Developing classroom interaction toward the goal of the lesson in primary mathematics classes

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This presentation

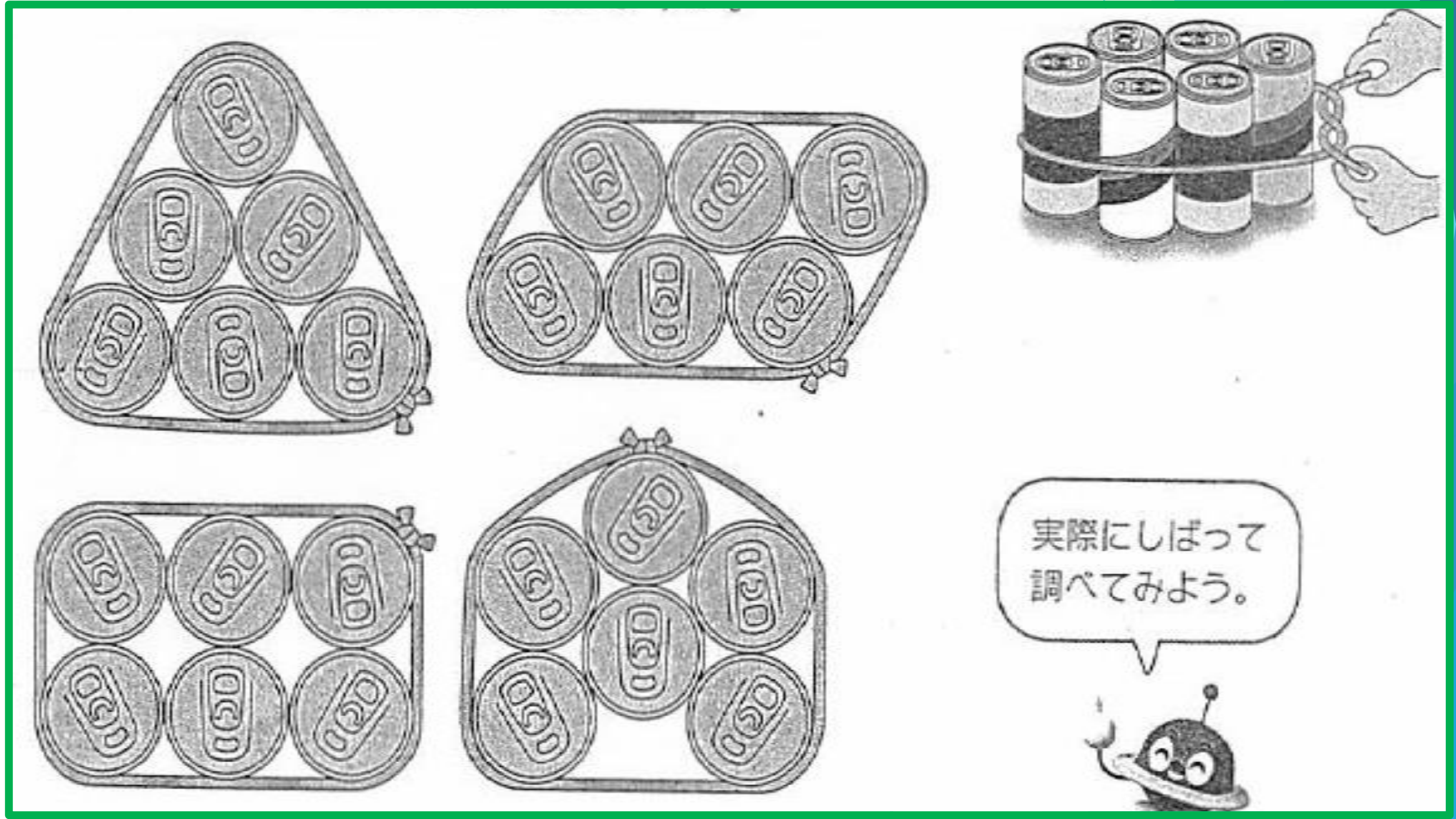
- Mathematics instruction needs to be understood from multiple perspectives and involves a variety of factors. One of these factors is the goal (or objective) of the lesson(s) that the teacher has in mind.
- This presentation will examine (i) the importance of goals of the lesson by drawing on several studies, (ii) illustrate classroom interactions in terms of how they are channelized toward the goal and what roles the teacher plays.
- The analysis will provide information on teacher's goal as well as teaching actions influencing students' learning opportunities.

Content

- Introduction
- Goal of the lesson in studies related to teaching mathematics
  - The TIMSS curriculum framework and selected results
  - Lesson study
  - Study on classroom interaction
- An analysis of classroom interaction: how students are channelized toward the goal of the lesson
- Conclusion

