The Mathematics Educator
2003, Vol.7, No.1, 80-95

Using Anchors to Revise Probability Misconceptions

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Abstract: *Prevalent in everyday non-technical use, probability is highly prone to misconceptions which have been the object of much research. One way to deal with these misconceptions is through anchoring situations, as advocated by Clement (1987) and Fast (1997). This paper uses a similar framework, with two versions of a questionnaire and interviews conducted with college science majors. Misconception-prone situations were given in Version A of the questionnaire, and possible anchors in Version B. Results showed that although erroneous notions existed, anchors could be effectively used to overcome them.*

Introduction

Probability is double-edged. On one hand, it is undeniably close to everyday experience, where terms such as "randomness" and "chance" are used by lay people to justify one action over another. Elementary probability ideas appear in economic forecasts, weather prediction, sports analysis, media commentary, among others. On the other hand, everyday probability concepts are used loosely, and in the process, lend themselves to fallacies (Tversky & Kahneman, 1973) and errors (Konold, 1989). These fallacies have been well-documented in cognitive psychology and mathematics education.

Theoretical (mathematical) probability is a different case altogether - a "scientific, rigorous concept in contrast to the fuzzy idea of chance which pervades everyday settings" (Pratt, 1998, p. 2). The rigorous foundations of probability have been built through the years, with well-defined terms and logically proven rules. However, since well-founded concepts frequently go against ordinary intuition and gut feeling (such as the so-called "gambler's fallacy"), students (even *science* students) often find the topic complex. "Thinking mathematically demands more. It presupposes that learners have a more or less rich pool of intellectual tools at their disposal [which are] available to a mathematician - and precisely those lacking in the naïve learners" (p. 2).

Here lies the challenge. Given that probability misconceptions are so pervasive, what methods can be utilized to counter these false beliefs and replace them with mathematically appropriate ones? Researchers have long focused on the specific reasons for such misconceptions (the most celebrated ones are those of Daniel

Kahneman and Amos Tversky, whose work in this area was awarded the 2002 Nobel Prize in Economics).

But only recently have *efforts* to overcome misconceptions been made, efforts such as utilizing appropriate mental models (Hong & O' Neil, 1992) and rewording of anomalous information (Graesser & McMahen, 1993; Lee-Chua, 2001a). Another promising strategy to deal with misconceptions is the use of anchors. Pioneered by Clement in the mid-1980s (Brown & Clement, 1987; Clement, 1987) to study physics misconceptions, the use of anchoring situations in other areas has attracted interest in the last decade. The efficacy of anchors has been studied vis-à-vis cognitive science (Perkins & Salomon, 1989; Glaser, 1991), the representativeness fallacy (Cox & Mouw, 1992), artificial intelligence (Crews, Biswas, Goldman, & Bransford, 1997), basic stochastic misconceptions (Fast, 1997), elementary statistics misconceptions (Lee-Chua., 2002), and most recently, interactive learning environments (Cognition & Technology Group at Vanderbilt, CTGV, 2002).

However, using anchors to study more *advanced* probability concepts has still not been explored. The most recent research along these lines has been done by Fast (1997) on secondary mathematics' teachers' views on elementary probability, where anchors indeed have proven to be effective in revising false beliefs. Fast indicated that "[b]ecause of Clement's success in using analogies in physics, it seemed worthwhile to attempt his approach in the area of probability" (p.326).

Hence, and because of Fast's success in using anchors in basic probability, it seemed worthwhile to examine if this approach also works for more advanced mathematics concepts such as the Monty Hall paradox. Moreover, as far as it can be ascertained, anchoring research so far has only been done in the United States and Europe. In this study then, the replication occurs in a Southeast Asian context, where often drill and routine are emphasized over conceptual understanding

Schema Revision and Anchoring Situations

Why is dealing with (and revising) misconceptions difficult? Many cognitive psychologists believe that acquiring these concepts in the first place requires much effort, thus changing the concepts in the face of contradictory evidence is therefore resisted by the learner (e.g., Brown & Clement, 1987). Revision requires change of concept construction - in cognitive psychological terms, a *change of schema* (Davis, 1984).

