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Mathematics Education Research: Designing, Implementing and Concluding

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Abstract: Mathematics education, as a field of study, comprises numerous and diverse research traditions including those of philosophy, sociology, psychology, history, and, of course, mathematics. The nature of mathematics educational research, as a result, is therefore complex. What questions are asked and what constitutes "fact" are aspects of this research. To study the appropriateness of questions and fact, Uprichard (1982) proposed a hierarchy of levels of educational research ranging from theory construction to validation. In this editorial and commentary, his levels in the context of current research initiatives such as action research are revisited, with a view to determining characteristics required for appropriate matches between research methods and the nature of research questions posed.

A Field of Study

Academically, education is a field of study that examines issues related to teaching and learning, curriculum and schooling, as well as, for example, political and economic components. Each of these areas of focus necessarily is influenced by the disciplines which comprise this field – disciplines of psychology, sociology, philosophy, history and of the teaching-subject components themselves such as mathematics. Educational research is therefore complex given the collection of diverse research traditions inherent to the component disciplines.

With respect to this complexity, in a 'forum for researchers' paper related to quality issues in mathematics education research, Simon (2004) stressed that all fields of study within education from time to time need to reconsider their standards for research quality. In mathematics education over the last 20 years, he noted that "sweeping changes in the acceptance and subsequent predominance of qualitative research" (p.157) have occurred and indicated that "with the challenges from outside, particularly in political arenas", much discussion is now crucial to ensure growth and strength of the field. Further, the strengthening of research in mathematics education rests not on an acceptance of a set of criteria, but rather on a dynamic and ongoing discussion of what constitutes this research quality. Thus, in this paper I intend to examine selected relevant concerns in the context of a hierarchy of levels of research proposed by Uprichard (1982).

Preliminary comments

Many believe that educational research has to date produced little dependable research that is of use to guide school policy (Hirsch, 2002). Hirsch indicated that

> despite the high claims being made for statistical techniques like regression analysis, or experimental techniques like random assignment of students into experimental and control groups, classroom-based research has not been able to rid itself of uncontrolled influences that have made it thus impossible to tease out the relative contributions of the various factors that have led to "statistically significant" results (p.51).

Hirsch (2002) reasoned that we might reduce some of this uncertainly by "placing less reliance on traditional educational research that makes inferences from school data and applies those inferences back to schools" (p.55). He argued in favour of cognitive-science approaches, in contrast to school-based research, where consensus arises not just from classroom educational research but principally from laboratory studies and theoretical considerations unconnected to the classroom. "Classroom based research rarely converges on a consensus view" (p.63). Whether one entirely agrees or not with Hirsch's views, his comments may be taken as challenge for us to re-examine how we design our research, how we choose and implement our various methods, and how we make responsible conclusions.

When discussing changes to the way we consider mathematics education research Shulman (1988) reminded us of a quote from Hamlet: "Though this be madness, yet there is method in it" (p.3). He suggested that to characterize something or assert that something has "method", is to maintain that there is an order or regularity to it. This quote is a useful analogy for us in that when we examine educational-research methodology (or, more to the point, methodologies) we must recognize the contextual implications of the recommendations. Research is not merely about observation and speculation, but also about method. Within the family of methods that is basic to forms of educational research is the central concept of disciplined inquiry (Shulman, p.3). Disciplined inquiry may be formal, or in the early stages perhaps more free-wheeling. Of importance, however, is that there must be responsible attention given to the nature of the inquiry and hence to the selection of method and thus to the selection of facts, formulation of questions, and principles of discovery and verification.

Analyses of Dimensions of Educational Research

Objective versus Subjective

Burrell and Morgan's (1979) classic text on social paradigms and organizational analysis begins with a discussion of two conceptions of social reality: the established *traditional* view which has as its primary purpose one of discovering natural or universal laws (essentially a view parallel with that of natural science) and an *interpretive* view which argues that people differ from naturally occurring phenomenon. To address this dichotomy in more detail, Burrell and Morgan further

analyzed this objective-subjective dimension into four sub-dimensions or assumptions – ontological, concerned with the nature of the phenomenon (dependent or independent existence); epistemological, addressing whether or not knowledge is real or subjective; human nature, characterized by determinism versus free will; and methodological assumptions relating to measurement of universal laws (nomothetic) versus explanation and understanding of individuals (idiographic). A summary of these sub-dimensions is given in Figure 1.

