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The Practice of Information Technology in Mathematics Education: A Critical Look

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Abstract: Over the past two-plus decades there have been numerous publications asserting the virtues of information technology in mathematics teaching and learning. Despite these claims a broad look at technology supported mathematics classroom practice suggests that implementation is not always smooth and results may not match intentions. This article, using the context of Canadian mathematics classrooms, explores the consequences of mandated ICT use, unanticipated outcomes when calculators and mathematics software are employed, opportunities for expanding student experience, shifting images of mathematics, and the university's role as a model for ICT use in mathematics research and education and in the preparation of future teachers.

Introduction

> I have suggested that the absence of a suitable technology has been a principle cause of the past stagnation of thinking about education. The emergence first of large computers and now of the microcomputer has removed this cause of stagnation. (Papert, 1980, p. 186)

In the two-and-a-half decades since the publishing of Papert's *Mindstorms: Children, Computers, and Powerful Ideas*, computers have become a common physical presence in the classrooms of the developed world (OECD Directorate for Education, 2004; Mullis, Martin, Gonzalez, & Chrostowski, 2004). The initial excitement, and in some cases trepidation, that met their arrival appears to have subsided. The computer's slide into the background of schooling and the relatively high comfort level with respect to information technology felt by mathematics education researchers has resulted in a recent academic literature that focuses on small case studies of information technology use. Not surprisingly, these projects, involving enthusiastic researchers and teachers and careful programming, yield mainly positive results. Present critiques of computer use in mathematics teaching and learning come from economists and policy analysts (Angrist & Lavy, 2002; Fuchs & Wößmann, 2005) and not from the mathematics education community.

When data gathering shifts from counting computers to reporting actual usage in mathematics lessons a somewhat different picture emerges. Information technology has become pervasive in our daily lives, but computer and calculator use in mathematics lessons is much less frequent (Mullis et al., 2004). The impact of computers has not reached the levels imagined in the early 1980s. Papert, along with his prediction, provided a caveat.

> The computer by itself cannot change the existing institutional assumptions that separate scientist from educator, technologist from humanist. Nor can it change assumptions about whether science for the people is a matter of packaging and delivery or a proper area of serious research. To do any of these things will require deliberate action of a kind that could, in principle, have happened in the past, before computers existed. But it did not happen. (Papert, 1980, p. 189)

It would appear that Papert provided an appropriate warning. Significant assumptions and issues concerning technology and mathematics teaching and learning continue to remain open for examination.

Wishing to ground discussion on real classroom experience, in this paper the conversation concerning selected issues is initiated by looking at examples of student use of computers and graphing calculators in the learning of mathematics. Each example surfaces one or more issues which are then isolated and explored.

Persuasion versus Compulsion

Over the past two decades, professional associations for mathematics teachers have issued documents recommending the use of computers and calculators in teaching and learning. In some cases official curriculum documents produced by ministries or departments of education have weakly echoed this call: "Alternative strategies for teaching and learning mathematics using the computer should be explored" (Ontario Ministry of Education, 1985, p. 20). Recently a number of state agencies responsible for education have issued curriculum guidelines that are more directive concerning the use of information technology (Mullis et al., 2004). For instance, Ontario's new mathematics curriculum for Grades 9 and 10 (Ontario Ministry of Education, 1999) contains 76 references to the classroom application of information technology, including many that make it clear that computer and calculator utilities must be employed. For example, in Grade 10, students are to "determine some properties of similar triangles (e.g., the correspondence and equality of angles, the ratio of corresponding sides) through investigation, using dynamic geometry software" (Ontario Ministry of Education, 1999, p. 31). Thus it appears that a change from "recommending exploration" to "must be employed" has occurred. In Ontario these curriculum demands have been accompanied by increased support in the form of funding for the purchase of graphing calculators and provincial licenses for computer software. Further, teachers are definitely making increased efforts to employ technology (Lock, 2001). But, given the compulsory adoption requirements, do the increased funding and teachers' increased efforts always lead to productive classroom experiences?