Anchors can prove useful in changing schemas. Echoing Clement (1987), Fast (1997) defined anchoring situations as "problems which are designed to draw out beliefs held by individuals which are in agreement with accepted theory and which are therefore expected to receive correct responses" (p. 326). Just as a metal anchor holds the ship steady and prevents it from running aground, psychological anchors help individuals construct correct schemas and either revise false beliefs or prevent them from committing the errors in the first place.

Specifically, according to constructivist theory, when new topics are introduced, a scaffolding or framework is helpful. Anchors serve as scaffolds where learners can elaborate and correctly draw upon their existing knowledge (Glaser, 1991). Well-chosen or well-prepared anchors can bridge the gap between what learners already know and what they still have to learn. "Bridging examples which have some of the characteristics of the anchor… and some characteristics of the target are used as supports in conceptually bringing the two analogous [situations] closer together" (Fast, 1997, p. 328).

For instance, Clement (1987) detailed a popular physics misconception which concerns Newton's Third Law (action and reaction). Consider a book lying on a table. Many students believe that the book exerts a downward force on the table, but (mistakenly) do not believe that the table exerts a corresponding force upward on the book. To revise this false belief, Clement used as anchor the situation of using a hand to push down on a spring. Practically all students believe that the hand exerts a downward force on the spring, and *at the same time* the spring exerts an upward force on the hand. This situation served as an effective anchoring bridge - from the readily-seen concrete example of the spring, to the less visible forces of the book and table, and finally to the abstraction of Newton's Law.

Aside from acting as scaffolds and bridges, anchors also help in activating the correct schema. Sometimes students may hold "parallel but different schemas, one which is appropriate for solving the problem, and the other which is not" (Fast, 1997, p. 328). Which schema is finally chosen by the student depends on several factors, such as problem representation (wording the problem differently, for instance), the familiarity of the context, and the degree of concreteness (Perkins & Salomon, 1989).

How then were the anchors in this study generated? The process was two-fold. First, the misconception-prone situations were examined to discover possible reasons for such false beliefs. Previous literature (Tversky & Kahneman, 1977; Piattelli-Palmarini, 1994) was heavily considered. Do the misconceptions occur because of too much abstraction, a failure to use multiple perspectives, or other factors? Second, anchors were constructed to attack the specific reasons for the misconceptions. For instance, in Question 1, the framing fallacy has been found to occur among learners who fail to view problems from various perspectives. Thus, the appropriate anchor is one which spells out different perspectives in a clear manner.

Fast suggested three techniques he has found useful for constructing anchors: “presenting the problem from a different perspective, utilizing concrete situations, and changing the numerical quantities in order to present an extreme case” (Fast, 1997, p. 329). These three techniques are illustrated in the examples that follow.

For this research then, possible anchors, conceptually isomorphic to misconception-prone situations, were generated. As discussed previously, these anchors serve to draw out beliefs held by students which agree with formal probability theory, and which therefore are expected to generate correct responses (Clement, 1987). The misconception-prone situations in this study have been deemed by researchers (e.g., Tversky & Kahneman, 1977; Piattelli-Palmarini, 1994) as often receiving wrong answers, which indicate mathematically incorrect concepts. Specifically, it was asked whether or not the generated analogous situations act as anchors and help students overcome previous misconceptions?
The tested hypotheses are as follows:

- Possible anchoring situations are more likely to result in more correct answers (thus, more correct concepts and schemas) rather than misconception-prone situations.
- Anchoring situations can be used to help students overcome previously held probability misconceptions.

