Designing lessons to foster mathematical thinking in children

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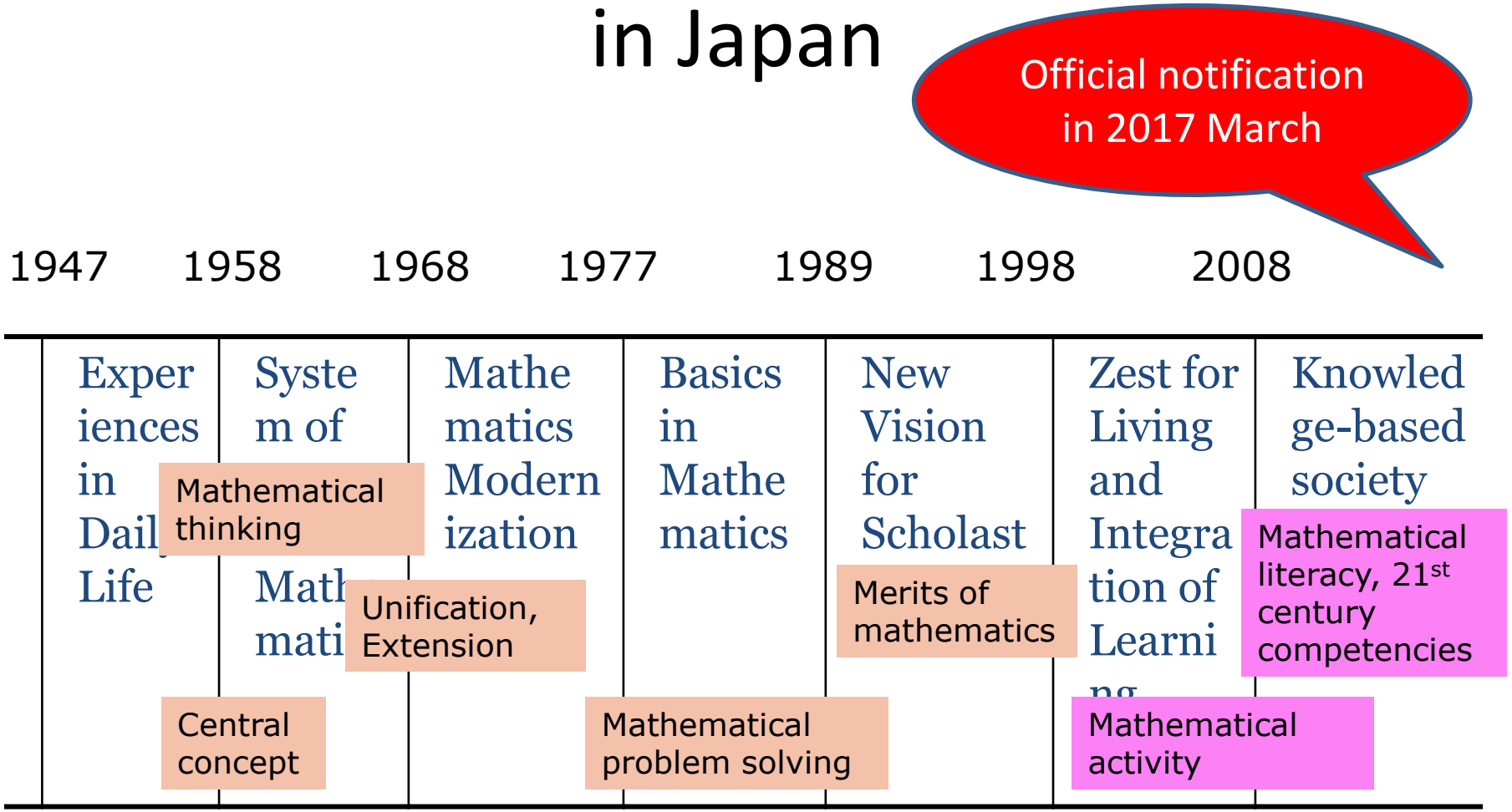
Teaching mathematics in the classroom: Problem solving approach

What is “problem solving approach” to teach mathematics

- Problem solving approach is the ways of teaching of mathematics in which children solve mathematically/educationally rich problems and discuss different solutions. Mathematical content is introduced to students by building on these activities in the lesson.