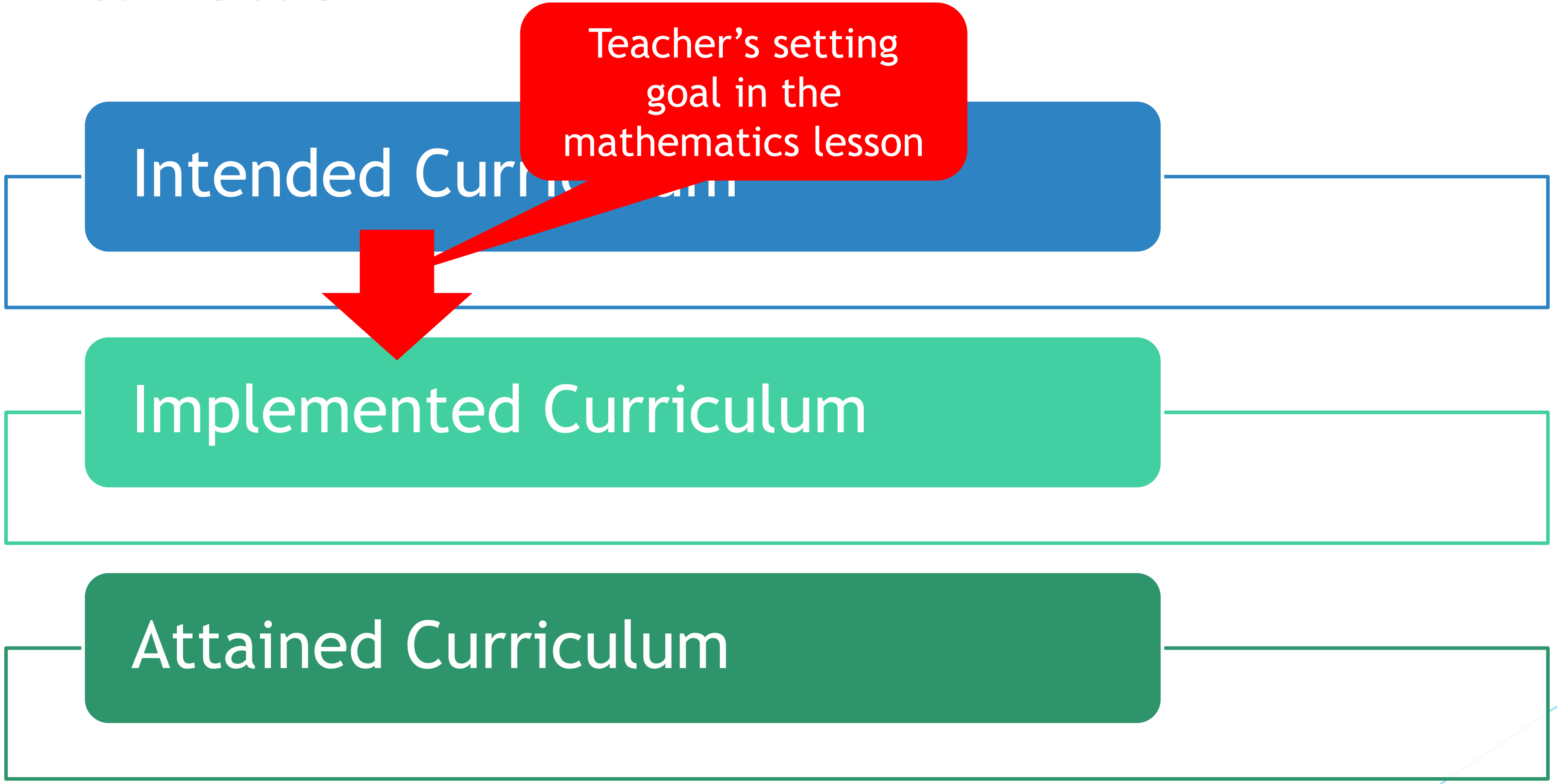
A question from a 8th grade teacher

- "In the four cases, the last one needs the Pythagorean Theorem in order to find the length of the rope. (The students will learn the Theorem in 9th grade.) But, if I remove the last one, the lengths of the ropes are the same. I think my students will get the answers easily. So I am wondering whether or not I should include the fourth one in my task of the lesson."



Goal of the lesson in studies related to teaching mathematics

Goal of the lesson in the TIMSS curriculum framework



Relationship between goal and lesson organization (e.g., Stigler et al., 1999)

Goal of the lesson

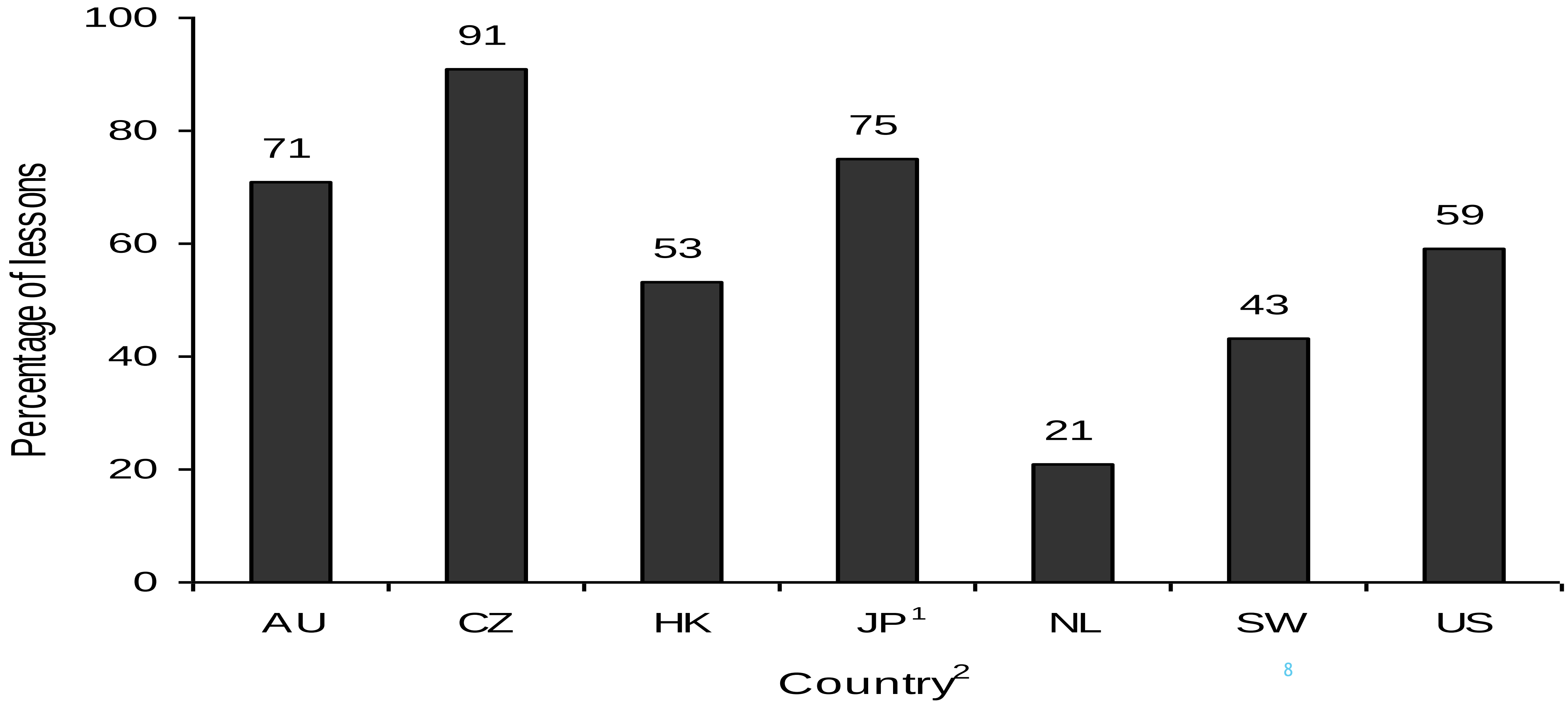
	Mathematical skill	Mathematical thinking	Others
Germany	55%	31%	14%
United States	61%	21%	18%
Japan	25%	73%	2%

Number of lessons: G (100), US (81), J (50)