Methodology

Instruments

Ten misconception-prone situations (mostly adapted from Piattelli-Palmarini’s 1994 text on Tversky and Kahneman’s work) were matched with conceptually isomorphic researcher-generated possible anchoring situations. The former were compiled as Version A, and their analogous counterparts as Version B. Problems were given in multiple-choice format. Justifications for student answers were also requested.

Two sample questions (both versions) are stated below. The first concerns the framing fallacy, modified from one of Tversky and Kahneman’s (1977) experiments.

***Question 1 (Version A, no anchor)*:** Imagine that our country is preparing for the outbreak of a disease that is expected to kill 500 people. Two alternatives to combat the disease have been proposed. Assume that the consequences of the program are as follows:

a. If Program A is adopted, 250 people will be saved.
b. If Program B is adopted, there is 1/2 chance that 500 people will be saved and 1/2 chance that no one will be saved.
c. It does not matter if you choose a or b.

Which will you choose and why?

Research has shown that most people are conservative and will choose the first scenario of making sure that at least half the people will be saved (Piattelli-Palmarini, 1994). In short, they fall prey to the so-called "framing fallacy", which means that when faced with a problem concerning choice, they "frame their decision"; that is, they (emotionally and rationally) accept it in the terms in which it is presented and neglect to look for another alternative formulation. Piattelli-Palmarini described it thus:

> Thanks to our cognitive sloth we become prisoners of the frame we are offered. We isolate the problem from its global context; the problem itself becomes the immediate and exclusive center of our attention. We do not take into account all the pros and cons of our choice and its consequences (p. 57).

The mathematically correct answer utilizes the concept of *mathematical expectation.* Using the definition, there is no difference between the first and second scenarios. In the long run, the number of people expected to be saved is 250. $\frac{1}{2}(500) + \frac{1}{2}(0) = 250$.

Let us now look at the analogous (anchoring) counterpart in Version B.

Question 1 (Version B, anchor) Imagine that our country is preparing for the outbreak of a unusual disease that is expected to kill 500 people. Two alternatives to combat the disease have been proposed. Assume that the consequences of the program are as follows:

a. If Program A is adopted, 250 people will be saved.
b. If Program B is adopted, the number of people who will be saved is the average of 500 and 0.
c. It does not matter if you choose a or b.

Which will you choose and why?

Put this way, the identical scenarios posed by both "a" and "b" are readily apparent. The second statement is a direct (identical) alternative formulation of the first one, and the framing fallacy should not occur. It does not require much mental shift work, so people should be more readily capable of shifting from one scenario to the other.

How was this situation developed as an anchor? Recall that one function of an anchor is to enable the learner to choose the correct schema from several possibilities, and that a useful technique to build an anchor is to present the problem from a different perspective. Because the false belief inherent in the framing fallacy is *precisely the failure* to use various perspectives, it seemed logical to investigate whether this technique (presenting the problem in a different way), attacking this fallacy head-on, so to speak, works.

The second question is perhaps the most famous conundrum of all - the Monty Hall paradox, first discussed by Gardner (1961). The paradox stemmed from the game show "Let's Make a Deal" hosted by Monty Hall in mid-1900s America.

***Question 2 (Version A, no anchor)*:** As a contestant in a game show, you are asked to choose one door among three closed doors. Behind two of the doors are goats, and behind the third is a car. The host knows which doors contain which prizes. You choose a door, say A. The host then opens one of the other two doors (say B), revealing a goat. The host then asks, "Do you want to stick with your choice, A? Or do you want to switch to C?" What will your strategy be? Why?

a. You stick with (and finally open) A.
b. You switch to (and finally open) C.
c. You don't think it matters—half of the time you stick with A, half of the time you switch to C.

Much research has shown that most people would choose "c," that is, they don't believe that the host's opening of a door with a goat has any effect on the final outcome. Another erroneous supposition would be to believe that after a door has been opened by the host, the probability of your choosing the right door (among the two remaining doors) must be 50%. Thus it doesn't really matter if you stay or switch.