- Teaching mathematics through problem solving is a widely preferred method within the community of mathematics educators in Japan. Generally, mathematics teaching is associated with solving mathematics problems (Hiebert et al., 2003), but quite often, solving problems is regarded as the application of knowledge acquired in a lesson. In Japan, having students solve problems is deeply connected with the goal of fostering mathematical thinking, which has been the goal of mathematics education for more than 50 years. Here solving problems is not only regarded as application of learned knowledge but also used as a vehicle for imparting new knowledge (Hino, 2007).

History of National Course of Study in Japan



Mathematical activity and problem solving

- In principle, mathematical activities are carried out as problem solving. That is, they are a sequence starting with generating wonder and questions, formulating problems by formalizing them, understanding the problems, planning, implementing and reflecting on solution processes, generating new wonder and questions, generating conjectures, and formalizing problems.
- Experiencing these series of activities provides opportunities for students to feel the joy of thinking and learning mathematics as well as the necessity for and the usefulness of mathematics. In addition, because these activities require students to persist, they provide opportunities for students to heighten their own self-esteem.
- Furthermore, by listening to and adopting different ideas from others, these activities can promote students to understand each other better. (MEXT, 2008)

Japanese lesson pattern
(Stigler & Hiebert, 1999)

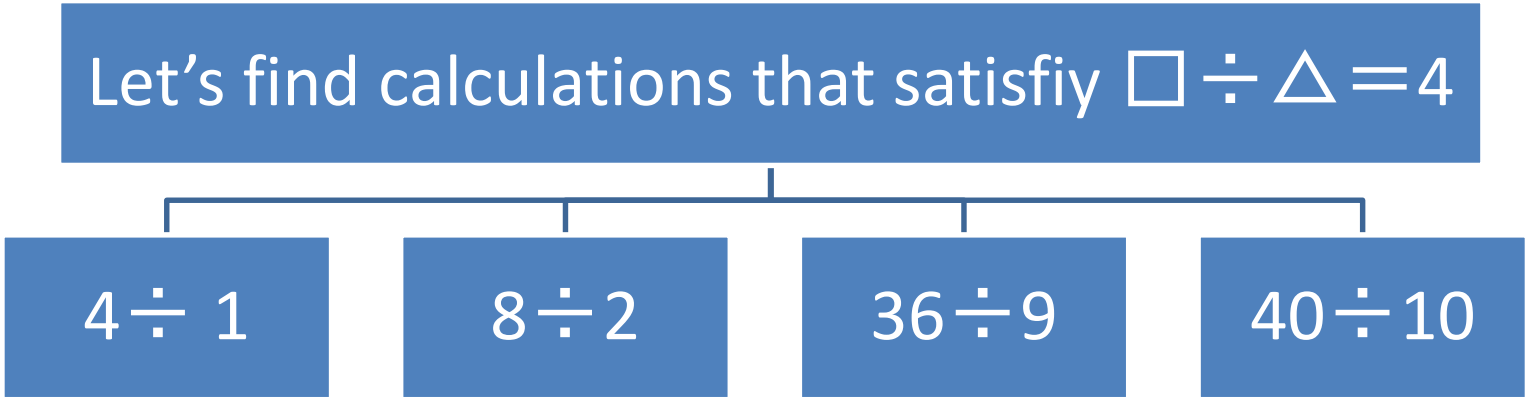
- Reviewing the previous lesson
- Presenting the problem for the day
- Students working individually or in groups
- Discussing solution methods
- Highlighting and summarizing the major poin

Structured Problem Solving

Two points for constructing mathematics lesson

1. Use of open ended problems

- ***Japanese mathematics lessons*** include a variety of open ended problems (problems that have several correct answers). This approach provides students with the opportunities to think and apply their own knowledge to solve problems.