Pursuing the Doable: Meeting the challenges

Teachers clearly need support to understand the potential impact of technology in their classrooms and how to go beyond rather trivial applications to meet curriculum dictates. Once an official curriculum demands the use of information technology there arises a need to provide support sufficient to ensure that all teachers can and will take the necessary implementation steps. This can lead to "cookbook" style directions for teachers and students with accompanying truncated learning opportunities.

In Ontario, Grade 10 students, while studying quadratic functions, are expected to "collect data that may be represented by quadratic functions, from secondary sources (e.g., the Internet, Statistics Canada) ... fit the equation of a quadratic function to a scatter plot ... and compare the results with the equation of a curve of best fit produced by using graphing calculators or graphing software" (Ontario Ministry of Education, 1999, p. 26). The Fathom software[1] (Key Curriculum Press, 2002), provided for all schools in the province, makes this task relatively simple and the instructional resource, *Parabola Power* (Professional Development Center, Key Curriculum Press, 2001), provides teachers and students with detailed instructions. Students are directed to capture the data for 1983-1999 youth crime using the Statistics Canada E-Stat facility and to produce a scatter plot (Figure 1).

The scatter plot does have a somewhat parabolic pattern and the activity instructions continue with: "The number of youth crimes goes up and comes back down in a roughly symmetrical way. One way to quantify this variation is to fit a parabola through the points" (Professional Development Center, Key Curriculum Press, 2001, p. 3). Following this advice students plot and adjust a parabola and find the quadratic function of best fit. But, why a quadratic model? Is there an underlying reason to postulate a second order relationship between year and the number of youth crimes? The activity instructions do not raise such questions. Students are not asked to explore and critique this model. Posing the simple question, "What was the state of youth crime prior to 1983?", leads to an obvious problem. Expanding the Year and Number of Charges limits of the graphing window shows that prior to 1978 the number of youth crimes should have been negative (Figure 2). Similarly, in about 2007 a negative number of youth crimes can be predicted. What interpretation is to be given to negative counts of crime?

[1]For readers not familiar with the software noted in this paper, the reference section provides bibliographic details including URLs from which additional information or evaluation versions may be obtained.

Youth Crimes

	Year	NumberOfCharges
1	1983	80525
2	1984	69448
3	1985	98271
4	1986	113027
5	1987	111731
6	1988	114079
7	1989	120277
8	1990	131155
9	1991	146906
10	1992	140378
11	1993	132983
12	1994	127095
13	1995	128809
14	1996	128542
15	1997	120208
16	1998	117542
17	1999	111474

