Integrating Problem Posing into Mathematical Problem Solving: An Experimental Study

Chunlian Jiang
University of Macau
Email: cljiang@umac.mo

Outline

1. What?
2. Why?
3. How?
4. An experimental study
5. Conclusion

What's Mathematical Problem Posing?

- The generation of new problems about a situation or the reformulation of a given problem (Silver & Cai, 1996).

- **A try**

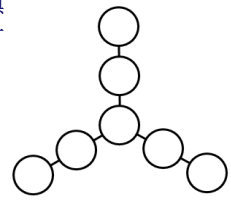
Ann has 34 marbles, Billy has 27 marbles, and Chris has 23 marbles. Write and solve as many problems as you can that use this information (Silver & Cai, 2005.)

Can you pose problems for students at different grade levels?

What's Mathematical Problem Posing?

Second try:

- 把1、2、3、4、5、6、7七个数填在图1的圆圈里（每个数只能用一次），使每条直线上的三个数相加的和都等于12。



- Translation: Put 1, 2, 3, 4, 5, 6, and 7 into the circles (one number for one circle) so that the sum of three numbers on the same line will be added up to 12.

What's Mathematical Problem Posing?

Discussion:

Can we make up similar problems from it?

Why?

1. From mathematicians' point of view
2. One of the objectives of mathematics education postulated in curriculum standards, e.g., USA, China, etc.
3. Historical development of mathematics education in China
4. Literature in mathematics education

W1: Mathematicians' Point of View

- The formulation of the problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill.
- To raise new questions, new possibilities, to regard old problems from a new angle, requires creative imagination and marks real advance in science.

— Einstein & Infeld (1938).

W2: The objectives of mathematics education

1. USA (NCTM, 2000, p.258):

students must be given opportunities to "formulate interesting problems based on a wide variety of situations, both within and outside mathematics.

W2: The objectives of mathematics education

2. China (Ministry of Education, 2001, 2010):

知识与技能，数学思考，解决问题，情感与态度

a) 经历将一些实际问题抽象为数与代数问题的过程，掌握数与代数的基础知识和基本技能，并能解决简单的问题。

b) 经历探究物体与图形的形状、大小、位置关系和变换的过程，掌握空间与图形的基础知识和基本技能，并能解决简单的问题。

c) 经历提出问题、收集和处理数据、作出决策和预测的过程，掌握统计与概率的基础知识和基本技能，并能解决简单的问题。

W2: The objectives of mathematics education

2. China (Ministry of Education, 2003, 2010):

知识与技能，数学思考，解决问题，情感与态度

- 初步学会从数学的角度提出问题、理解问题，并能综合运用所学的知识和技能解决问题，发展应用意识。
- 形成解决问题的一些基本策略，体验解决问题策略的多样性，发展实践能力与创新精神。
- 学会与人合作，并能与他人交流思维的过程和结果。
- 初步形成评价与反思的意识。

W3. Historical development of mathematics education in China

Problem posing has been integrated into mathematics curriculum in China since 1980s.

王华的妈妈在工厂里织袜。上午，织大人袜 32 双，儿童袜 48 双。

（1） 口头提出一个要用加法算的问题，再算出来。

（2） 口头提出一个要用减法算的问题，再算出来。

先把有关的条件和问题连起来，再计算。

3 个人做 15 面小旗，	还要做几面？
3 个人做小旗，每人做 5 面，	一共做多少面？
要做 15 面小旗，已经做了 5 面	平均每人做几面？

W3. Historical development of mathematics education in China

"Juyifansan" (举一反三), a kind of mathematical tasks

(1) After the teaching and learning of a topic

(2) When a student fails to solve a problem correctly.

W3. Historical development of mathematics education in China

Mathematical problem posing has become one of the main factors contributing to Chinese students' good performance in mathematics.

"Juyifansan" (举一反三)
《论语·述而》："举一隅，不以三隅反，则不复也。"

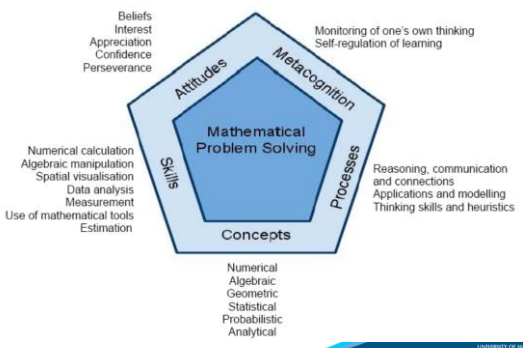
English translation: If I have presented one corner of the square and they cannot come back to me with the other three, I should not go over the points again.

W4. Literature in mathematics education

Group Discussion

What role can mathematical problem posing play in students' mathematics learning?

W4. Literature in mathematics education



How?

- What problem-posing tasks can you give to your students?
- How to integrate mathematical problem posing into mathematics teaching and learning?
- How to integrate mathematical problem posing into mathematical problem solving?

H1: What problem-posing tasks can you give to your students?