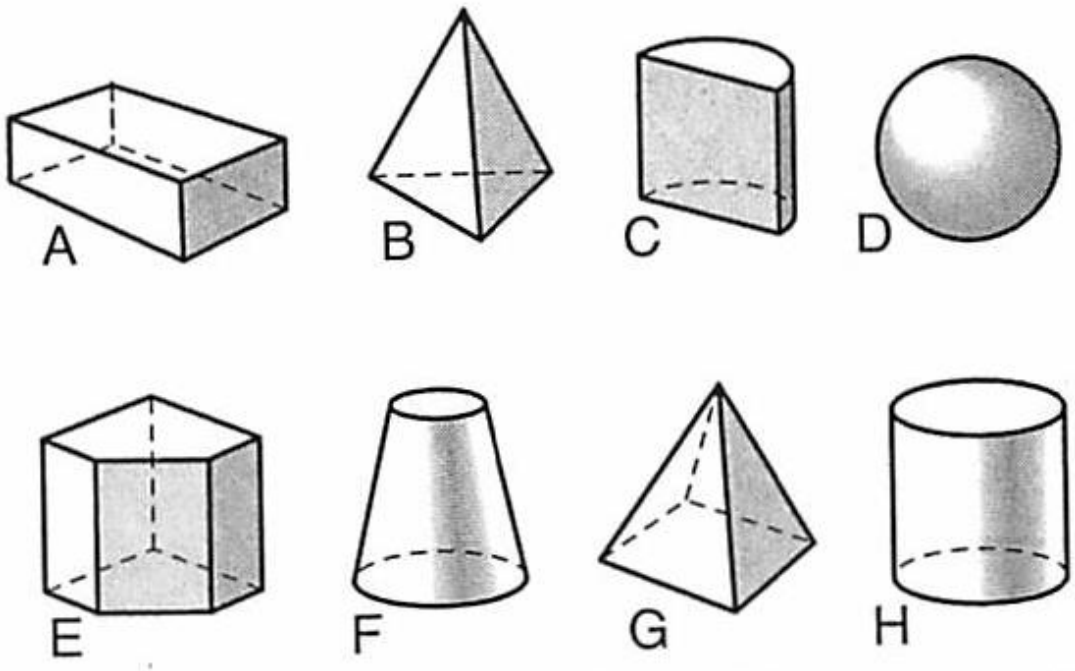
Types of open-ended problem
(Becker & Shimada, 1997)

- *Type 1. Finding relations*: Students are asked to find some mathematical rules or relations.
- *Type 2. Classifying*: Students are asked to classify according to different characteristics, which may lead them to formulate some mathematical concepts.
- *Type 3. Measuring*: Students are asked to assign a numerical measure to a certain phenomenon.

Finding relations

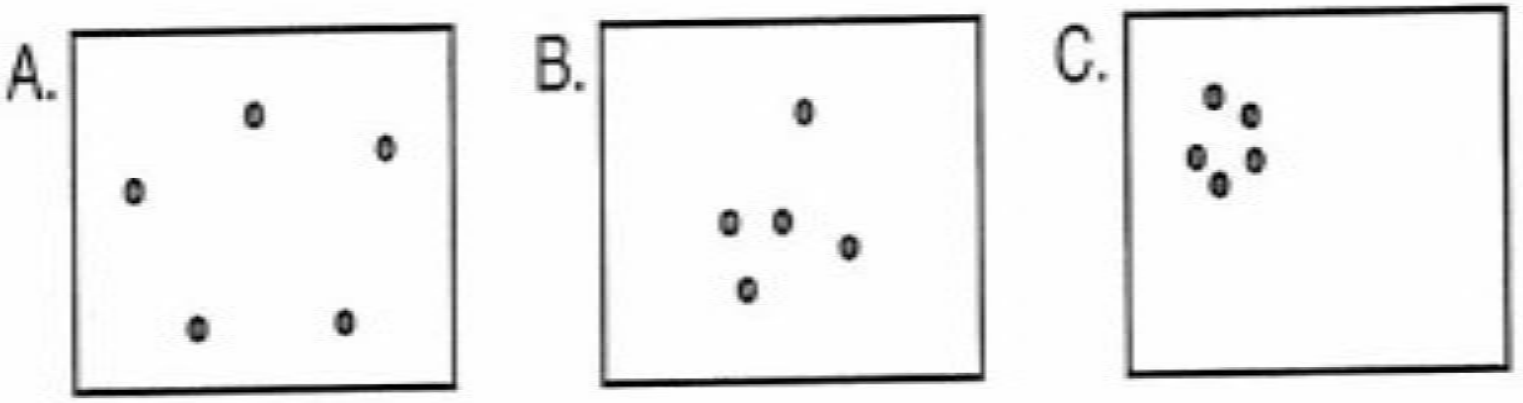
1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Classifying



Consider the solid figures as shown. Choose one or more figures that share the same characteristics with figure B and write down those characteristics. Next, choose one or more figures that share characteristics with figure H and write down those characteristics.

Measuring



Three students, A, B, and C, each threw five marbles, which came to rest as shown. In this game, the winner is the student with the smallest scattering of marbles. The degree of scattering seems to decrease in the order A, B, C. Devise as many ways as you can to express numerically the degree of scattering.