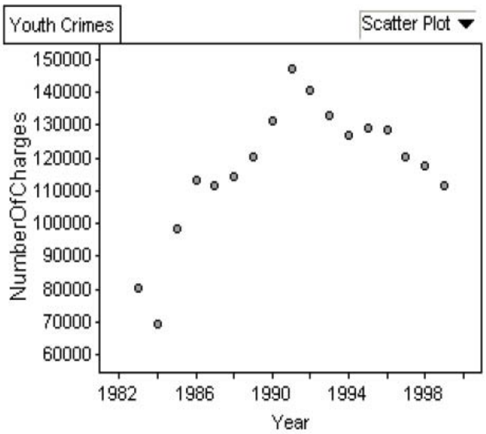


Figure1. Youth Crime versus Year Scatter plot

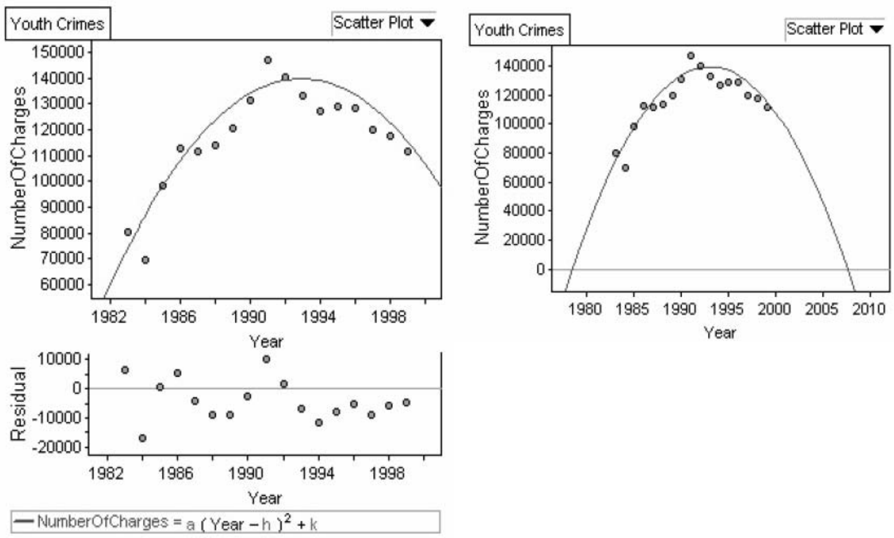


Figure 2. Youth Crime versus Year Parabolic Model

Adopting Technology: Not Always a Simple Step

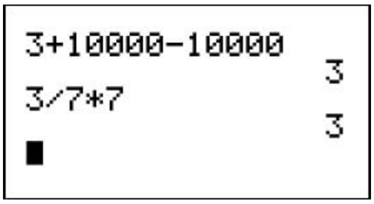
Teachers are encouraged to allow students to use calculators whenever the focus of study is problem solving rather than arithmetic skills. “The computational capacity of technological tools extends the range of problems accessible to students and also enables them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modeling” (National Council of Teachers of Mathematics, 2000, p. 25). The foregoing is true, but calculator use can also raise issues that require time-consuming detours.

Present primary-grade instruction in arithmetic emphasizes the inverse nature of the pairs of operations: addition and subtraction, and multiplication and division. Students know that the operations

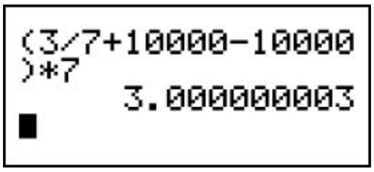
$$3 + 10000 - 10000 \text{ and}$$

$$3 \div 7 \times 7$$

must both yield answers of 3, and on a calculator such as the TI-83 Plus these results hold.



But, if these operations are combined as shown on the image of the calculator screen below an unanticipated result occurs.



Calculators can record only a finite number of decimal places and thus values such as $\frac{3}{7}$ can not be stored exactly. The floating point arithmetic employed by calculators introduces rounding errors that produce the results above.

In most applications such slightly inaccurate answers pose few problems, but when first observed by pupils’ questions that should not be ignored arise. At some point the class needs to investigate the calculator's decimal representation and accuracy and discuss the source of errors.

Experimental Mathematics

Benoit Mandelbrot (1992), one of the founders of the new mathematics of fractal geometry, called for an experimental approach to mathematics. He argued that, with the aid of computers, mathematicians should be exploring and testing new theories rather than focusing on carefully constructed rigorous proofs. In Mandelbrot's view, the "compulsive housecleaning" of formal work should give way to more exciting and constructive "house building". Students generally appreciate this point of view. Establishing results via repeated testing appears to be easier than constructing proofs, but such an approach can be problematic. An example provided by Muller (1989) illustrates this point.

When students are asked to determine the limit of the sequence recursively defined by

$$t_{n+2} = 111 - \frac{1130}{t_{n+1}} + \frac{3000}{t_n t_{n+1}} \text{ with } t_0 = \frac{11}{2} \text{ and } t_1 = \frac{61}{11}$$

there are two possible approaches: develop a closed form for the sequence and take the limit, or calculate a large number of terms of the sequence and look for a trend. With computer algebra systems the second approach is relatively easy. If the first 40 terms of the sequence are generated employing the RECURRENCE utility in Derive (Texas Instruments, 2002) along with 'Simplify' using 20 decimal place accuracy, the follow output is obtained.