Three categories (Stoyanova, 2000):

- Free situations, e.g., *write a (difficult) problem for friends to solve*.
- Semi-structured situations, e.g., *Write a story problem for 65×35.*
- Structured situations, e.g.,

12.	东庄民兵练习射击，分成 3 组，每组 6 个人，一共有多少个民兵？
13.	把第 12 题改编成一道除法应用题。

H1: What problem-posing tasks can you give to your students?

Cai & Jiang (2016):

- Problem-posing tasks in Chinese mathematics textbooks over three decades
- Problem-posing tasks included in Chinese and US elementary mathematics textbooks in 2010s

- Cai, J. & Jiang, C. (2016). An analysis of problem-posing tasks in Chinese and US elementary mathematics textbooks. International Journal of Science and Mathematics Education. DOI 10.1007/s10763-016-9758-2

H1: What problem-posing tasks can you give to your students?

- Posing a problem that matches the given arithmetic operation(s), e.g. Make up a word problem orally for 14 + 8 = ?
- Posing variations of a question with similar mathematical relationship or structure. Example: The distance between two cities, A and B, is 2590 km. A plane is flying from A to B at a speed of 650 km/h. Another plane is flying from B to A at a speed of 645 km/h. If they set out at the same time from their respective airports, after how many hours will they meet up? First solve it, then change it to a problem with meeting time as one of the givens and the distance of the two cities as the unknown^ (PEP, 1990, p. 62).

H1: What problem-posing tasks can you give to your students?

- Posing additional questions based on the given information and a sample question. Example: On weekends, a father and his son went climbing. The distance from the ground to the top of the mountain is 7.2 km. It took them 3 hours to climb up and 2 hours to walk down. What are the speeds going up and going down? Can you pose additional mathematical questions? (PEP, 2000, p. 20).
- Posing questions based on given information. Example: The average floor areas per person in three cities are shown in the following table, what problems can you pose? Can you solve them? (PEP, 2010, p. 100).

Cities	A	B	C
Floor area (m^2)	14.6	16.7	17.6

H1: What problem-posing tasks can you give to your students?

- Chinese mathematics textbooks over 3 decades

Table Percentages of types of problem-posing tasks in the three mathematics textbook series

Types of problem-posing tasks	1990s (n = 138)	2000s (n = 132)	2010s (n = 149)	1990s vs. 2000s	2000s vs. 2010s
1. Posing a problem that matches the given/specific kinds of arithmetic operation(s)	26.81	3.79	.67	5.22***	1.80
2. Posing variations on a question with the same mathematical relationship or structure	27.54	0	2.01	6.50***	−1.64
3. Posing additional questions based on the given information and a sample question	2.17	65.91	79.19	−11.11***	−2.50**
4. Posing questions based on given information	43.48	30.30	18.12	2.24*	2.39*

The numbers in the last two columns are the z scores

*p < .05; **p < .01; ***p < .001

H1: What problem-posing tasks can you give to your students?

- Chinese & US mathematics textbooks in 2010s

Table Percentages of types of PP tasks in the Chinese and US mathematics textbooks

Types of problem-posing tasks	Chinese	US	
	PEP ($n = 149$)	Investigations ($n = 52$)	*Everyday* ($n = 80$)
1. Posing a problem that matches the given/specific kinds of arithmetic operation(s)	.67	84.62	68.75
2. Posing variations on a question with the same mathematical relationship or structure	2.01	13.46	23.75
3. Posing additional questions based on the given information and a sample question	79.19	1.92	5.00
4. Posing questions based on given information	18.12	0	2.50

H2: How to integrate it into mathematics teaching and learning?

- Wu Zhengxian (吴正宪)

H2: How to integrate it into mathematics teaching and learning?

- Wu Zhengxian (吴正宪)
- 一群小朋友站成一排，从前往后数小明排在第五位；从后往前数，小明还是排在第五位。可以提什么问题？
- Translation: A group of children are lining. From the front, Xiaoming was counted as fifth. From the end, Xiaoping was counted as fifth too. Can you pose a problem?

H2: How to integrate it into mathematical problem solving?

- How many children are lining?
- How to solve it?

- Act it out
- Draw a picture
- Mathematical expressions 5+5-1

Why -1?

H2: How to integrate it into mathematical problem solving?

如果班上参加语文兴趣小组的有5人，参加数学兴趣小组的有9人，两个小组都参加的有2人。问参加语文或数学小组的共有多少人？

Translation: In a class, there are 5 students participating in the Chinese interest group, 9 students participating in the mathematics interest group, and 2 participating in both. How many students are there participating in either Chinese or mathematics interest groups?

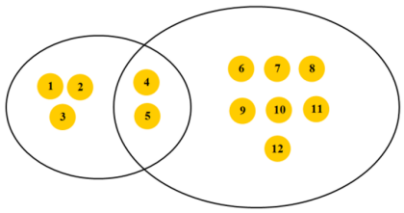
H2: How to integrate it into mathematical problem solving?

Discussion

Multiple solutions:

H2: How to integrate it into mathematical problem solving?