2. How to deal with multiple solutions (Koto and colleagues, 1998, 2010)

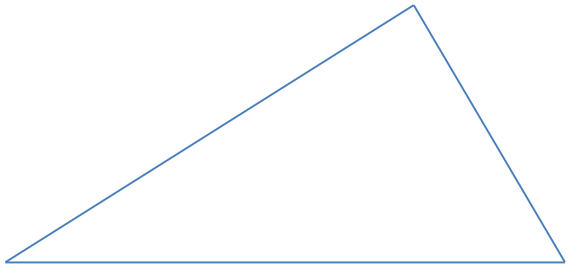
- Four principles of organizing discussion
  - Examination of the validity of each solution
  - Examination of the relationship among solutions
  - Comparison of different solutions from the point of view of relevance, generality, or utility
  - Looking back the solutions from self-evaluation by each student

- Four types of multiple solutions
  - *Independent type*: Each solution method has its own validity as a mathematical idea, but the relationship among the solutions is weak or non-existent. Each solution method is considered equally valid to the other methods.
  - *Ordered type*: Solution methods can be ordered from the perspective of mathematics or the objective of the lesson, such as, mathematical relevance, generality, or utility. For example, calculation method A may be judged better than calculation method B because A can apply to other numbers, while B can only apply to specific cases of numbers.
  - *Unified type*: By focusing on similarities, different solution methods can be unified under a common mathematical idea or principle.
  - *Connected type*: By focusing on the relationships among the solution methods, they can be synthesized into several interconnected groups.

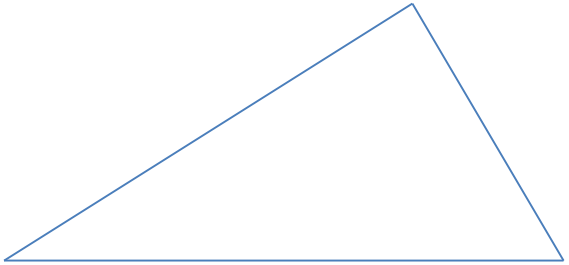
Ordered type:
Which is larger, 3/4 or 4/5?

- Use of diagrams
- Use of number line
- Change fraction to decimal number
- Make fractions that are the same size
- Method of finding a common denominator

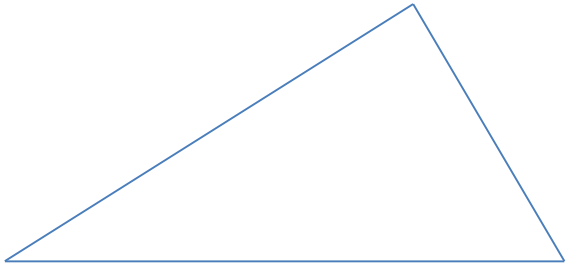
Unified type
Teaching formula of the area of triangle



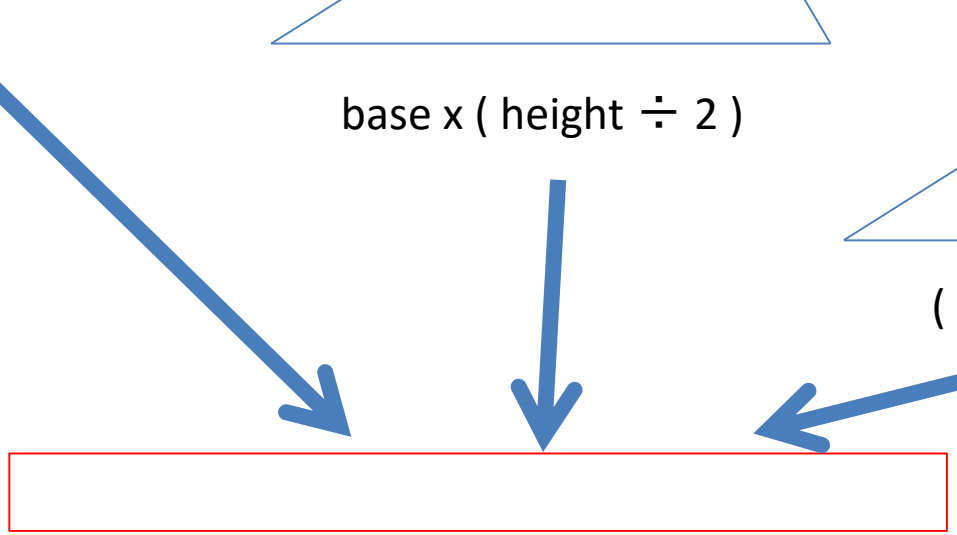
(base x height) ÷ 2



base x (height ÷ 2)



