Mentoring Mathematics Projects

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HWA CHONG INSTITUTION
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Questions that may be in your mind ...

- What is a Mathematics Project?
- As a teacher, I have never done a Mathematics Project before, can I still mentor a Mathematics Project?
- How to Mentor a Mathematics Project?
- Can my students come up with a Mathematics Project?
A Mathematics Project is ...

→ **NOT** a compilation of cut and paste facts of certain Maths topics/concepts/ideas
→ **NOT** a collection of Maths formulae/quizzes
→ **NOT** a slide show about Mathematican(s) and his(their) achievements
A Mathematics Project can range from simple demonstrations of problems, techniques, principles, or well known results to exciting discoveries of new concepts or theorems.
In a Mathematics Project

- Maths is the king
- Evidence of reading of Mathematics
- Encapsulates mathematical thinking
- Written mathematically
- Have a complete solution to a small problem
- Problem should be extensible
Mathematics Project - RECIPE

(1) Introduction
(2) Literature Review
(3) 3 Research Problems
(4) Methodology
(5) Results
(6) Further Extensions
- As a teacher, I have never done a Mathematics Project before, can I still mentor a Mathematics Project?

YES, YOU CAN!
Give a man a fish and you feed him for a day. Teach a man to fish and you feed him for a lifetime.

Chinese Proverb
Fears are stories we tell ourselves
How to mentor a Mathematics Project?

(1) Where do I get good Mathematics Ideas for my students?

(2) How do I develop their Mathematics Idea into a Mathematics Project?
(1) Where do I get good Mathematics Ideas for my students?

Read
- Books
- Mathematics Magazines
- Websites

• Every new idea is inspired by an older one!
Books

*Solve this: math activities for students and clubs*  
By James Stuart Tanton

*Writing Math Research Papers: A Guide for Students and Instructors*  
By Robert Gerver
Mathematics Magazines

Plus Magazine
http://plus.maths.org/content/

NRICH Math
http://nrich.maths.org/frontpage
Math Website & Videos


Numberphile

https://www.youtube.com/user/numberphile
(2) How do I develop their Mathematics Idea into a Mathematics Project?

- Ask “What if” Questions
- Use “7 Basic Ways to Change the Problem”
- Use “Other Questions to Ask...”
Ask “What if”? 

Fibonacci Numbers:  
1, 1, 2, 3, 5, 8, 13 ...

What if we change the starting numbers? 
What if we add more than two terms at a time? 
What if we change the operation at each step? 

• Fibonacci at random 
  • http://www.sciencenews.org/pages/sn_arc99/6_12_99/bob1.htm 
• New Mathematical Constant Discovered 
  • http://www.maa.org/devlin/devlin_3_99.html
7 Basic Ways to Change the Problem

- Change the numbers.
- Change the geometry.
- Change the operation.
- Change the objects under study.
- Remove a condition, or add new conditions.
- Remove or add context.
- Repeat a process.

From “Making Mathematics: List of Mathematics Research Projects and Student Work”
A Carnival Game

Richard has devised a carnival game.

He has a large square board with 2500 one inch squares, in a 50 by 50 pattern.

The squares are red or black and are in a checkerboard pattern.

A person who wants to play the game tosses a dime on the board. If the dime lands totally within a red square, the player wins.

If it lands in a black square or overlaps several squares, the player loses the dime.

What is the player’s probability of winning?
Change the numbers

What if there were only 10000 squares?
What if the board contained infinitely many squares?
Change the geometry

Change the **shape**
What if the board used triangles? Hexagons?

Change the **dimension**
What if we change the size of the dime?
What if it were a $n$ by $n$ pattern?

Change the **location** of elements in a problem
What if the dime totally lands with a certain area of the board?
Remove a Condition or Add new ones

What if we add more colours to the board?

What if the board were three dimensional and the player threw a sphere instead of a circle?
Other questions to ask ...

What Is the Minimum Possible Value?
What is the Maximum Possible Value?
Can we Generalize the Problem?
Explain why that Pattern occur?
Permutable Primes

A **permutable prime** is a prime number can have its digits switched to any possible permutation and still spell a prime number.

1 digit examples are: 2, 3, 5, 7

2 digit examples are: 11, 13, 17, 31, 37, 71, 73, 79, 97

3 digit examples are: 113, 131 etc
Challenge

Can you find the rest of the 3 digit examples?
How about 4 digit, 5 digit?
How about $n$ digits?
Is there a pattern?
How many permutable primes are there? Make a guess.
How do you know if you are right? Can you prove it?
Research Problems
(from project “Permutable Primes (2008-2009)”) 

Objectives:

To develop an efficient algorithm for the search of permutable primes.

To find out the existence of permutable primes with $n$ digits for each integer $n$

- If permutable primes exist for all $n$, then prove it.
- If permutable primes only exist for certain $n$, then find these specific $n$.

To check the existence of permutable primes
SET Game

Game introduced to students when teaching Set Theory
Set!
Problems with the SET Game

Unable to obtain set of 3 cards within 12 cards
Unable to form exact sets of 3 within 81 cards
Research Problems
(from “Discrete Mathematics in the game of Set (2006-2007)”)

1. How many Sets under the first definition are there in the deck such that \( r \) attributes have different values and \( A-r \) attributes have the same value?

2. What is the total number of Sets in the deck?

3. What is the probability that in a Set under the first definition, \( r \) attributes are different and \( A-r \) attributes are the same?

4. What is the maximum number of cards in the general game not containing a Set?
Mentoring, to me ...

Give the students the opportunity to think
Believe and Support the students’ willingness to learn more about Mathematics
Challenge students in informal situations

Be a Learner all over again
Go through the Research Process
Patience
Hard Work
Frustration ...
Mathematics Research Process

1. A Question
2. Understand the Question
3. Generate Examples and Make Observations
4. State Conjectures
5. Create Representations and Look for Connections
6. Ask More Questions
7. Test Further
8. A counterexample is found. Try to reformulate the conjecture
9. Reconsider
10. Prove Either Your Claim or Smaller Results that Support it
11. Extend Your Work with Related Questions

Education Development Center, Inc. 2000
Passion: Origami

Conducted Math and Origami course in HCI

Link topics like Pythagoras Theorem, Geometrical Concepts, Symmetry with Modular Origami
Research Objectives
(from “PHiZZy Donuts (2006-2007)”) 

To gain a better understanding of topological surfaces e.g. torus

To Investigate how the construction of topological surfaces can be achieved through modular origami
Products
(from “PHiZZy Donuts (2006-2007)”)
Products
(from “PHiZZy Donuts (2006-2007)”)

Genus 2 Torus
<table>
<thead>
<tr>
<th>Research Objectives</th>
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<tr>
<td><em>(from “PHiZZy Cola (2007-2008)”)</em></td>
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<td>Create a Klein Bottle</td>
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<td>To make use of origami PHiZZZ units</td>
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<td>Refine Design Process</td>
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<td>To gain a greater understanding of topological</td>
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<td>concepts via the entire process and explain</td>
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<td>Understand Topological Concepts</td>
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Products (from “PHiZZy Cola (2007-2008)”)
When the tough gets going..

Many Brains make Light Work

Seek Expert Help

I’m sorry ...