PrecisionDigits := 20

$$\text{RECURRENCE}\left(111 - \frac{1130}{t_2} + \frac{3000}{t_1 \cdot t_2},\ t,\ \left[\frac{11}{2}, \frac{61}{11}\right],\ 40\right)$$

```
[5.5454545454545454545, 5.5, 5.5901639344262295081, 3.1194348173820314582,
  80.895815606907297443, -239.35682024223658322, 96.876480997321883726,
  115.59160817047541863, 99.603564585698901345, 101.48477122669736956,
  99.951811924912529124, 100.16107783156264557, 99.994214067246320485,
  100.01770739758246868, 99.999310394709141965, 100.00194953735188097,
  99.999918294356522186, 100.00021469004037508, 99.999990368302609426,
  100.00002364474877329, 99.999998869578948610, 100.00000260431301247,
  99.999999867840744019, 100.00000028687090166, 99.999999984601869119,
  100.00000003160199348, 99.999999998211399622, 100.00000000348158508,
  99.999999999792809203, 100.00000000038297435, 99.999999999976060089,
  100.00000000004212717, 100, 100, 100, 100, 100, 100, 100, 100]
```

The results certainly provide evidence that the limit of the sequence is 100. If the calculation accuracy is increased to 100 digits these last terms are seen to be marginally different from 100.

```
99.99999999997239602568050489915968922298770626142720128894077763255565150275 13~
151394859372854849387,

100.00000000004657692750006235465606624144852284303449400166988208875708911790 7~
132445633855438226348,

99.999999999968238320423552706769694046138820831749637559100874788284313577648~
697337623461103890858,

100.00000000000513299052887955415147278490128743136548078594238934960951723168 5~
287355335817343315601,

99.99999999999963522255307244003246055285875759074470385602009550825525637010 20~
177228893786519551483,

100.000000000000565723290517530733111520244864571540121953723497258093432320925~
194403207900638789773,

99.999999999999958177310966415797075481048714582307849762388186392091782568393 1~
065938489998940505073,

100.00000000000006235503002402909798059543022979400891662261969857127272365187 5~
432430471353950945908]
```

But, when one continues to evaluate terms with this accuracy, the value 100 re-appears at t_{116}. Such an experiment in Derive or other computer algebra system is likely to convince students that the limit is in fact 100. Switching to 'exact' calculation mode, where rational fractions rather than decimals are employed, gives a different picture. If the calculations are allowed to run for many hours the terms can be seen to be converging on 6. In fact, it can be shown by mathematical induction that the sequence can be expressed in closed form by

$$t_n = \frac{5^{n+1} + 6^{n+1}}{5^n + 6^n}$$

and 6 is the limit of this expression.

Technology supported experimental mathematics can hold traps, and when the underlying mathematical processes are hidden from view these may be missed. Students and teachers need to be aware of these potential events and treat displayed results with some skepticism.

Building on Strange Events

Although inaccurate or questionable results obtained from calculators and computers can sometimes lead students astray, they can also, when noted, be starting points for productive mathematical explorations. When completing TI Interactive! (Texas Instruments, 2003) supported activities exploring the graphs of power [$y = x^a$] and exponential [$y = a^x$] functions, some class members decided to combine these expressions and graph $y = x^x$. The resulting curve seemed to be in conflict with some of the observations made earlier (Figure 3).

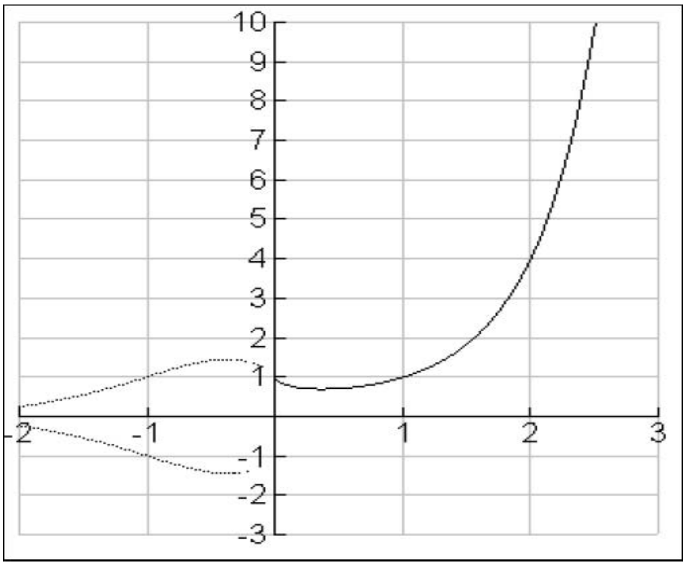


Figure 3. TI Interactive! graph of $y = x^x$

Students had noted that they could not obtain a curve for $y = (-1/2)^x$ since there were infinitely many values of x for which the expression was undefined, but here it appears that there is a point plotted for $x = -1/2$. In fact, there are two points plotted, one vertically above the other, which contradicts the definition of a function. Fortunately the pupils conducting this investigation recognized the potential problems and their questions initiated a new class exploration - How is TI Interactive! generating the plotted points and is the approach correct?