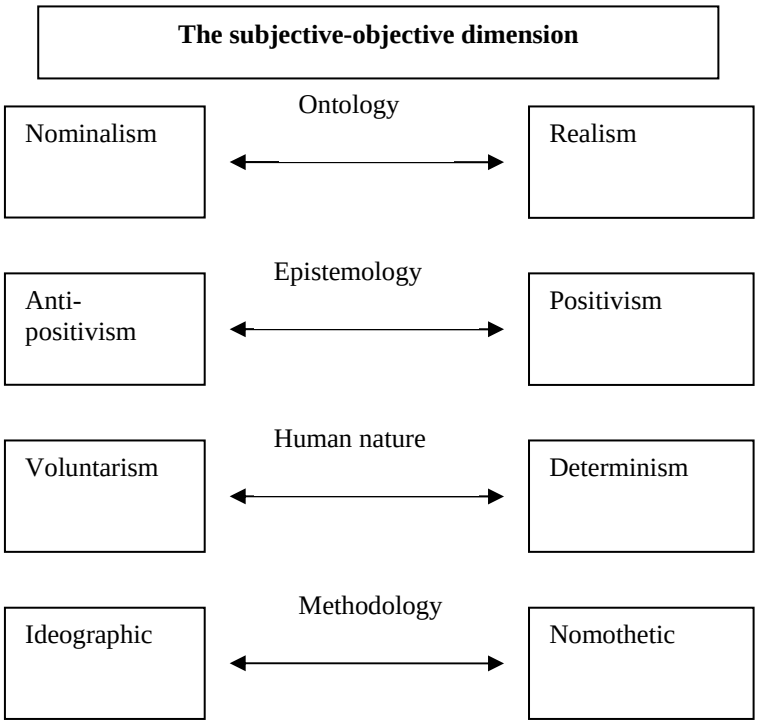

Figure 1. Assumptions about the nature of social science (Burrell & Morgan, 1979).

Burrell and Morgan's (1979) discussion and subsequent analysis of these sub-dimensions resulted in a description of three principal paradigms of research: normative, interpretive, and critical theory. The *normative* (or *positivist)* paradigm

holds that knowledge is based on experience and is advanced by means of observation and experimentation. The assumptions about the nature of science result in empirical processes that involve experience, classification, quantification, discovery of relationships, and approximations of truth (Burrell & Morgan, p.8). The tools of science include concept and hypothesis formations. The i*nterpretive* (or a*nti-positivist)* paradigm on the other hand holds that understanding of the world must be individualistic and subjective. Meaning and interpretation are two key purposes of this paradigm and phenomenology and ethnography are two principal methodologies employed by such researchers.

A third category, *critical theory*, generally has as its focus political and ideological issues. Its purpose is not only to understand situations and phenomena but also to cause some sort of change, often of an emancipation or equalitarian nature (Burrell & Morgan, p.29). The "Mathematics for All" research of Kilpatrick, Swafford, and Findell (2002) is an example of work that perhaps relates closely to critical theory conceptualizations.

Of interest, Kilpatrick, in a 'reflecting on criteria for research in mathematics education' seminar presented in Singapore in February 2002, indicated that approaches to mathematics education research have now become multi-dimensional. Paralleling the earlier views of Burrell and Morgan (1979), Kilpatrick (2002) labeled these approaches as behaviourist, interpretist, and critical-theorist.