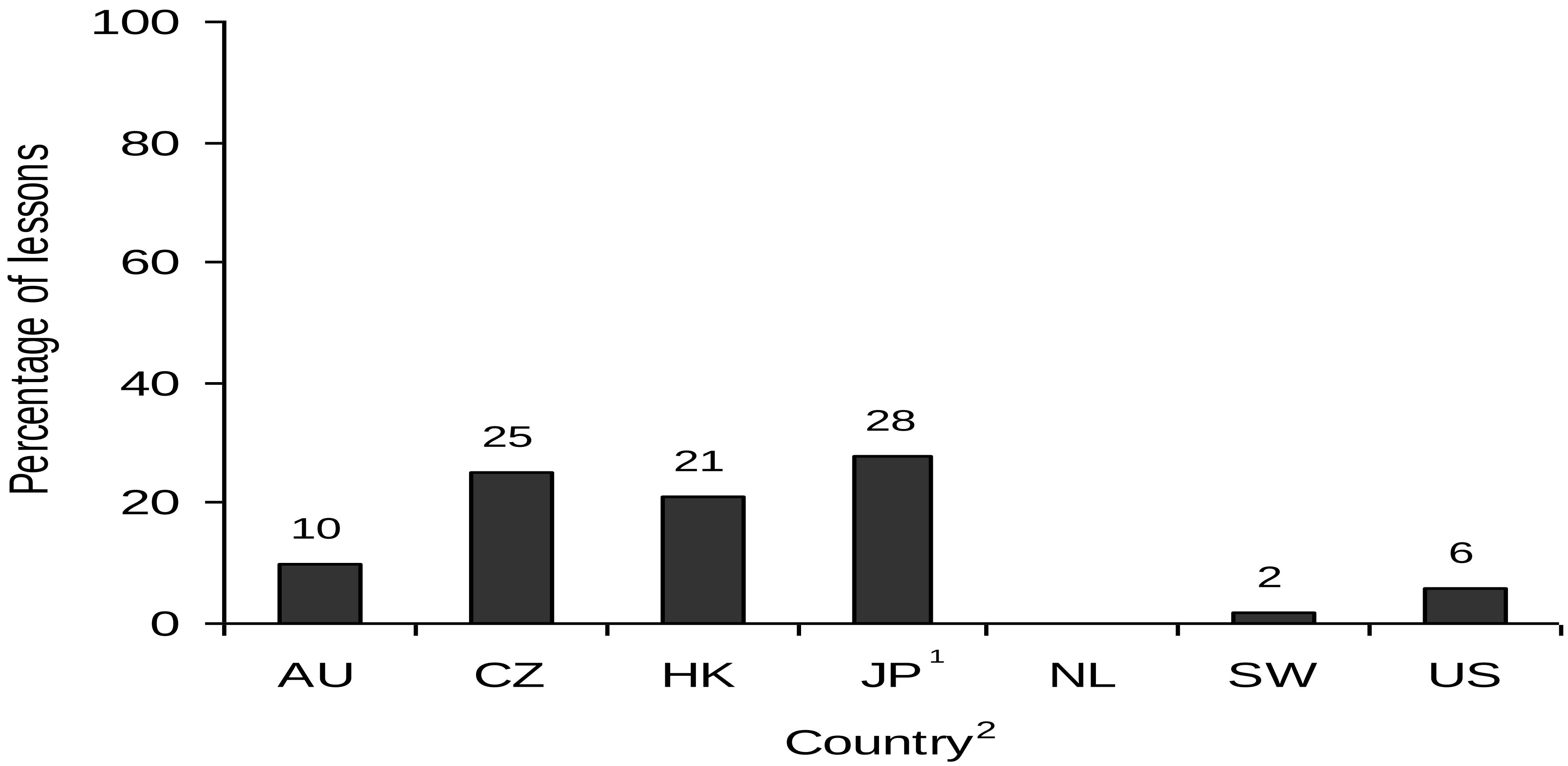
Lesson pattern

- U.S
  - Reviewing previous material
  - Demonstrating how to solve problems for the day
  - Practicing
  - Correcting seatwork and assigning homework
- Japan
  - Reviewing the previous lesson
  - Presenting the problem for the day
  - Students working individually or in groups
  - Discussing solution methods
  - Highlighting and summarizing the major points

Percentage of lessons in which teacher states the goal of the lesson (Hiebert et al., 2003)



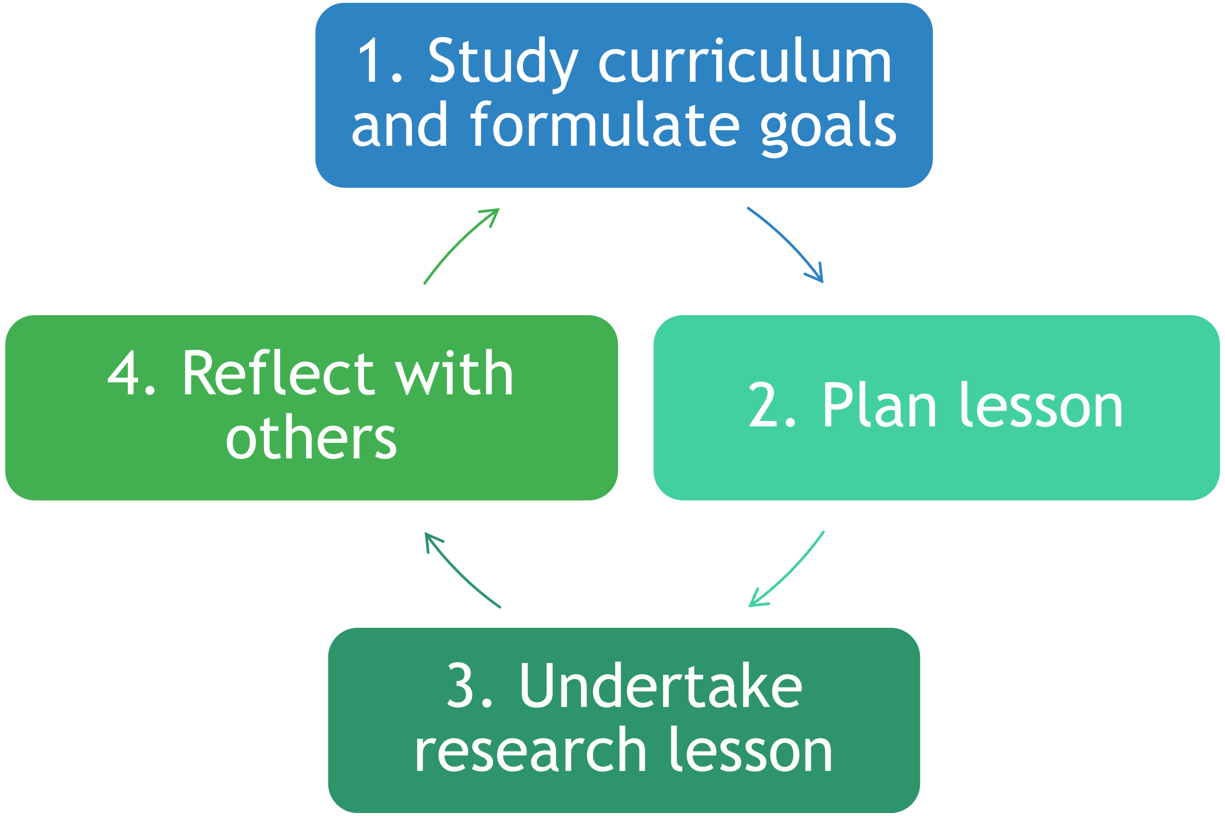
Percentage of lessons in which teacher summarized content of the lesson (Hiebert et al., 2003)



How do teachers access curricular information and incorporate it into their classroom teaching?

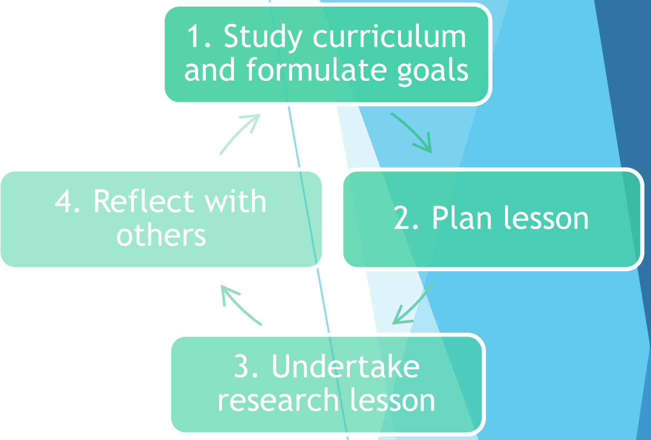
Some of the resources that mediate Intended Curriculum and Implemented Curriculum in Japan

The Lesson Study cycle (Fujii, 2015; Lewis & Hurd, 2011)