(base ÷ 2) x height



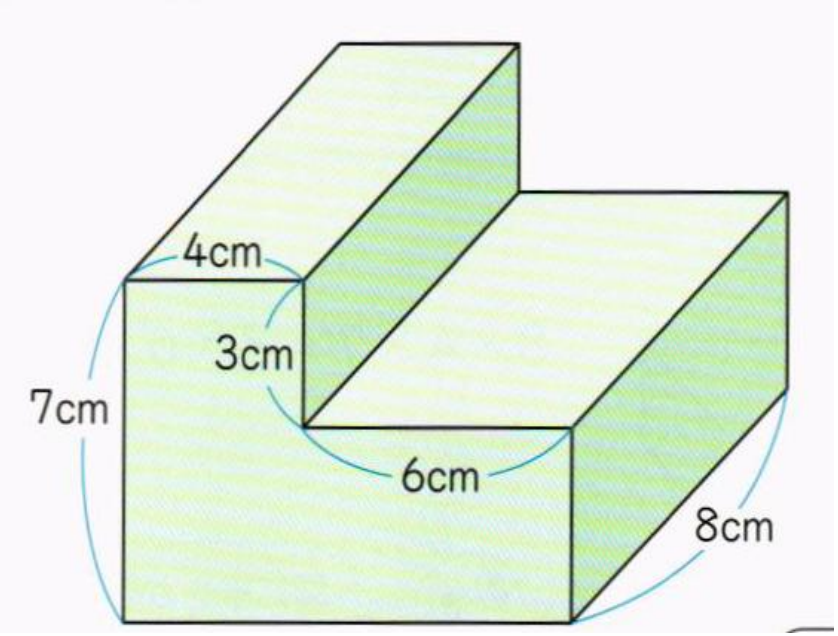
Connected type
Which room is the most crowded?

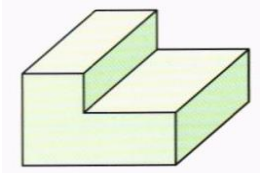
Room	A	B	C
Area (m^2)	10	12	12
Number of Students	7	7	9

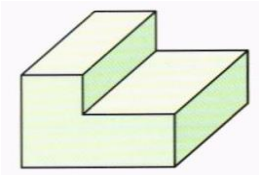
Examples of classroom activity

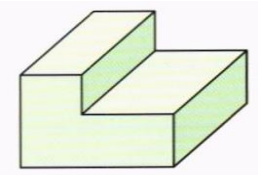
Let’s try and discuss

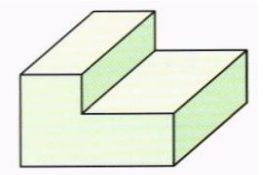
1. Find the volume





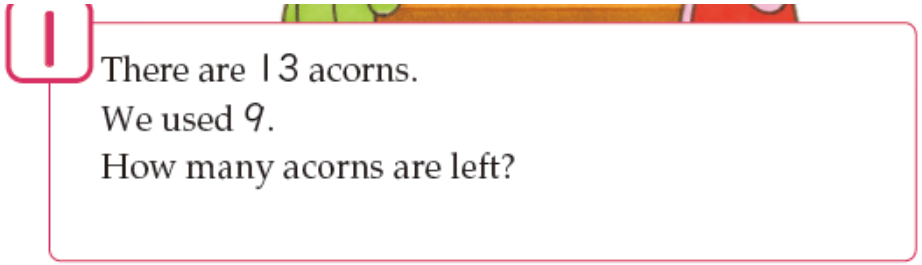






1. Let’s anticipate students’ responses to this problem
2. Set the goal of the lesson and plan how to discuss and summarize multiple solutions to meet the goal

2. To make open question to facilitate student's learning



1

There are 13 acorns.
We used 9.
How many acorns are left?

① Write the math sentence.

Math Sentence

② Think about how to calculate.

Let's make it open to facilitate student's learning

1 Division of Fractions

1 With $\frac{3}{4}$ dL of paint, we could paint $\frac{2}{5}$ m^2 of boards. What is the area of boards that we can paint with 1 dL of this paint?

Let’s think about the problem

1. What math sentence should we write by using division?
2. Let’s think about how to calculate.

Summary

- Using open-ended problem
- Eliciting and organizing different ways of thinking
- Proposing and making sense of mathematically-significant focus
- Using symbolic representations (e.g., words, mathematical expression, students' self-developed symbols)

Thank you very much

Major references

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