The Mathematics Educator
2000, Vol. 5, No.1/2, 205-215

Pre-service Teachers' Conceptual Understanding of Volume

Yeap Ban Har
Cheong Ngan Peng Christina

Abstract

Teaching for conceptual understanding is one aim of the problem solving curriculum. The first part of the article describes a study that aimed to identify potential factors that may prevent pre-service primary school teachers from teaching the topic of volume for conceptual understanding. The study involved ninety-five non-graduate pre-service teachers. Their responses to a paper-and-pencil instrument were used to identify possible obstacles to teaching for conceptual understanding. The possession of a narrow range of experiences related to a concept, the possession of procedural knowledge based upon weak conceptual knowledge and the reluctance to form mental representation of a concept emerged as possible obstacles. The second part suggests ways to facilitate pre-service teachers teaching the topic of volume for conceptual understanding.

Introduction

Teaching for conceptual understanding is one aim of the problem solving curriculum (Ministry of Education, 2000). One who possesses conceptual, or relational, understanding (Skemp, 1976) has a rich schema of a concept. This schema (Piaget, 1952) is capable of becoming both more extensive as well as more complex. The schema includes links to other concepts. One who possesses procedural, or instrumental, understanding (Skemp, 1976) knows way or ways to get from a given to the goal. An example of procedural understanding is knowing how to use a formula to compute the volume of a cuboid. An example of conceptual understanding is being able to acknowledge that two solids have the same volume because both are built using the same number of unit cubes.

The first part of the article describes a study that aimed to identify potential factors that may prevent pre-service primary school teachers from teaching the topic of volume for conceptual understanding. The second part suggests activities that pre-service teachers can be engaged in to facilitate them teaching the topic of volume for conceptual understanding.

The Study

Ninety-five non-graduate pre-service teachers in the diploma programme from four intact classes were asked to complete four test items in a paper-and-pencil instrument. Pre-service teachers in the diploma programme are trained to teach in primary schools.

Figure 1. An item used in data collection (JUG)

Name of item	Item
JUG	You have Jug A (capacity of 4 litres), Jug B (capacity of 20 litres), Jug C (capacity of 59 litres) and lots of water. All the jugs have no marking. How can you measure out exactly 31 litres of water using only Jugs A, B and C using the minimum number of steps? List down the steps clearly.

The item JUG (Figure 1) was designed to assess the ability to conserve volume. For example, when water from Jug C (capacity 59 litres) is poured into Jug B (capacity 20 litres), 39 litres remain in Jug C.

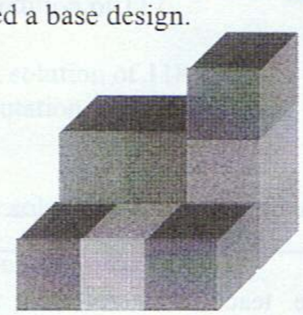
Almost all of the ninety-five pre-service teachers solved this problem correctly. Two pre-service teachers satisfied the required task but not with the minimum number of steps. Two others did not understand that the containers, which have no marking, only measure fixed volumes.

The majority of the pre-service teachers have internalized the principle of conservation of volume. The few who were unable to solve the problem failed because of imperfect problem solving skills. Some did not spend enough time understanding the problem conditions while others did not question themselves if their solution was an optimal one.

The item STRUCTURE (Figure 2) was designed to assess the ability to visualize a three-dimensional structure and an understanding of volume in terms of unit cubes that fill a space.

Figure 2. An item used in data collection (STRUCTURE)

Name of item	Item
STRUCTURE	The diagram on the left shows a figure. The diagram on the right shows the number of cubes on each base square. This diagram is called a base design.



2	2	3
1	1	1

Draw the base design for this three-dimensional figure. Hence state its volume.

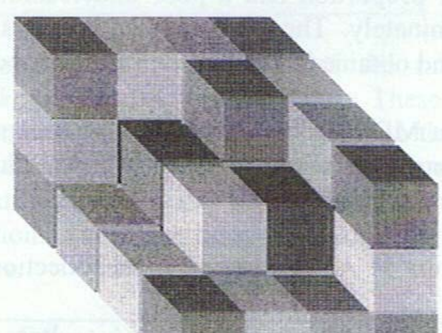


Table 1 describes the categories of responses to this item.

Table 1. Categories of responses to STRUCTURE

Solution	Frequency
Able to obtained the correct solution	75
Unable to draw a base design	11
Able to obtain the base design but unable to obtained volume	9
Total	95

Most of the pre-service teachers were able to perform this item satisfactorily. However, there was a significant proportion (more than 10%) who either were unable to visualize in three dimensions or have poor comprehension ability. A similar proportion had a poor understanding of volume and applied a formula indiscriminately. These pre-service teachers used the formula *length* × *width* × *height* and obtained 48 cubes, despite a correct base design.