Checking TI Interactive! calculations confirmed that in fact the software did know that $(-½)^{(-½)}$ is not a real number, and zooming in on the graph for the domain -0.75 to 0 showed that the curve was plotted as discontinuous (Figure 4). This led to a

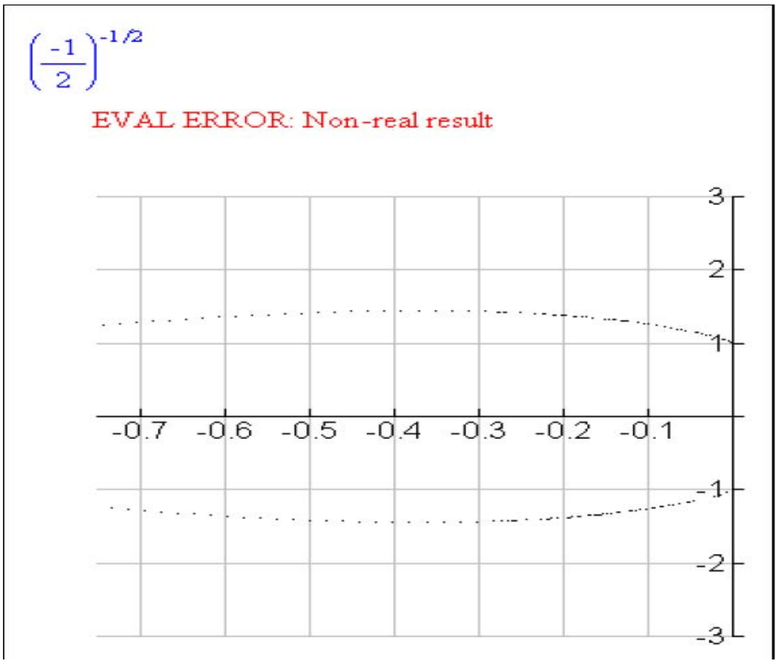


Figure 4.

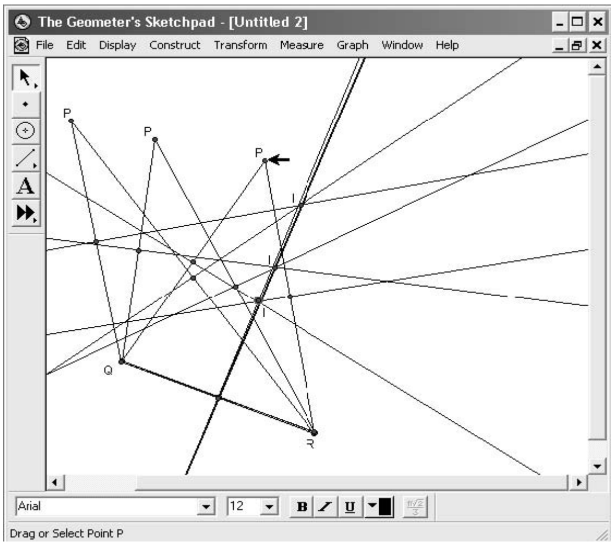
discussion concerning the sizes of the sets of negative *x*-values for which the *y*-value is undefined, positive, or negative - a natural first look at issues concerning the cardinality of infinite sets. Looking at the distributions of these sets along the negative *x* axis suggested that it might not be strictly correct to show the curve as a collection of distinct points. Checking with some other computer algebra systems and graphing utilities showed that most plotted no points for $y = x^x$ for $x<0$. On the other hand, GrafEQ (Pedagoguery Software, 2004) gives a plot that shows two smooth continuous branches in the second and third quadrants and the software developers claim that this is a solution superior to that generated by other graphing packages (Pedagoguery Software, 2005). Such “confusion” is not really problematic for it makes clear to students that mathematics is not a collection of well defined truths - debates still exist.