But there is a mistake in the addition of probabilities. The sum of the probabilities of the two closed doors is indeed 1, but the two probabilities are not equal. The door you choose first always has a 1/3 chance of being the right one. The other two doors combined have a 2/3 chance of containing the car. But at the moment Monty Hall opens the door B (with the goat), the other one C alone has a 2/3 chance of containing the car. Therefore, contrary to intuition, switching increases the probability from 1/3 to 2/3. 1/3 added to 2/3 equal 1, just as ½ and ½ do, but in this game, the probabilities are not equal.

The above explanation is usually given to explain the strategy, but again research has shown that many people still reject it (the columnist Marilyn vos Savant, for instance, received many angry letters from mathematicians when she correctly explained the preceding strategy).

How should an anchor be constructed in this case? Recall that one of Fast's (1997) techniques was to change the numerical quantities to exaggerate the situation and to present an extreme case. Hopefully, the extreme obviousness of the situation will trigger the correct schema, making the solution readily apparent. For instance, Fast used this strategy to counter a statistics misconception concerning sample sizes. Consider two teams competing for the championship. Would a 5- or 9-game series be more likely to show which team is better? Or is there no difference? Fast discovered that most students fall prey to Tversky and Kahneman's (1977) representativeness heuristic; that is, they believe that the two sample sizes work equally well, that there is no difference between 5 or 9 games. However, statistically speaking, the 9-game series would be more likely to show which team is actually better.

To counter the misconception, Fast constructed the following anchoring situation: Instead of comparing 5- and 9-game series, Fast (1997) asked the students to choose between a sudden-death one-game and a 5-game series. Which is more likely to show which team is better? Practically everyone answered that a 5-game series is better. Fast explains, "Experience tells you that you do not win every game; therefore, a sudden death 1-game playoff is very risky whereas a 5-game series gives you a better chance to show your talent. It is the extremity of the numerical quantity '1' in the one-game series which makes the correct choice more obvious. Consequently, this situation provides an appropriate anchor" (p. 430).

In the Monty Hall paradox, a possible anchor utilizes a similar strategy. Fr. Bienvenido Nebres (Lee-Chua, 2001b) proposed the following situation, using extreme numerical quantities to make the correct answer more obvious - and convincing. Consider the anchoring counterpart of the Monty Hall paradox:

Question 2 (Version B, anchor)**:** As a contestant in a game show, you are asked to choose one door among 10 closed doors. Behind 9 of the doors are goats, and behind one door is a car. The host knows all which doors contain which prizes. You choose a door, say A. The host then opens one of the other 9 closed doors (say B), revealing a goat. The host then asks, "Do you want to stick with your choice, A? Or do you want to switch to another door?" What will your strategy be? Why?

a. You stick with (and finally open) A.
b. You switch and finally open another door.

c. You don't think it matters—half of the time you stick with A, half of the time you switch to another door.

Just as in Fast's sudden-death playoff example, in this exaggerated version of the original paradox, most of the subjects felt that the correct choice was quite obvious. They often decided (correctly) to switch. Why? Changing the situation from 3 to 10 doors often activated the correct schema, as described by many of the subjects in the following manner: Suppose for instance, that your original choice was the right one, when Monty Hall opens a goat-door, then for you to switch will definitely penalize you. But if your original choice was a goat-door anyway, you will certainly gain by switching. How often is your original choice correct (and thus penalized if you switch)? One out of 10 times. How often will your original choice be wrong (and thus you will be rewarded if you switch)? Nine out of 10 times.

The above explanation, triggered by the extreme case is more convincing than the traditional one (by Marilyn vos Savant and others). Nevertheless, viewing the problem in this way sheds light on this solution: The door that you choose first always has the probability 1/10 of being the right one. The other doors combined have a 9/10 chance of containing the car. At the moment Monty Hall opens a door containing a goat, the other remaining 8 doors will have a 9/10 chance, thus switching is advisable. Moreover, because there are more doors in this version, the possibility of a 50-50 chance does not arise, and does not mislead the subjects into thinking that neither strategy matters.