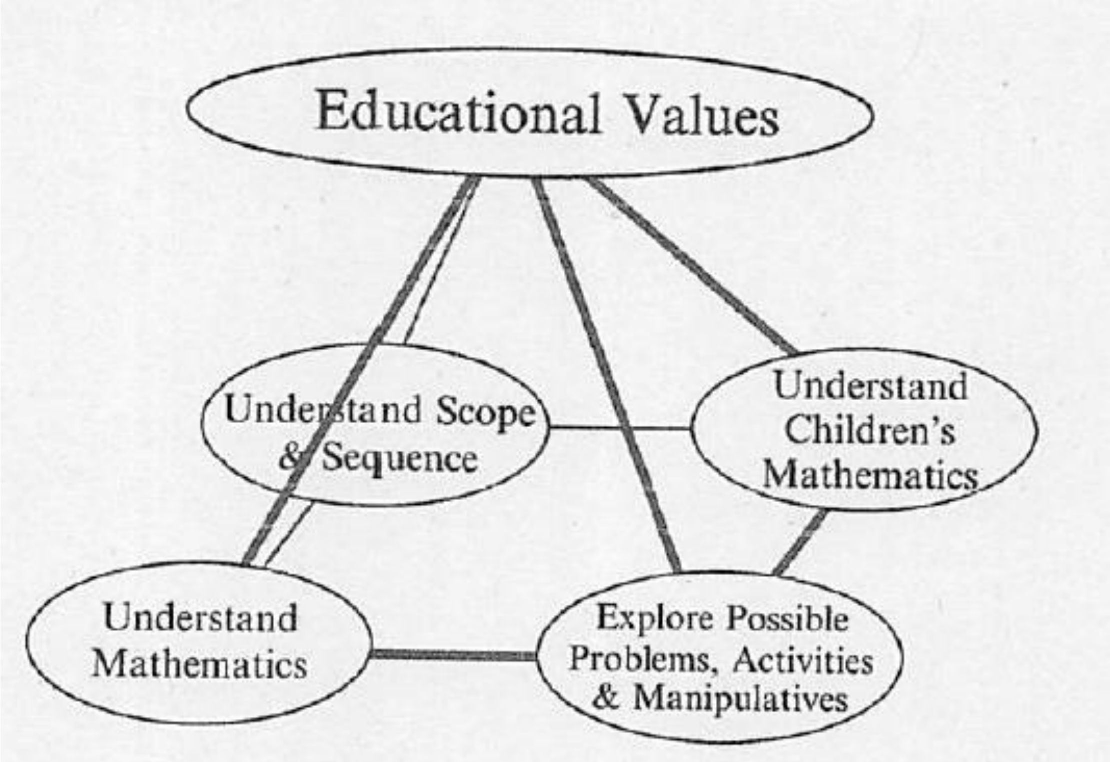
Teacher’s activity concerning goal of the lesson in lesson study

- Setting the long-term goal of lesson study (one whole year)
- Setting the goal of the lesson in lesson plans in relation to the long-term goal
- Setting tasks used in the lesson, sequence of activities, and/or method of evaluation: these are closely linked to the goal of the lesson
- Implementing the lesson: teacher’s actions and interactional moves toward the goal of the lesson
- Debriefing the lesson from the viewpoint of goal of the lesson
- Revising the lesson plan: modification of lesson and/or goal of the lesson
- Writing a report of the lesson study in one year



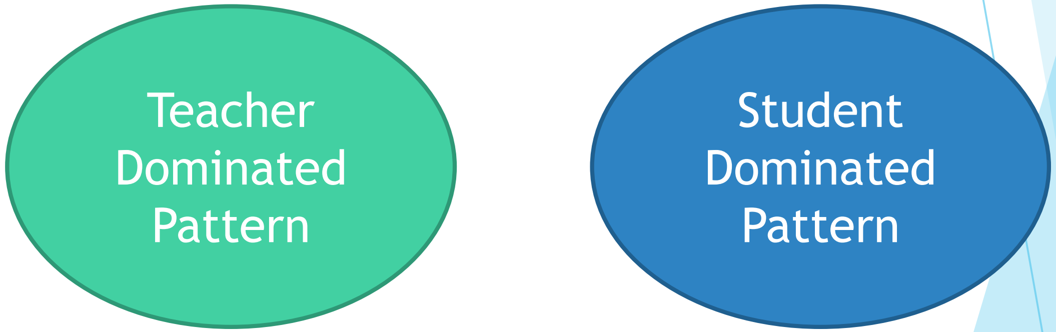
Goal and task design activity

- In Lesson Study, what kind of goal teachers set in the lesson is one of key issues. The quality of the goal is always examined in the lesson planning.
- Fujii (2015) states, “A common misconception … is that solving the task is the main point. Such misconception leads to a focus on goals such as ‘students can do X’ or ‘students understand X’. … It is well and good that students can do X, but X should contain some value, and what that value is needs to be considered” (pp. 284-285). He stresses the critical functioning of designing task and locates “educational value” at the top of the teachers’ activities.



Study on classroom interaction

- It has been shared that a high quality of discursive practice is essential to foster effective student learning. Some researchers have analyzed interlocking system of obligations and expectations in terms of interactive patterns in the classroom.
  - Funnel pattern (Bauersfeld, 1998)
  - Elicitation pattern (Voigt, 1985)
  - Focusing pattern (Wood, 1998)



- “A clear understanding of the connection between social interaction and children’s development of mathematical thinking is still not well understood” (Wood et al., 2006)

Study on classroom interaction: To balance the two patterns

- There is an advantage of interaction in which “children’s thinking was extended, pulled together, or strengthened by argument” (Wood et al., 2006, p. 248) .
- Lobato et al. (2013) states “... teachers can play an important role in directing students’ attention toward or (unintentionally) away from what is centrally important for students to notice for a given topic” (p. 845).
- Emergence of studies that address the dilemma of the teaching as telling and not-telling in mathematics education.
  - Beyond being told not to tell (Chazan & Ball, 1999)
    - Expand the definition of telling to include teaching actions, such as teacher’s rephrasing students’ comments for the whole class, or teacher’s inserting a new voice through questions and comments
  - A reformulation of telling as “initiating” (Lobato et al., 2005)
  - Teacher’s move to make the implicit mathematical practice explicit (Selling, 2016)
    - Eight types of teacher moves in the reprising talk turns, such as naming the mathematical practice in which students just engaged or highlighting aspects of student engagement in mathematical practices
  - Guided focusing pattern (Funahashi & Hino, 2014)

In sum,

- Teacher's goal of the lesson varies across countries. Whether teacher's makes explicit the goal in the lesson also varies. These observations suggest the intricate nature of the goal. Teacher's goal of the lesson affects both explicitly and implicitly his/her decisions in choosing mathematical tasks and organizing activities.
- In lesson study, importance of goal is well recognized and there are indeed numerous teacher activities with respect to the goal of the lesson. However, we need more studies that pay enough attention to the role of the goal in the actual classroom teaching and interaction with students.
- In the studies on classroom interaction, even though the researchers do not use the term "goal of the lesson," there is an emergence of perspectives that place key role of teacher in stimulating and moving forward students' mathematical thinking in the classroom. In these studies, teachers' intention is considered as an important feature of telling and interactional moves.