Methodologies and Methods

Typical forms of analyses of educational research list and discuss characteristics of a selection of methodologies and methods employed by social science researchers. Cohen, Manion, and Morrison's (2000) book, now in its fifth edition, is an exemplary example of these texts. In establishing the context of educational research, Cohen et al. initially distinguish between *methodology* with its aim to assist in understanding, describing, analyzing, noting limitations, and so on, and *method* with its range of techniques and procedures to be used in the data-processing aspects of research. Although they highlight similarities among common components within forms of educational-research methodologies such as the need to provide clear statements of purpose, need to operationalize these purposes, and need to address principles of research design (p.38), they also stress that social scientists have now come to abandon the spurious choice between qualitative and quantitative data collection:

> The social scientists are concerned rather with that combination of both qualitative and qualitative data which makes use of the most valuable

> features of each. The problem becomes one of determining at which points they should adopt the one, and at which point the other (p.45).

From their analysis, what is clear is that although educational research may take various forms, this research must be systematic, and must demonstrate scholarly application of disciplined inquiry. Once preliminary decisions have been made with respect to the nature and purpose of the research and thus perhaps to which questions to ask, and so on, the researcher must then and only then consider what kind or style of research to undertake; that is, which model will best operationalize the research. Typical models include experimental designs, survey, ethnography, case study, history, action research, and testing and assessment (Cohen et al., p.80).

When the research requires data collection, the specific instruments researchers adopt for the data-collection component of these models are necessarily varied and include interviews, questionnaires, systematic observation, biographies, and narrative inquiry. As each of the models involves certain requirements in terms of process, similarly each of the instruments require application of certain techniques perhaps relating to discourse, account, or statistical analysis. That is, the operational decision-making process occurs at three levels: model determination, instrument selection, and data-analysis technique choices. Of course, these models, instruments and techniques are often themselves subject to detailed study. One may refer to books, for example, on single-subject research (a model), on participant-observer activity (an "instrument"), and on factor analysis (a technique).

Levels of Research

Consistent with the analysis of Burrell and Morgan in 1979, Edward Uprichard (1982) observed that educational research was shifting from a "one-method orientation" to a "problem orientation", one that employs many methodologies. The one-method orientation referred to the then dominant empirical (positivist) approaches most closely associated with psychological research. Universities were only just beginning to introduce courses labeled, for example, qualitative research methodologies. His purpose at that time was to stimulate discussion about appropriate ways to study the teaching / learning process. He argued that this process was *dynamic* in that it cannot be easily controlled; was *complex* in that it was affected by an array of variables; and was *extensive* in time in that cognitive and social changes occur over periods of months and years, not days nor weeks (p.1). To respond to this shift in orientation, and to help organize his own thinking, Uprichard developed a taxonomy of research methodologies that "relates these methods in a logical manner" (p.2). He proposed a paradigm that began with an articulation of a theoretical rationale moving progressively through stages involving exploration and inquiry, ultimately to validation by statistical techniques. For a

complete listing of his paradigm see Figure 2. In the figure note that at each level Uprichard specifies both the purpose of research at that level and a likely associated methodology.

	Level	**Purpose**	**Methodology/ Method**
1	Theory	Identification and articulation of theoretical base or rationale	Analytic-synthetic review
2	Reality	Application of theory to real world	Ethnographic study
3	Clarification	Formulation of general questions related theory	Non-structured review
4	Refinement	Development of specific hypothesis	Structured interview
5	Intervention	Initial testing of instructional hypotheses	Teaching experiment
6	Validation	Confirmation or negation of statistical hypotheses	Group experimental designs

Figure 2. Purposes and research methods for levels of the paradigm (Uprichard, 1982)

In light of comments by Hirsh (2002), Simon (2004) and Kilpatrick (2002) relating to the move toward using multi-dimensional approaches when planning educational research, it may be appropriate to revisit Uprichard's levels with a view to considering the relevancy of his paradigm and the matches between research methods and, for example, the nature of the research questions posed. Certainly I am not advocating that today we accept and use Uprichard's levels as he wrote and presented them. They were written for a 1982 audience of mathematics education researchers. Rather, I am suggesting that we consider his work from an historical perspective and from a more general perspective where educational research can be thought of in terms of levels with characteristic objectives and methods associated with each of these levels. One point concerning the paradigm is important to note. Uprichard (1982) stated that the levels in the paradigm represent only a general hierarchy. Ideally a researcher would move through the paradigm systematically over a number of years pursuing a particular line of research. Realistically, he added however, that entry level into the paradigm would be contingent on several factors including previous research or knowledge.