Sample

The subjects for this study were 30 junior science majors from a university in the Philippines. All have taken an introductory probability and statistics class one semester before this research was conducted.

Procedure

The subjects were first given the ten questions of Version A to complete. Their answers were then collected - to prevent the possibility of returning to Version A after answering B. They were then immediately given Version B. No time limit was given, but all the subjects completed both versions of the questionnaire (including written justifications for their choices) within one hour.

When the subjects submitted Version B, they were asked whether they could be interviewed. The purpose of the interview was to determine whether the anchors in Version B could be used to help overcome misconceptions in A. All subjects agreed, but it was found later that six got perfect scores (on both versions) with appropriate justifications for each question, thus they were excused from the interviews.

For the remaining 24 subjects, each was individually interviewed by the researcher the day after completing the questionnaire. They were presented with situations in Version A in which they had *wrong* answers (misconceptions) and the analogous counterparts (possible anchors) in B wherein they had *right* answers. In the interview, the researcher guided the subjects in a reasoning process where hopefully they would be led to correct their incorrect answers in A. Following Fast (1997), this change from a wrong to a right response would be interpreted as evidence of overcoming that particular misconception.

The results were categorized as either "Success," "Partial Success," or "Failure," coded 1, 0.5, or 0, respectively. "Success" means that during or after the interview, the subject changed his or her wrong response in Version A to the correct one, with the help of the anchor in Version B. "Partial Success" means that the subject changed his or her wrong response in Version A to the correct one, but without the help of the counterpart in Version B (e.g., the change was due to other factors). It can also mean that the subject changed his or her wrong response in Version A to the correct one, but he or she was not convinced about the reasons for the change. "Failure" means that the subject did not change his or her incorrect response, or changed it to still another wrong one.

Results and Discussion

Responses in Both Versions

In Version A, 30 subjects answered 10 questions each, for a total of 300 answers. However, six subjects obtained perfect scores with appropriate justifications for both versions, thus were excused from the interviews and deleted from further analysis. Of the remaining 240 responses, 108 were mathematically correct, thus, there were 132 wrong answers. In Version B, out of 240 answers, 198 were correct, leaving 42 wrong responses. Based on the proportion of correct answers, therefore, correct schemas were used 108/240 or 45% of the time in Version A compared to 198/240 or 82.5% of the time in B. This difference is highly significant ($Z = 16.3$, $p < 0.01$). Moreover, each question in Version B resulted in more correct responses than its counterpart in A, and *no subject had a correct answer in Version A but a wrong one in B.* For instance, Table 2 shows the number of correct responses for the three previous questions and possible anchors. (The Version B situations were constructed by the researcher to function as possible anchors, but their actual effectiveness still has to be investigated, hence the use of the word "possible." Actual success rates of the possible anchors appear in Table 1.)

Table 1.
Number of Correct Responses for Sample Questions in Both Versions

Question Number	Version A (24 subjects)	Version B (24 subjects)	Possible Anchors
1	15	22	7
2	9	19	10

These results indicate that the situations in B rightfully served their purpose to encourage students to use correct schemas in understanding hypothesis of the study, that possible anchors are more likely to result in more correct answers rather than misconception-prone situations, has thus been supported.

Anchoring Situations
The subjects who did not get perfect scores for both versions were interviewed, focusing on situations where possible anchors in Version B appeared for the misconceptions in A. For instance, from Table 1, there are seven possible anchors for the first question, and 10 for the second.

Based on the coding method presented previously, the points that best describe the subjects' subsequent decision were computed. The proportion of these points to the total possible number of anchors was defined as the Success Rate. These results are shown in Table 2. In general, the situations in Version B successfully anchored students to overcome their misconceptions in A. Success rates ranged from 0.75 to 1.00. The hypothesis that anchors can be used to help students overcome previously held misconceptions has also been supported.