An analysis of classroom interaction: How students are channelized toward the goal of the lesson

Data source

- Two 5th grade lessons on comparing fractions, in a university-affiliated primary school in Tokyo, 2010. (Shimizu, 2011; Fujii, 2013).
- The lesson was conducted as part of the Learner's Perspective Study-Primary. The LPS-P collected data from the lessons and from interviews with the teacher and four focus students.
- The transcribed data were sectioned according to the framework of guided focusing pattern, and examined in terms of how the shared foci of class discussion were developed and shifted and what roles teacher play in making the shift of focus toward the goal of the lesson.

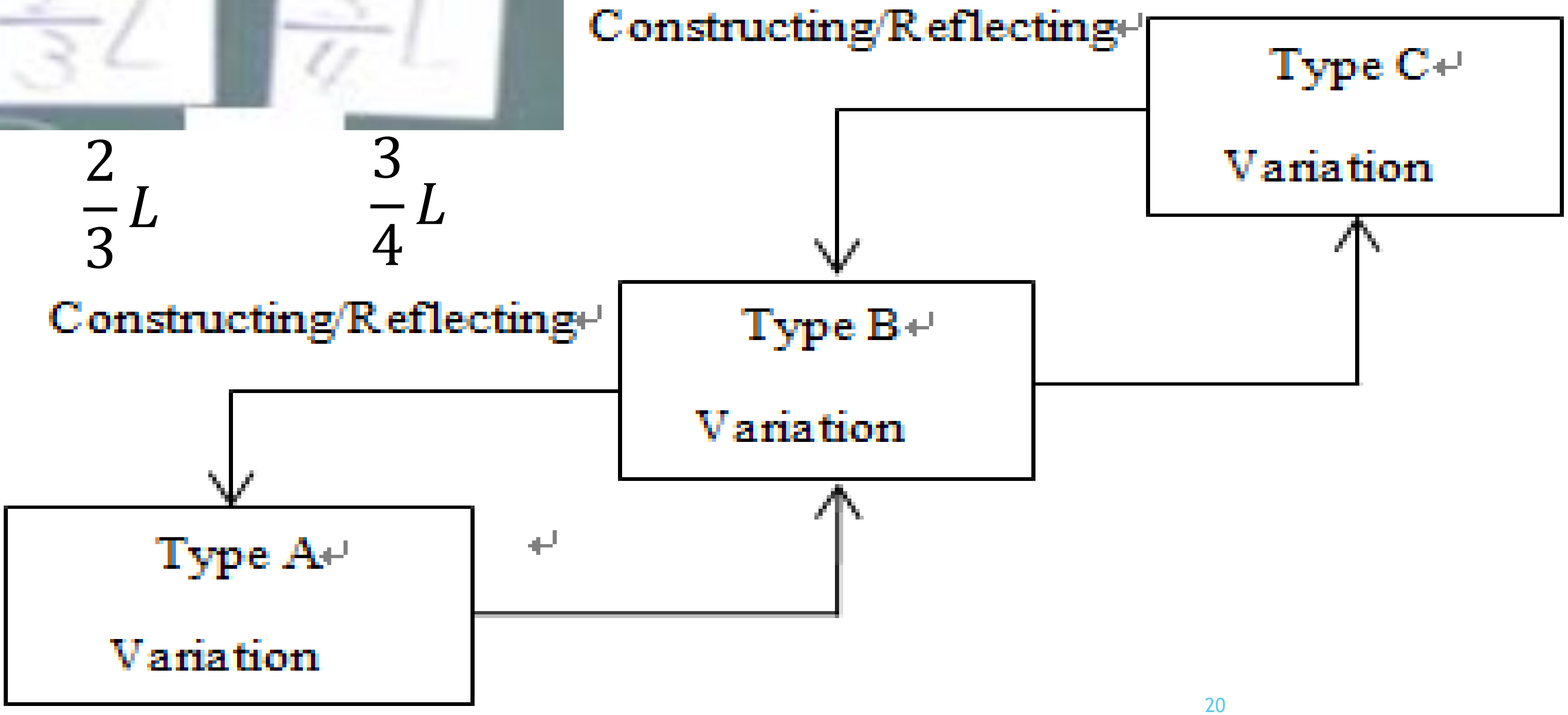
The teacher (Mr. Taka)'s goal of the lessons

- According to Mr. Taka, the goal of these two lessons were to understand that fractions can be compared if a common "unit fraction" is found and to understand the reasons for comparing fractions by finding a common denominator or numerator.
- In the teacher interview, Mr. Taka emphasized repeatedly the idea of finding a common unit fraction because once it is found, one can compare the fractions and add or subtract fractions in the same way as was previously learned with whole numbers.
- He said that these concepts connect with building students' understanding of fractions as numbers. In his teaching, Mr. Taka consistently focused on the "unit" or "unit fraction."

Presenting problems with variation

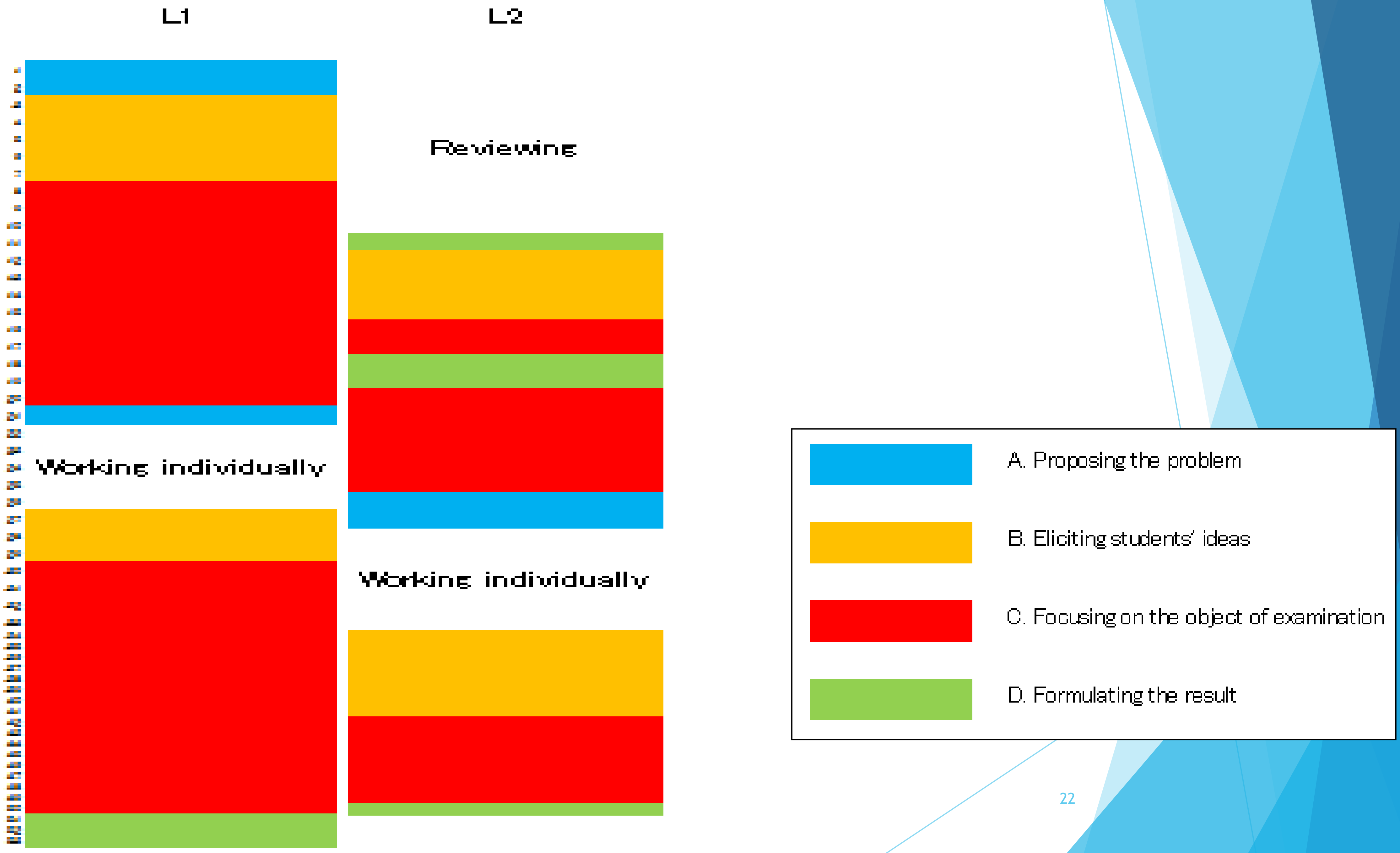
Which one is bigger?