Theory, level 1 of the paradigm, dominates the research from planning and establishing of the rationale stages through interpretation of results (Uprichard, 1982, p.3). Without theory, the research lacks direction. Whether one chooses to study mathematical literacy issues, or the teaching of statistics to pupils in primary schools, one must begin with a thorough understanding of the existing literature.

The purpose of the reality level, level 2, is to contrast theory with (classroom) reality, and typically involves an ethnographic methodology to examine the construct both in its setting and from the subjects' perspectives (p.5). According to Uprichard, observing teachers in their natural setting is thus essential.

Having read (level 1) and made some observations (level 2), the researcher then seeks to clarify and explain apparent agreements or discrepancies that exist between the theory and the reality (level 3). During this clarification level researchers may use non-structured, focused, interviews to ascertain subjects' interpretations, beliefs, and so on, in the context of their personal experiences. "Unstructured yet focused" implies that an interview protocol would be developed by the researcher but the interviewer would deviate from this protocol as the need and opportunity arose. At this level, general questions are prepared with respect to the relationship between theory and observation.

The goal for level 4, refinement, is to focus on and develop these questions to the point where hypotheses or specific research questions may ultimately be formulated (p.9). Often questionnaire or structured interview techniques are developed and little deviation from the protocol is anticipated or even permitted.

At the intervention level, level 5, there is an attempt at manipulation of variables and generally requires some form of teaching experiment. The experiments are frequently small scale and specific with respect to both sample and instructional process. At this level the researcher is likely to be studying how, for example, children learn. Subjects are often used as their own "control group" as performance is judged or compared to some initial reading of performance (p.10). Critically, results of this level of research are situation specific and are not generalizable.

Level 6, validation, as one might expect, is focused on the "logic of proof" (p.11). Statistical techniques to confirm or reject hypotheses, using group experimental designs characterized by the control of independent variables, are the norm. Notions of bias, sampling and random assignment are all key components of research designs at this level. The following is an example of a typical 'validation' question: "Does the subtractive method or distributive method of long division result in

consistently higher scores for students working long division examples with more than a one-digit divisor?"

Discussion

In selected comments about the paradigm, Uprichard (1982) wrote that "no single level is more important or subservient to another" (p.12). This point is particularly worthy of note. One level of research does not have more status than another. Each may be placed on a continuum of quantitative to qualitative, or subjective to objective, but not so with respect to rigour. That is, each level is characterized by 'disciplined inquiry' - by formulation of purpose, methodology, and reflective interpretation. Uprichard (1982) also questioned whether it was appropriate or not to publish research findings from levels other than level 6. He provided a tentative sounding answer suggesting that if the research at any level was well done then it should be worthy of publication (p.13). At any level, so long as a researcher understands the nature of the research, what kinds of questions may be asked, what kind of methodologies are suitable, what methods may be employed, and what kinds of results and generalizations are permitted, then sharing this new "knowledge" is responsible research dissemination.

In this paper I have been discussing levels of research explicitly highlighting notions and purposes of hierarchy. A few related comments however may be appropriate. First, a technical reading of Uprichard's levels of research may leave one with the impression that if one is conducting research at say level 5, then it is no longer necessary to do an extensive reading and reporting of the relevant and related literature. Such a reading would be unfortunate in that the discussion of Uprichard's levels has not been to imply that one must follow these levels on a step-by-step basis but rather to show that these levels provide a reasonable set of guidelines within which to analyze relevant research where relevancy implies contextualization, justification, validity and usefulness. It is expected that people begin research by selecting a topic, doing some preliminary reading and observation, posing questions, and then and then only, opting for particular methodologies and methods. Validity refers to the conclusions of the study and how these conclusions build bridges from the original literature discussion to the study's interpretations and generalizations (Kilpatrick, 2002).