Table 2.
Results of Interviews with 24 Students

Question	Possible Anchors	Resulting Points after Interview	Success Rate
1	7	5.5	0.78
2	10	7.5	0.75

Successful Anchors
To elaborate on the coding process, transcripts from interview sessions follow. (R stands for researcher, S for subject). These sessions were done one day after the questionnaires were answered.

The following interview was coded "Success" and given a score of 1.

R: *Here is Version A of Question 1. Your answer was "a." Will you stick with this?*

S: *I think so.*

R: *Why?*

S*: Because the other option is too risky. The chance that everyone might die is too high. It is not worth taking.*

R: *But so is the chance that everyone might be saved.*

S: *What are you trying to say?*

R: *Here is Version B. Let's look at Question No. 1. You chose "c." Do you still agree?*

S: *I think so. The results of "a" and "b" are the same. Both are 250.*

R: *Now look at these two versions again carefully. Can you see any similarities?*

S: *(studies for a while, then smiles) I think this is a good analogy (points to Version B) to this one (Version A). I get it. The answer to the earlier one should also be "c."*

R: *Can you explain why?*

S: *There is a 50% chance that everyone will die, but there is also a 50% chance that everyone will live. In this case, when you add the two probabilities, which is the case with Version B, you will get 250 people saved or 250 people dead. I should be using the formula in Version B for the problem in Version A. I think we studied this last time.*

R: *Do you remember?*

S: *Yes. Mathematical expectation.*

Clearly, in this situation, the anchor in Version B helped the subject change his wrong answer in Version A and justify it well. Most of the interviews ended along the same lines, with most anchors becoming successful at helping students revise misconceptions.

What accounts for the high success rates? Let us return to the notion of changing knowledge construction (revising schemas). According to constructivist theory, students come to comprehend thoroughly concepts through personal construction of knowledge (Davis, 1984). This "genuine" understanding is contrasted to mere "instrumental" knowledge, where the student knows how to go about particular routines but without thorough understanding of the concepts involved or how to apply them to new situations.

Anchors become useful in revising schemas. Fast (1997, p. 327) explained, "Since misconceptions have been acquired through constructivist activity, it is reasonable to apply the constructivist approach in the concept reconstruction process to acquire mathematically correct concepts, or schemata." When situations where students are encouraged to answer correctly are generated, then anchors are established, and revision of schemas can proceed.

Partially Successful and Unsuccessful Anchors

The majority of the anchors in this study have been successful, as we have noted. However, some interviews were coded only "Partial Success," like the following:

R: *Here is Version A of Question 1. You chose "a" yesterday. Will you stick with this?*

S: *Of course. It is inhuman to choose something wherein there is a chance that the whole population will die.*

R: *Look at the question from the mathematical point of view. Now look at Version B.*

R: *You chose "c" yesterday. Do you stick with this answer?*

S: *Yes. This is not like the first one. There is no uncertainty here, only averaging.*

R: *So you mean there are no probabilities in getting the average?*

S: *I don't think so. An average is just getting the mean of things, but probabilities are related to odds and things like that [sic], more complicated.*

R: *Look at these two questions again carefully. Do you see any similarities?*

S: *Well, both have the same numbers there, and in some way seem to be alike.*

R: *In what way?*

S: *(after a few minutes) I don't really know how to explain it, but they seem the same.*

R: *Do you recall the concept of mathematical expectation?*

S: *(after a few minutes): Oh. So the answer to Version A is also "c"?*

R: *What do you think?*

S: *I think so. If we are going to use expectation, then it's like a series of averages, isn't it?*

R: *It's actually the average in the long run.*

Unlike the preceding case, here the subject finally revised his misconception, but not wholly with the help of the anchor. Though helpful, the anchor was not convincing enough. The researcher had to point out and clarify the relationship between expectation, average, and probabilities. But since the subject finally changed to the correct answer, his effort could not be considered a failure either, thus was coded "partial success" (0.5).