$\frac{2}{4}L$ $\frac{2}{3}L$ $\frac{3}{4}L$



Guided focusing pattern (Funahashi & Hino, 2014)

Phase	Brief description
A. Proposing the problem	Posing the task for the day. Sharing approaches for exploring the task.
B. Eliciting children's ideas	Asking a question that may elicit multiple ideas from children. Accepting and/or rephrasing children's ideas.
C. Focusing on the object of examination	Focusing on the important idea that students proposed in phase B. If children do not spontaneously produce the idea expected by the teacher, the teacher facilitates or leads children to the idea or, eventually, provides it.
D. Formulating the result on the basis of the object	Formulating results and/or approaches as generally as possible, not being confined to the given problem.


L1
L2
Reviewing
Working individually
Working individually
A. Proposing the problem
B. Eliciting students' ideas
C. Focusing on the object of examination
D. Formulating the result

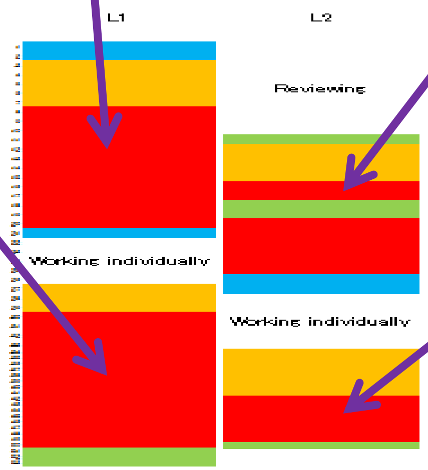
Focusing on the object of examination

Lesson 1 (2/4 < 3/4, 2/4 < 2/3)

- Explaining why 2/4 < 2/3 in terms of “area of one part”
- Examining a student’s method: Explaining why we need 12 to compare 2/4 and 2/3

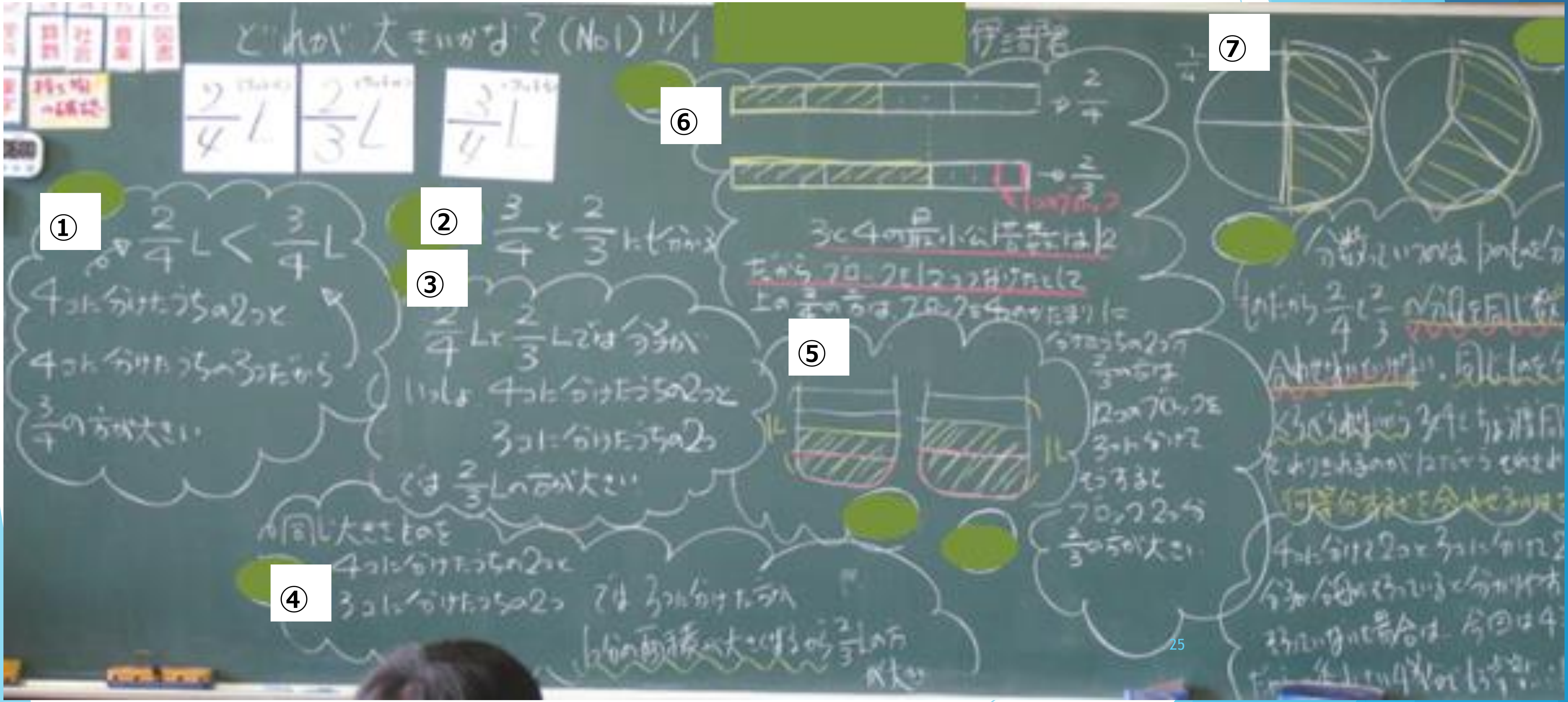
Lesson 2 (2/3 < 3/4)

- Examining a student’s method: Making connection with the previous methods
- Justifying a student’s method of finding common numerator



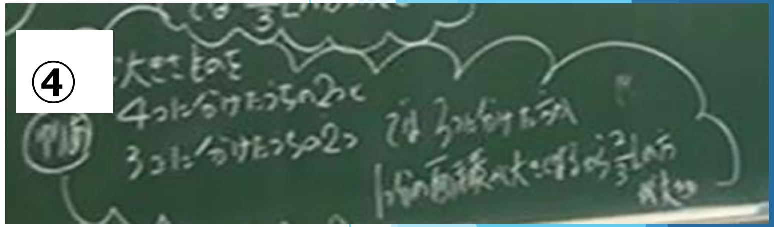
Focusing on the object of examination (Lesson 1)

