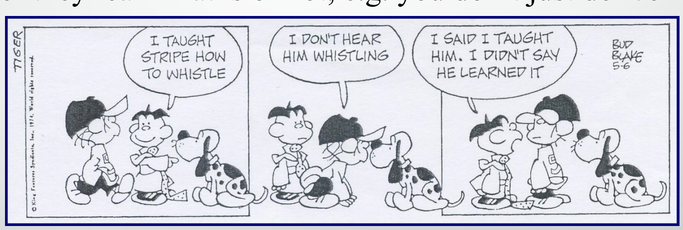
- Similar triangles: lengths of corresponding sides are proportional (video on application of similar triangles in hospitals can be found here: http://math.nie.edu.sg/bwjyeo/videos)
- Statistical graphs (e.g. bar chart, pie chart): length of bar or size of sector proportional to frequency

Conclusion for Lecture

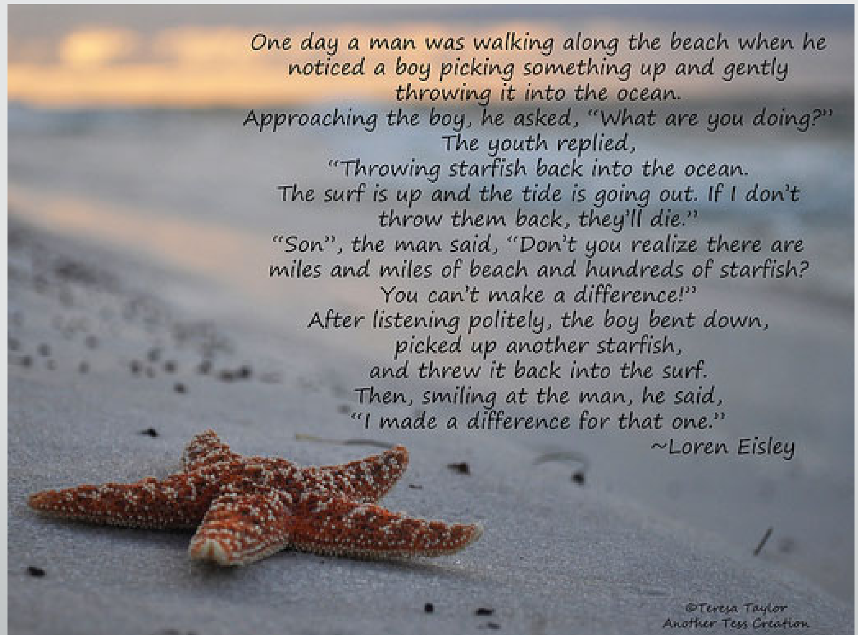
- Connections among concepts of ratio, rate, proportion and variation
- Proportionality is common in maths and in real life: most people may not use equality of ratios or form equations to solve proportional problems, but they actually use proportional reasoning (via Unitary Method) to solve

My Teaching Philosophy

- As a maths teacher, you don't teach maths. You teach …?
- You teach **students**. What is the difference?
- Teaching students means:
  - you **care** whether they learn maths or not, e.g. you don't just deliver lessons to cover syllabus

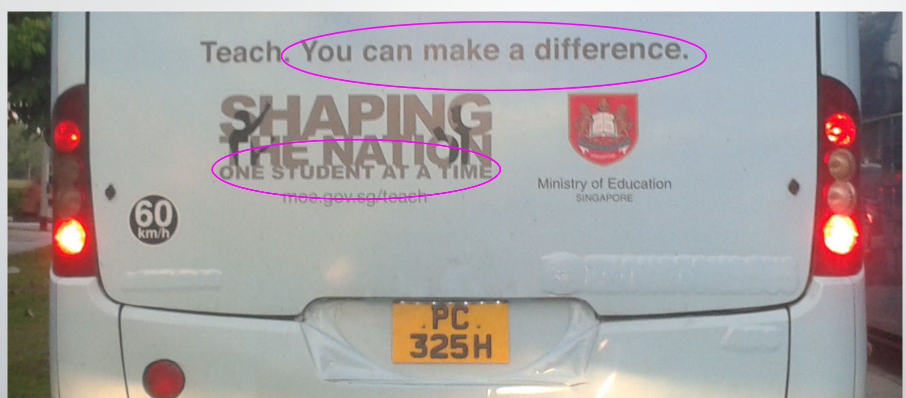


  - you **care** whether they learn other subjects
  - you **care** about **their entire well-being**, not just their academic performance



Always remember this

- Key idea from starfish story: **one student at a time**



Quotation by William Arthur Ward

- The mediocre teacher tells.
- The good teacher explains.
- The superior teacher demonstrates.
- The great teacher **inspires**.