Mathematics software that supports exploration can be described as empty technology (Zucchermaglio, 1993) in that it does not contain course content nor provide instruction. But, such tools are not neutral with respect to the discipline. They do paint particular pictures of mathematics as a subject.

Changing Images of Proof

As well as expanding the range of mathematical applications, computers are altering the nature of the discipline itself. The computer's ability to rapidly test many examples of a conjecture challenges the strength of deductive proof as the sole arbiter of mathematical 'truth' (Horgan, 1993). This certainly appears to be the case when dynamic geometry software [Cabri Geometry (Laborde & Bellemain, 2004), the Geometer's Sketchpad (Key Curriculum Press, 2001)] is employed during the study of Euclidean geometry, the traditional location of pupils' introduction to formal deductive reasoning.

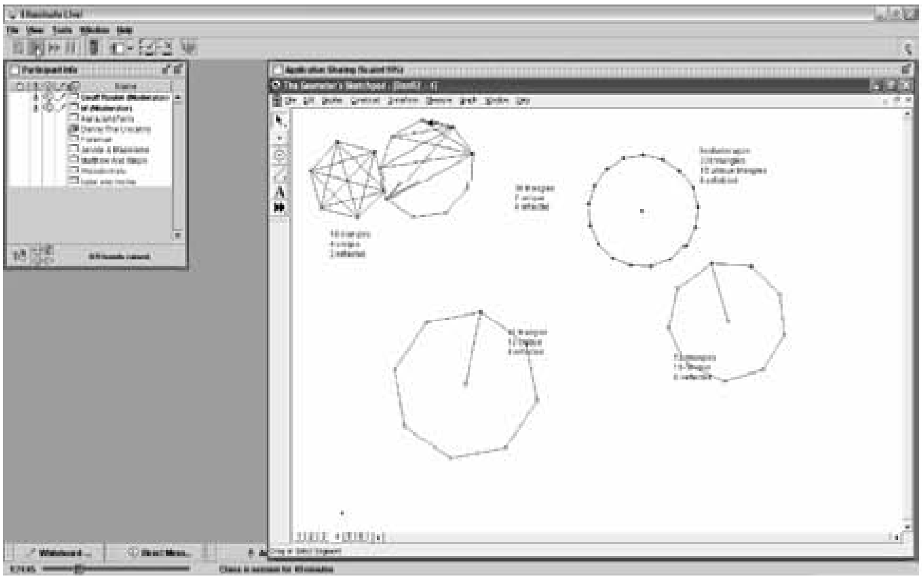
Dynamic geometry software provides the ability to grab and drag geometric objects while maintaining constructed geometric relationships. This presents teachers and students with a powerful tool for exploring, discovering, and demonstrating geometric principles. Students, if asked to construct a triangle and the three perpendicular bisectors of the sides, will observe that the bisectors are concurrent. Grabbing and dragging the vertices to create a range of triangles will go on to show that this concurrency is always present.

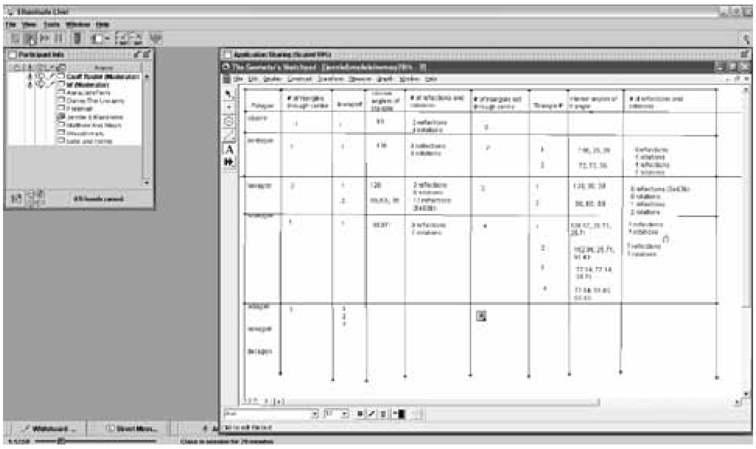


An important geometric property has been discovered, but problems arise when the teacher now asks students to prove that the perpendicular bisectors of the sides of a triangle always pass through a single point. The class already knows that this is true for all triangles. They have observed it for an infinite number of examples as they moved the triangle's vertices. Why is there any need to construct a deductive proof?