Make stories for the following picture.



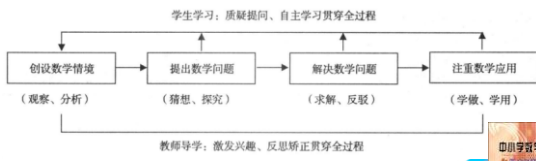
H2: How to integrate it into mathematical problem solving?

- 如果班上参加语文兴趣小组的有5人，参加数学兴趣的有7人，问两个小组都参加的可能有多少人？
- Translation: In a class, there are 5 students participating in the Chinese interest group, 7 students participating in the mathematics interest group. How many students are participating in both interest groups?

H2: How to integrate it into mathematics teaching and learning?

- 4 steps:

  创设数学情境（前提）→提出数学问题（核心）→解决数学问题（目标）→应用数学知识（归宿）



H2: How to integrate it into mathematics teaching and learning?

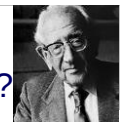
Applying Mathematical Knowledge Newly Learnt

Solve Mathematical Problems

Pose Mathematical Problems

Create a mathematical situation

H2: How to integrate it into mathematical problem solving?

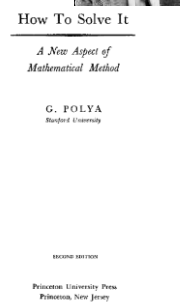


Look back

Carry out the plan

Make up a plan

Understand the problem



H2: How to integrate it into mathematical problem solving?

Look back

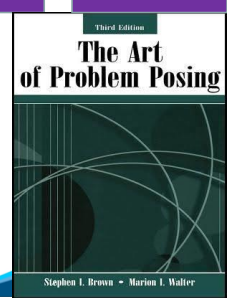
Problem Posing

What if not?

Carry out the plan

Make up a plan

Understand the problem



An Experimental Study

- No. (%) of students involved in the study

	Boys	Girls	Total
Experimental group	18 (64.3%)	10 (35.7%)	28 (100%)
Control group	16 (57.1%)	12 (42.9%)	28 (100%)

An Experimental Study

Pre-test:
(1) PS: 13 items on linear equations in one variable
(2) PP: 1 problem-posing task in linear equation in one variable

Post-test:
(1) PS: 7 problem-solving items
(2) PP: 1 problem-posing task in simultaneous linear equations in two variables

Experimental group: Teaching of mathematical problem solving with problem posing integrated
Control group: Traditional teaching

Scoring – Pre-test

Description	Item No.	Scoring
Solve equations, Use algebraic expressions to express quantitative relationships	1-5	1 – Correct; 0 - Incorrect。
Problem solving - translation and planning skills	6-9	1 – Correct; 0 - Incorrect。
Problem solving - Build simultaneous linear equations in two variables	10-12	1 for each correct equation, the total is 2 marks.
Problem solving - Build linear equations in one variable	13	1 – Correct equation; 0 – Incorrect equation.
Problem-posing task	14	Pose an appropriate word problem that can be solved with given equation – 1; others – 0.

Scoring – Post-test

Description	Item No.	Scoring
Problem solving		
Build equations	1-7	1 for each correct equation, the total is 2 for each item.
Solve equations	1-7	1 for correctly finding the value for one variable, the total is 2 for each item.
Problem posing	8	Pose an appropriate word problem that matches one of the two given equations – 1; the total is 2.

Results – Pre-test

No Difference
PS & PP

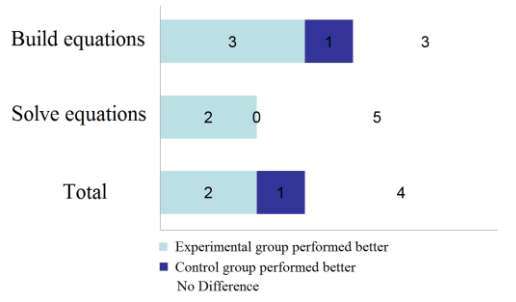
Experimental Group ⟷ Control Group

Results – Post-test

No Difference
in PS

Experimental Group ⟷ Control Group

Results – Post-test



Results – Post-test

Significant difference in PP

Experimental Group ⟷ Control Group

$p < .05$

d = 0.528

Conclusion - To improve students' problem-solving skills

- Overall no differences
- In more items, the experimental group performed better than the control group, especially in skills to build up equations and to solve equation.

Conclusion - To improve students' problem-posing skills

- The experimental group performed better than the control group in the difficult problem-posing item (Cai et al., 2013).

Reference:
Cai, J.; Moyer, J.C.; Wang, N.; Hwang, S.; Nie, B.; & Garber, T. (2013). Mathematical problem posing as a measure of curricular effect on students' learning. *Educational Studies in Mathematics*, 83(1), 57-69.

Conclusion - To improve students' problem-posing skills

- If students could have a longer period of experiences in mathematical problem solving with problem posing integrated, I am sure that they can improve not only their problem-solving skills, but also problem-posing skills.

Thanks!

Questions are welcome!