On several occasions I have met people who state they wish to do, for example, narrative-inquiry research. I then ask them about the purpose of their study. They answer that they haven't yet decided! To my mind these individuals do not recognize that method results from a (clear) understanding of purpose. I say 'clear' parenthetically because I do understand that some good research begins with only general notions of direction and that with some reading and perhaps some action-

based experience that method gets developed and adapted as the research becomes more focused. Nevertheless, perhaps some individuals are math-phobic and thus wish to shy away from statistical analysis. They do not appear to understand, or are ignoring, a key element of the research process. Selection of method follows from and corresponds to the kind of research questions posed.

Second, recognizing that the application of concepts of bias, sampling, formation of control groups, and so on, are primarily characteristic of the validation level research (level 6) should also now be more apparent. Although some of these characteristics may be applied at earlier levels, even implemented, that they are not all implemented would suggest a weakness in an experimental research design and thus likely would relegate the research to quasi-experimental status. The implication is that research conducted at levels 1 to 5 is no longer "validation" in the sense that Uprichard (1982) used the term. This does not mean that such research is not valuable. It does mean that the results are not generalizable.

Similar comments regarding "action research" may be considered. For example, some action research has as its purpose to "solve" problems in specific (classroom) situations. The intention is that the results do not need to be generalizable. The four part cyclical process of planning, action, observation and evaluation, and reflection may all be specific to addressing a particular problem. The focus of the research in this case is thus at level 5. Specific adherence to bias, control group, and sampling is not expected. Whether or not these constructs are discussed by the researcher as limitations to the study is normally left to the discretion of that researcher.

It is also possible to use "action research" as an example where strict adherence to Uprichard's taxonomy may not be appropriate. Some action research may actually start with the recognition of classroom-based problems (level 2) and informal subsequent attempts to address those problems (level 3). Planned interventions may then be undertaken and evaluated (level 5). Eventually the researcher / teacher may decide to do some reading and reflection (level 1) before embarking again at some higher level in the taxonomy.

Third, it is important to distinguish between a report and a research study. A well-crafted and well-written report may share some of the characteristics of a research study, but it is the "research study" that provides the interpretation of the report. The report exists separately from the study (Kilpatrick, 2002). Reports may be data based, or may take other forms such as presenting a narrative, or describing a set of classroom interactions. The data or the stories by themselves however do not constitute research. They need to be contextualized, rationalized, and interpreted. Relating a narrative, for example, without the analysis may be somewhat akin to

telling a joke without the punch line. Note also, although rigorous criteria may be applied to the reporting and discussion of the research, the fault may be with the original report.

Conclusion

Educational research can be characterized as a collection of diverse research traditions perhaps extending from and relating to its component disciplines. Within these traditions, levels of research exist that progress from theoretical and observational stages to validation ones. As well, each of these levels is distinguished by methodology, method and technique. Each level has its own form of thoroughness and thus, although these research traditions may seem particularly varied, each level has characteristic features of disciplined inquiry.

Whether attending to large-scale research initiatives perhaps extending to several years duration, or whether, developing a relatively small-scale modest study, the mathematics education focus must be worthwhile, must have a set of rationalized research questions and associated methods, must be implemented with high levels of ethical standards, and must conclude with significant levels of reflection.

Admittedly much of what has been written and discussed in this article may be said to apply to educational research in general and is not specific to mathematics educational research in particular. In one sense that would be a fair statement. However, the issues considered in the article do apply to mathematics education research as well. Uprichard's, Simon's, and Kilpatrick's comments are specific to their observations and understanding of what has and is happening in mathematics education research. Hence, with respect to this journal, *The Mathematics Educator*, and requests for research contributions, my hope is that readers and potential authors will have not only a better understanding of the current state of mathematics education research but also a better appreciation of our stated purpose: to provide publication opportunities for disciplined inquiry into 'trends, issues, and developments in curriculum and instruction in mathematics education' and that these contributions may include 'for example, experimental, case study, survey, historical or philosophical studies'.

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