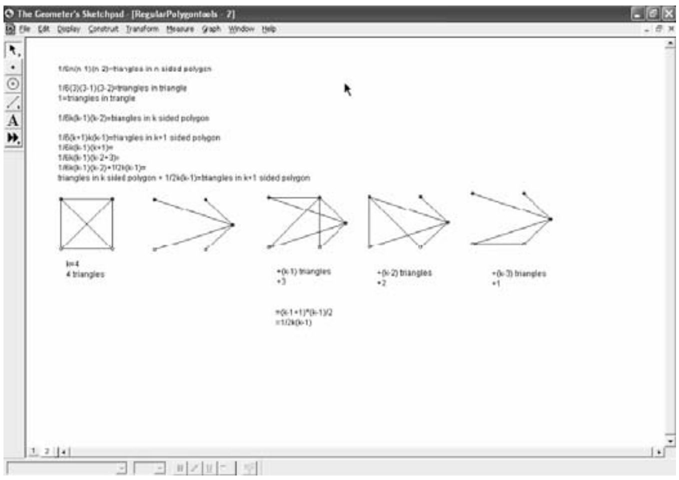
On the other hand, technology supported geometry investigations that leave questions open can motivate the study of traditional proof methods. In a recent study (Roulet, Mackrell, Taylor, & Farahani, 2004), students in a senior Geometry and Discrete Mathematics course (Ontario Ministry of Education, 2000) employed The Geometer's Sketchpad to explore the question, "How many different triangles can you make by joining only the vertex points of a polygon?". A regular polygon construction tool, provided for the class, and The Geometer's Sketchpad transform utilities helped student pairs to categorize triangles and organize their data for particular polygons. But, technology, in the form of dynamic geometry software, could provide no further support in this exploration. Individual polygon cases could be adjusted, but there was no way to dynamically move from an n-gon to (n+1)-gon.

In addition to The Geometer's Sketchpad the project employed Elluminate Live!, web-based software that supports desktop sharing. Student pairs, when demonstrating their work and sharing conjectures, could display their constructions on the screens of all computers in the room. Beyond this, if any other students had suggestions to offer, control could be passed to them so that they could manipulate the original sketch. With this visual communication channel in place, the potential for collaborative learning was greatly increased. Patterns in the triangle counts and conjectures for the number in the general n-gon case were shared.





Complexities in the sketches and problems with counting led to a variety of conjectured patterns and general formulas. Sharing of these results along with reasons in support of individual hypotheses highlighted the conflicting views and produced the uncertainty that can motivate efforts to construct a proof (Hadas, Hershkowitz, & Schwarz, 2000). Some pairs of students, seeking to strengthen their arguments, resorted to careful checking of previous cases and production of additional data. Others, without teacher prompting, decided to explore a general case and worked on the construction of a proof. For this they reached back to the discrete mathematics section of the course and developed a sophisticated proof by mathematical induction.



Mathematics, Pedagogy and Technology: Balance in Teacher Education

Beginning secondary school mathematics teachers need technology supported teaching and learning experiences, but these are difficult to provide within the limited time available in teacher education programs. In Canada most faculties of education include a sampling of technology applications within their mathematics instructional methods course, possibly augmented by an optional course dealing with information technology and education in general.

The issue of classroom use of information technology is connected to all other aspects of curriculum: course content, student learning activities, teacher planning, and classroom management. Workshops that focus on accessing software and calculator features do not provide models of effective classroom applications. Working through typical secondary school technology supported activities is a step in the right direction, but still does not provide new mathematics teachers with authentic investigation experiences. There is a need for mathematical exploration activities that present challenges to beginning mathematics teachers and at the same time illustrate and address parallel curriculum issues. Queen's University has for some years run a "problem of the week" activity for mathematics teacher candidates. Recently the Problem-of-the-Week committee has been posing questions that invite technology application and, increasingly, participants have been submitting problem solutions in the form of electronic output from computer software presently available in Ontario schools. Hopefully, through these technology-supported investigations, teacher candidates will develop the skills and more importantly the inclination to bring computers and calculators into their future classrooms.