Let us now look at the third sample transcript, one of the few times when an anchor failed (coded "Failure" or 0):

R: *Here is Version A. Your answer yesterday was "a." Would you stick with this?*
S: *Yes.*
R: *Why?*
S: *At least if I choose "a," then half of the people will survive.*
R: *Now here is Version B. For Question No. 1, you chose "c." Do you stick to this?*
S: *Yes.*
R: *Why?*
S: *There is no difference between "a" and "b". When you compute it right, the average in "b" is also 250.*
R: *Look at the two questions carefully. Can you see any similarities?*
S: *Obviously they are all about people surviving a disease. The numbers are also alike.*
R: *Anything else?*
S*: (shrugs) The difference between the two questions though is also a lot [sic]. In A, there are probabilities, so we can never be really sure how many people will survive. In B, there are no probabilities, so we are sure 250 will live, the same for "a" and "b."*
R: *Do you remember the concept of mathematical expectation?*
S: *I think so. Isn't there a long formula for it?*
R: *What do you think of the formula in "b" for Version B?*
S: *Getting the average?*
R: *Yes.*
S: *You mean expectation is just getting the average? I don't think so. At least that's not how the teacher explained it to us.*
R: *What do you think expectation means then?*
S: *Well, it is a number or an event that you expect will happen. I remember something about tossing dice and coins, but I don't think they have anything to do with average.*
R: *Perhaps you should review mathematical expectation. So you are not changing your answer for Version A?*
S: *Should I?*

Clearly, in this case, the anchor failed. Not even when the researcher spelled out the concept to be utilized (expectation) did the subject overcome her misconception. She was a victim not only of the framing fallacy, but also by a wrong understanding of expectation. Again we turn to constructivism. In establishing schemas, students have to exert considerable time and effort, which make previously acquired concepts difficult to eradicate. Indeed, erroneous ideas are difficult to erase, especially in personalized contexts (Myers, Hansen, Robson, & McCann, 1983).

Conclusion

This study investigated college science majors' conceptual understanding of various probability situations, and whether anchors can help them deal with misconceptions. The researcher-generated counterpart situations served as anchors. As we have noted, these anchors drew out beliefs from students - beliefs which coincided with formal probability theory and which therefore resulted in a majority of correct responses. They also succeeded in guiding students to overcome false ideas. However, as shown by different success rates, not all anchors worked equally well for all students. Students who still held onto their wrong beliefs even after encountering analogies might most likely need to hone other skills, such as basic understanding of fundamental concepts.

Using anchors to overcome mathematical misconceptions is a fruitful field for research. The causes for such misconceptions have already been analyzed (Tversky & Kahneman, 1977). Now the burden shifts from understanding the reasons for these false beliefs to ways of dealing with them, and anchoring is one of the posited methods. Recommendations for further research include:

1. Focus on analyzing which probability situations serve as the most effective anchors, and the reasons behind their effectiveness.
2. Hypothesize certain situations where anchors may fail, and test these hypotheses. For instance, anchors may not succeed in situations where questions are presented ambiguously.
3. Study the factors which make students more readily accept anchors in revising their former erroneous ideas. Such factors may include an ability to see connections between different topics, openness of mind, and an interest in mathematics in general.

Probability may indeed be double-edged, but correctly applying its concepts to academic and everyday situations is necessary. With the use of anchors, misconceptions and errors will not only likely be minimized, but when introduced correctly, may also prevent them from happening in the first place.

References

Bar-Hillel, M. (1980). The base-rate fallacy in probability judgments. *Acta Psychologica 44*, 211-233.

Brown, D. E., & Clement, J. (1987). Misconceptions concerning Newton's Law of action and reaction: The underestimated importance of the third law. In J. D. Novak (Ed.), *Misconceptions and educational strategies in science and mathematics. Proceedings of the International Seminar.* (Vol. 3, pp. 39-53). Ithaca, NY: Cornell University, Department of Education.