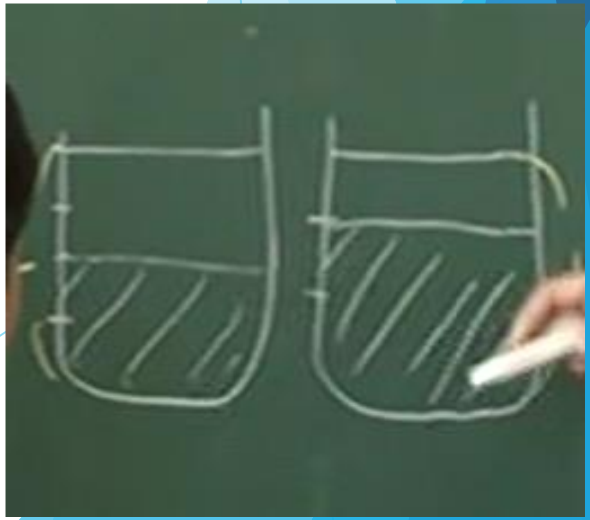
Blackboard Writing (Part of Lesson 1)



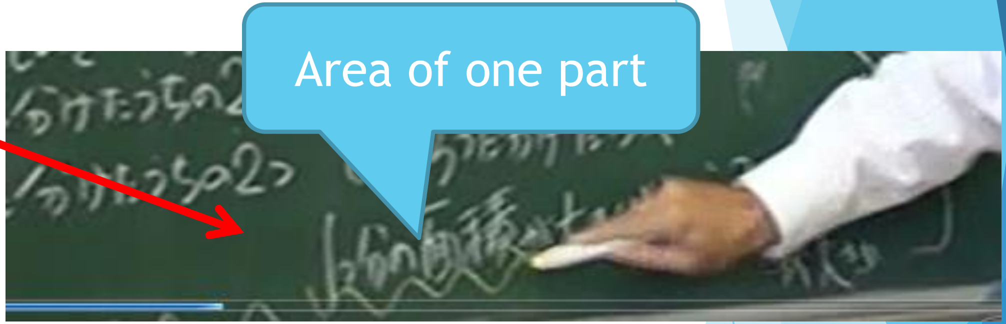
Explaining why 2/4 < 2/3 in terms of "area of one part"

- S1(④): The one that is divided into three [is larger], because the "area of one part" is larger, so we know 2/3L is larger.
- S2: [I can make it] in detail. Mine is in detail!
- T: Alright. So you can make what S1 said more detail, can't you?
- S2: Yes, yes. [He walks in front of the blackboard and draws a figure.] I used a figure. Two of four equal parts are this part, and … 2/3L means, well, divide this into three equal parts, and take two of them, they are here. [Mr. Taka added lines.] So it means that 2/3L is [larger] …
- T: What you are saying is the same as what S1 just said? I understand your explanation itself very well. Very good explanation.
- S: S2 said area. I think that is different [from what S1 said.]
- S2: I am not saying area.





- T: What S1 said was the same amount was divided by four, she divided it into four, and then take two of them. This is what S2 did. And the amount was divided by three and take …
- S1: S2’s figure is different.
- T: The figure is different? You [S2] said, take two of three, so this part is missing. [Mr. Taka draws an underline with a yellow chalk.]
- S2: What? Area? … [Looking at the underlined part], but this is “litter”.
- T: It says “area of one part.” [To S2,] What? Does it say about a litter?
- S3: No, it isn’t. “The area of one part” is this part. [She walks in front the blackboard and colored the 1/4 part and 1/3 part red.]



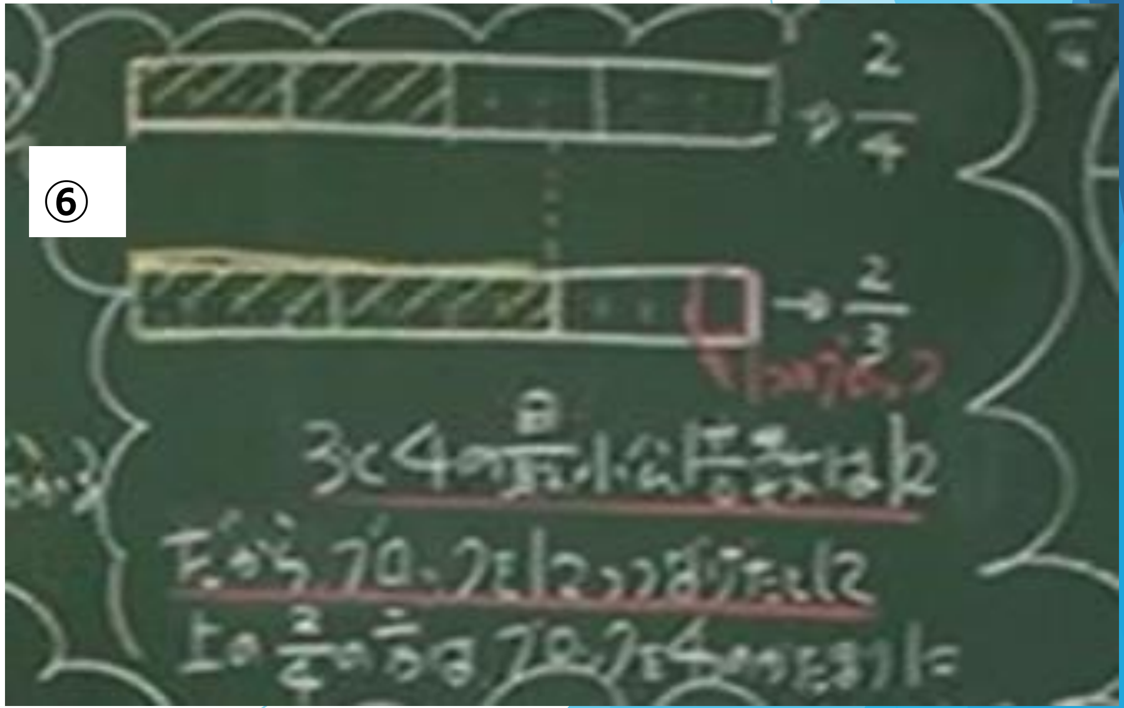
Teacher’s role

- The students’ various explanations were refined toward more viable understandings with a clearer focus on the idea of finding a common unit fraction, which was mediated by their informal language “area of one part.”
- The driving force of this progression was Mr. Taka’s question about a student’s explanation. He encouraged the students to give additional detail. Moreover, he directed students’ attention toward checking if their way of discussing something has logical consistency.
- Thus, noticing important mathematics in the student’s language, highlighting it on the blackboard, and requesting all students clearer explanations are all important component of reactions by Mr. Taka to guide students’ focus toward the goal of the lesson.

Explaining why we need 12 to compare 2/4 and 2/3

- S4: Mine is easier to know the difference. The least common multiple between 3 and 4 is 12. So, I divided a rectangle into 12. I connected 12 blocks. This is one block [pointing to 1/12 part]. [Mr. Taka marked it with red chalk and wrote “a block.”] For 2/4, I divided the blocks into 4 chunks, and 1, 2, well, I marked here [pointing at the area of 2/4]. [She explained 2/3 in the same way.] Then we know that 2/3 is larger by the difference of 2 blocks.