The item MEDICINE (Figure 3) was designed to assess ability to make sense of a computation. The rounding off procedure will not yield a correct solution.

Figure 3. An item used in data collection (MEDICINE)

Name of item	Item
MEDICINE	A large bottle contains 707 cm^3 of cough mixture. It is dispensed into smaller containers, each containing 6 cm^3 of the cough mixture. What is the largest number of smaller container of cough mixture that can be dispensed?

Table 2 describes the categories of responses to this item.

Table 2. Categories of responses to MEDICINE

Solution	Frequency
Obtained the correct solution of 117	83
Obtained the incorrect solution of 118 despite having done the computation correctly	9
Obtained the incorrect solution of $117\frac{5}{6}$ despite having done the computation correctly	1
Others	2
Total	95

The results indicated that the majority of the pre-service teachers demonstrated looking back (Polya, 1957) behaviour. These teachers, having performed the necessary computations, probably reflected on the original problem condition and made sense of the computation to give the correct response. Pre-service teachers who were unsuccessful in this item seem reluctant to form a mental image of the given situation. Their solutions were derived from a series of computations and procedures.

The item BLOCK (Figure 4) was designed similarly to assess ability to make sense of a situation. In this question a routine procedure will not produce a correct solution. Some amount of visualization was required.

Figure 4. An item used in data collection (BLOCK)

Name of item	Item
BLOCK	A wooden block (17 cm by 16 cm by 15cm) is cut into smaller cubes. Each cube has edges of 6 cm. Find the maximum number of cubes that can be cut from the wooden block.

Table 3 describes the categories of responses.

Table 3. Categories of responses for BLOCK

Solution	Frequency
Obtained the correct solution	
▪ Using a diagram	4
▪ Using computation	11
Obtained an incorrect solution due to inability to visualize in three dimensions	3
Obtained an incorrect solution by diving the volume of the wooden block by the volume of a cube	75
Others	2
Total	95

While the majority was successful in the item MEDICINE, many pre-service teachers were unsuccessful in this item. The use of formulae to calculate volume was rampant. The majority of the pre-service teachers used the idea of division inappropriately.

Possible Obstacles to Teaching for Conceptual Understanding

Three possible obstacles that may prevent the pre-service teachers in the present study from teaching the topic of volume for conceptual understanding were identified. The obstacles are (1) the possession of a narrow range of experiences related to the concept, (2) the possession of procedural knowledge based upon weak conceptual knowledge, and (3) the reluctance to form mental representation of the concept.

Many of the pre-service teachers had a narrow range of experiences related to the concept of volume. Many seemed too ready to connect the idea of a volume to the formula to compute volume. Such a narrow view of the concept will prevent these pre-service teachers from providing their students with a range of experiences essential for the development of a rich schema of the concept. In the item STRUCTURE, some pre-service teachers preferred, inappropriately, to determine the volume using a formula rather than counting the number of unit cubes that make up the structure. Such preference also suggests that the pre-service teachers possessed a procedural understanding without a strong conceptual understanding. Hence, despite the fact that they had the procedural understanding to find the volume of a figure using formula, the pre-service teachers were unable to use the formula under appropriate conditions. Pre-service teachers who possess procedural understanding without the necessary conceptual knowledge will more likely teach the topic by providing the formula without providing its conceptual basis. The preference for the use of a formula also contributed to the teachers' reluctance to form mental representations (Mayer, 1996) of a concept. In the item BLOCK, an alarming proportion of the pre-service teachers was not forming a mental image of the situation and used the division procedure inappropriately. This is likely to prevent these pre-service teachers from encouraging their students to form a mental representation of concept or situation. These pre-service teachers are more likely to see problem solving as a series of computation. Sense-making is likely to be relegated or ignored.

Facilitating Teaching for Conceptual Development

Pre-service teachers should understand the concept of a volume as the amount of space that an object occupies. This amount of space can be measured in terms of unit cubes that fill the space completely. The choice of unit cubes is by convention. There are many possible substitutes for the unit cube. Some examples include pebbles, beans and matchboxes. Figure 5 describes one activity used to facilitate the construction of the concept of volume. Pre-services teachers' responses during this activity have been edited and summarized.