One major exception to the blended ‘curriculum methods - technology applications’ approach occurs at l’Université du Québec à Montréal (UQAM), where students preparing to be secondary school mathematics teachers take four full-semester courses focusing on information technology applications in their subject. Here beginning teachers explore the features of mathematical tools such as graphing calculators, spreadsheets, graphing utilities, dynamic geometry software, statistics packages, and computer algebra systems. The focus is on both the software functions and the roles such resources can play in students’ learning. In addition, the teacher candidates learn how to employ information technology tools to aid them in the development of materials for student use. Activities involve the production of mathematical text using word processors, 3D dynamic graphics to illustrate mathematical concepts, a web site for student support, and utilities programmed in BASIC. Here the opportunities to explore and learn about information technology are significantly greater than in most other teacher preparation programs. Other faculties may wish to consider something parallel to the UQAM program and can obtain additional information from the web site for the Baccalauréat en enseignement secondaire, Faculté des sciences de l'éducation, UQAM. (http://www.regis.uqam.ca /prod/owa/pkg_wpub.affiche_prog_desc?P_prog=7951).

Arithmetic and Algebra Skills: University Expectations and Models

Discussions concerning the balance between procedural and conceptual knowledge in mathematics have been on-going since at least the 1970s when Skemp (1976) made the distinction between relational and instrumental understanding. More recently, the controversy has subsided, with new research showing the mutual support and value of the two forms of knowing (Hiebert, 1986). Today, with the increasing use of information technology, in particular graphing calculators and computer algebra systems at the secondary school level, a new aspect has been added to the debate. There are now calls from tertiary education levels for the maintenance of traditional paper-and-pencil skills (Herget, Heugl, Kutzler & Lehmann, 2000).

Further, university faculties are sending out mixed messages concerning their acceptance of information tools in the teaching and learning of mathematics. Some (Schramm, 1998) eagerly embrace the new tools in their teaching and do not fear the loss of manual skills. They support parallel technology-supported teaching and learning in the schools. Others (Klein, 1999) suggest that, even in the presence of information technology tools, students should be required to learn and demonstrate traditional pencil-and-paper skills. They argue that one can not fully understand mathematical operations without having performed them by hand. This inconsistency in messages from the tertiary level presents problems for school teachers conscientiously attempting to prepare their students for future education and careers.

In Ontario, new curriculum guidelines (Ontario Ministry of Education, 1999, 2000) state that, "The development of sophisticated yet easily used calculators and computers is changing the role of procedure and technique in mathematics" (p. 3, 5). Students in graduation-year, university-preparation courses are required to use graphing calculators. One year later, some of these students are enrolled in first year university programs where they meet course outlines stating, "You may use only non-programmable, non-graphing calculators for the tests and the final examination. We reserve the right to disallow any calculator" (Carleton University, School of Mathematics and Statistics, 2004). Secondary school teachers, knowing that their students will meet such restrictions, are reluctant to make technology a core aspect of their programs.

Other institutions deliver messages that are more encouraging. Visitors to the web-site maintained by the University of Western Ontario, Department of Applied Mathematics read that "Computers play an important role in the research activity of the Department" (2004) and are informed that high powered Hewlett Packard calculators with computer algebra systems are required for completion of course work. University faculty who value the use of technology in mathematics study need to ensure that their views are known at the high school level and must encourage their university colleagues to acknowledge the ICT experience and skills of newly arrived undergraduate students.

Looking to the Future

With the growing presence of computers in students' lives outside the school and the increasing application of information technology in pupils' future careers we can expect mounting pressures to employ technology in our mathematics classrooms. Research conducted within carefully crafted pilot programs shows that ICT applications hold significant potential for increasing student understanding. But, we need to keep in mind that ICT use also presents traps and problems. Each implementation needs to be considered carefully.

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