- T: I really don’t understand why this 12 comes out.
- S: What? Well… [Some students raise hands.]
- T: Some of you are raising hands. OK. So please write in your notebook the reason for 12 came out, so that everyone can understand easily. I will give you 5 or 6 minutes.



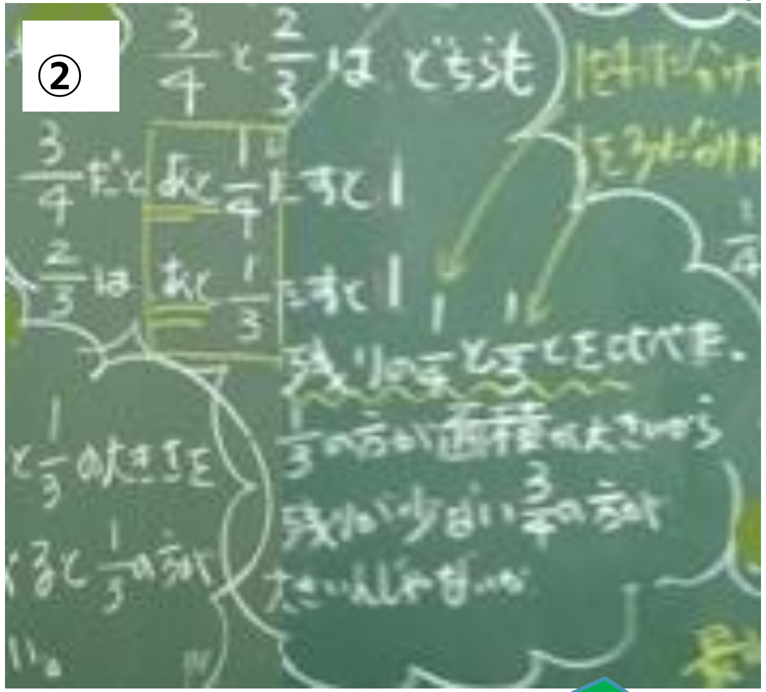
Teacher’s role

- By looking back a student’s explanation, Mr. Taka questioned her use of 12: “I really don’t understand why this 12 comes out.” Furthermore, he asked, “Why must you make the denominators the same?” and the discussion continued.
- In the teacher interview conducted after the lesson, Mr. Taka described his dissatisfaction with the students’ superficial understanding when they simply made calculations without thinking much about their meaning.
- In this way, Mr. Taka was consistently checking whether the students made sense of the idea of finding common unit fractions when comparing two fractions. He was especially sensitive to whether the students paid attention to meaning and quantity.

Focusing on the object of examination (Lesson 2)

T: Is this method different from the one S1 and S2 explained before?

T: Why can you compare these remainders, 1/4 and 1/3?



I think this has nothing to do with the previous ones. The other method is an easy one because either the numerator or denominator is the same, but this one is about the difference.

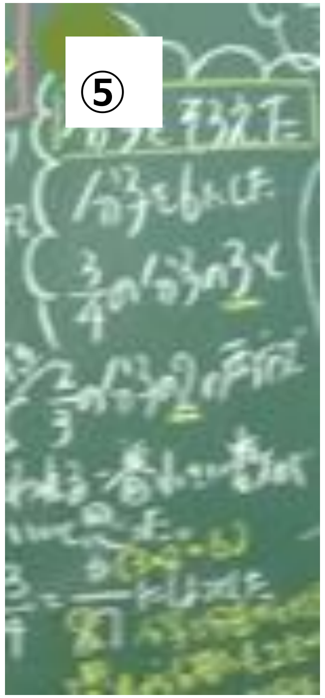
T: Yes, but after that, this method compares 1/4 and 1/3, doesn't it? [It compares] which result is larger. So it compares the remaining amount, 1/4 and 1/3.

I think it probably has something to do with... Making the denominator or numerator the same and add to make 1 and... well...

1/4 and 1/3 have the same numerator. So it has a relationship with the fact that we can compare fractions if either the numerator or the denominator is common.

I made the numerators same. I thought the smallest number that divide both 3 and 4 is the best one. To change 3/4 to 6/□, 3X2 is 6, and they are proportional, so 4X2 is 8. I got 6/8

⑤

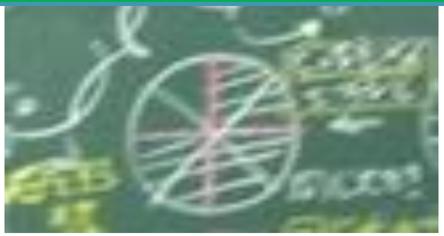


T: You thought since you multiplied by two for the numerator 3, you must multiply by two for the denominator. Why? You said “proportional.” Can someone say this part clearer so that everyone can understand? It is better if you draw figure.

I made the denominator same, instead of the numerator.

T: Wait! You are proposing different idea?

No, I am not. I want to talk about the table of proportional relationship. I only use the method of making the denominator the same.



Teacher's role when examining children's methods of why 2/3 < 4/3

- Mr. Taka intentionally attended students' non-standard methods as the object of examination, and guided them toward the goal of the lesson by asking for justification and refinement. He asked questions on the connections of their unique methods with the previously proposed method, which enabled the students to pay attention to the ideas they are learning.
- As in Lesson 1, he probed the students' use of term, such as "proportional relationship," only as a calculation and asked them to explain the meaning in this situation. He was also checking logical consistency in the discussion.
- In the interview, Mr. Taka said , "when I found that the student's response is incomplete, I thought, 'Oh, I am lucky,' because by elaborating it they can focus on the unit fraction." He also said, "when I walked around the desks I found a student who made numerators same. I though I would name her first. … I didn't want to elicit the method of making denominators same because it would direct the students' attention to calculation (the least common multiple)." Here, we can find Mr. Taka's intentions behind his instructional moves.

Summary: Teacher’s role for channelizing students toward the goal of the lesson

- Attending to and highlighting an important mathematical idea in students’ language, and furthermore, involving students in the activity of clarifying the idea/language
- Intentionally attending students’ non-standard methods as the object of examination, and guiding them, by asking for justification and refinement, toward the goal of the lesson
- Being alert in making connections with the previous solutions proposed by students; checking logical consistency in their discussion
- Consistently checking whether the students make sense of ideas; being sensitive to whether the students pay attention to quantity

Teacher’s role for channelizing students toward the goal of the lesson

Attending important mathematics

Highlighting

Naming

Collectively refining/Clarifying

Checking logical consistency in the discussion

Making connections

Making sense of idea and quantity

Conclusion

- In this presentation, I explored the importance of goals of the lesson by drawing on several studies. It was shown that even though goal of lesson is indispensable to conduct a lesson, setting the goal is not a trivial act but requires active and deliberate thinking for the teacher. Incorporating teacher's goal into the research framework of social interaction is a growing area, which also shows the intricate nature of such goal.
- In the latter part of this presentation, I analyzed two lessons taught by an experienced teacher who had a profound mathematical knowledge for teaching. An important observation of this case study is that the teacher had a clear goal of the lesson and moreover, a network of goals that cover the whole unit of study. Because of that, he can manage a variety of students' responses. He can highlight important mathematics and guided them to involve in the process of refining naïve method or language, which he claims is crucial in developing students' understanding.

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Thank you for your attention!