Figure 5. Discussion questions to facilitate the development of conceptual understanding

Discussion Question for Pre-service Teachers
You are given a small bowl and a paper cube of approximately the same size. How can you tell which has a larger volume? *Responses* ❑ *Fill both with water. Measure the volume of water used using a measuring cylinder.* ❑ *Fill both with sand. Measure the volume of sand using measuring cylinder. Water may damage the paper cube.* What if there is no measuring cylinder? *Responses* ❑ *Measure the length of one side of the cube. Use the formula to calculate the volume. Measure the radius of the bowl. Use the formula of a hemisphere to estimate its volume. But the bowl is not exactly a hemisphere.* ❑ *Fill both containers with water or sand. Pour the water or sand in each container into identical containers (such as paper cups) and compared directly.* What if you have the bowl and you friend has the paper cube, and you are trying to compare the volume through the telephone? There is no measuring cylinder available. *Responses* ❑ *Fill both containers with water or sand. Pour the water or sand in each container into identical containers (such as paper cups) and measure the height of the water or sand. Compare indirectly.* ❑ *Fill both with beans. Count the number of beans needed to fill each.* What if you use red beans (which are larger) and your friends used green beans (which are smaller)? How can we overcome this problem? *Responses* ❑ *Agree on the same type of beans.* ❑ *Use marbles that are identical in size. But there are gaps between marbles.*

Pre-service teachers also should understand the conceptual basis of the formula that they are teaching. They should understand why the formulae to find volume of cubes and cuboids are such. They must see the link between the formulae and the meaning of volume. Figure 6 shows one activity that can be used to help pre-service teachers do so.

Figure 6. An activity to facilitate the development of conceptual understanding

Activity
You are given a 8 units by 12 units by 5 units box.
It is given that the volume of the box is the number of unit cubes that can fill the box completely.
Show how you determine the volume of the box (a) If you are given 500 unit cubes, (b) If you are given 100 unit cubes, (c) If you are given 50 unit cubes, (d) If you are given 20 unit cubes. *The activity directs the pre-service teachers to predict the number of unit cubes required to fill the box by finding the number required to fill one row, one layer and eventually the entire box. For example, if one row has 8 unit cubes then one layer with 12 rows has 8x12 unit cubes. As 5 layers are required to fill the unit cube, the total number of unit cubes needed are 8x12x5. This provides a basis for the formula of volume of a cuboid.*

Pre-service teachers should be exposed to situations where it is necessary to engage in sense-making while solving a problem. It is not sufficient for them to engage in typical textbook problems that are often solved by the application of an operation or a series of operations. Figure 7 shows two of a series of word problems given to the pre-service teachers to increase the awareness of the importance of qualitative reasoning. The correct answers to the original problems can be obtained by performing an operation. The solutions to the alternate problems can be obtained only if qualitative reasoning is used. In the case of volume, qualitative reasoning may require some kind of visualization.

Figure 7 Problems modified to encourage qualitative reasoning and visualization

Original Problem	Alternate Problem
A cube with sides 8 cm is cut into cubes with sides 2 cm. How many smaller cubes are obtained?	A cube with sides 9 cm is cut into cubes with sides 2 cm. How many smaller cubes are obtained?
1000 ml of cough syrups are poured into containers each with capacity 10 ml. How many full containers of cough syrup are there?	1000 ml of cough syrups are poured into containers each with capacity 7 ml. How many full containers of cough syrup are there? How many containers are required?

Conclusion

There are some possible obstacles that may prevent teachers from teaching for conceptual understanding. In this article, potential obstacles that arise from teachers' lack of conceptual understanding are described. Specifically, the possession of a narrow range of experiences related to a concept, the possession of procedural knowledge based upon weak conceptual knowledge and the reluctance to form mental representation of a concept have been identified as possible obstacles.

We also offer several suggestions to facilitate the development of pre-service teachers' conceptual knowledge in one particular topic. There are other possible ways to help primary teachers teach for conceptual understanding. Material design is one of them. Textbook writers may implicitly influence teachers to teach for conceptual understanding through the activities the former include in instructional materials. Explicit explanation in teachers' manuals that accompany such instructional materials will consolidate the effort.

Further research to investigate the effectiveness of such intervention on facilitating teachers to teach for conceptual understanding will provide useful insights into the development of mathematics teacher training programmes.

References

Mayer, R.E., & Hergarty, M. (1996). The process of understanding mathematical problems. In R.J. Sternberg & T. Ben-Zeev (Eds.), The Nature of Mathematical Thinking (pp. 29-54). Mahwah, NJ: Lawrence Erlbaum.

Ministry of Education. (2000). Mathematics Syllabus - Primary. Singapore: Curriculum Planning & Development Division.

Piaget, J. (1952). *The child's conception of number*. New York: Humanities Press.

Polya, G. (1957). *How to solve it.* Princeton, NJ: Princeton University Press.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching,* 77, 20-26.