Clement, J. (1987). Overcoming students' misconceptions in physics: The role of anchoring intuitions and analogical validity. In J. D. Novak (Ed.), *Misconceptions and educational strategies in science and mathematics. Proceedings of the International Seminar.* (Vol. 3, pp. 84-97). Ithaca, NY: Cornell University, Department of Education.

Cognition & Technology Group at Vanderbilt. (CTGV, 2002). Adventures in anchored instruction: Lessons from beyond the ivory tower. In R. Glaser (Ed.), *Advances in instructional psychology, Vol. 5.* Mehwah, NJ: Lawrence Erlbaum.

Cox, C., & Mouw, J. T. (1992). Disruption of the representativeness heuristic: Can we be perturbed into using correct probabilistic reasoning? *Educational Studies in Mathematics 23*(2)., 163-178.

Crews, T. R., Biswas, G., Goldman, S. R., & Bransford, J. D. (1997). Anchored interactive learning environments. *International Journal of Artificial Intelligence in Education, 8.*

Davis, R. H. (1984). *Learning mathematics: The cognitive science approach to mathematics education.* Norwood, NJ: Ablex.

Fast, G. R. (1997). Using analogies to overcome student teachers' probability misconceptions. *Journal of Mathematical Behavior, 16*(4), 325-44.

Gardner, M. (1961). *The Second Scientific American Book of Mathematical Puzzles and Diversions.* New York: Simon & Schuster.

Glaser, R. (1991). The maturing of the relationship between the science of learning and cognition and educational practice. *Learning and Instruction, 1*(2), 129-44.

Graesser, A. C., & McMahen, C. L. (1993). Anomalous information triggers questions when adults solve quantitative problems and comprehend stories. *Journal of Educational Psychology*, *85*, 136-51.

Hong, E., & O' Neil, H.F. (1992). Instructional strategies to help learners build relevant mental models in inferential statistics. *Journal of Educational Psychology*, *84*(2), 150-59.

Konold, C. (1989). Informal conceptions of probability. *Cognition and Instruction, 6*(1), 59-98.

Lee-Chua, Q. N. (2001a). Rewording and anomalous information in the domain of statistics word problem solving. *The Loyola Schools Review, 1*, 89-102.

Lee-Chua, Q. N. (2001b). Do you want to be a contestant on the Monty Hall game show? *Philippine Daily Inquirer,* B5. 7 December.

Lee-Chua, Q. N. (2002). The role of analogies and anchors in revising misconceptions in statistics. *Journal of Science and Mathematics Education in Southeast Asia 25*(1), 117-135.

Myers, J. L., Hansen, R. S., Robson, R. C., & McCann, J. (1983). The role of explanation in learning elementary probability. *Journal of Educational Psychology*, *75*(3), 374-81.

Perkins, D. N., & Salomon, G. (1989). Are cognitive skills context-bound? *Educational Researcher, 18*(1), 16-25.

Piatteli-Palmarini, M. (1994). *Inevitable illusions: How mistakes of reason rule our minds.* New York: John Wiley.

Pratt, D. (1998). The coordination of meanings for randomness. *For the Learning of Mathematics, 18*(3), 2-11.

Tversky, A., & Kahneman, D. (1973). Availability: A heuristic for judging frequency and probability. *Cognitive Psychology, 5*, 207-32.

Tversky, A., & Kahneman, D. (1977). Judgment under uncertainty: Heuristics and biases. In P. N. Johnson-Laird & P.C. Watson (Eds.), *Thinking: Readings in cognitive science.* (pp. 326-33). Cambridge, London: University Press.

Acknowledgments

The author wishes to acknowledge the aid given by the FEBTC-Jose B. Fernandez Endowed Professorial Chair given by the Ateneo de Manila University for SY 2002-2003.

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