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Singapore Primary School TIMSS Data: Geometry And Measurement, Estimation And Number Sense

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Abstract

This article is concerning the performance of Singapore students in Third International Mathematics and Science Study (TIMSS). Singapore students' performance on Geometry and Measurement, Estimation and Number Sense show some clear trends. They are: (1) Primary four students outperform primary three students; (2) On most items Singapore students outperform their corresponding International cohort; (3) Girls outperform boys on Geometry but the results are mixed for Measurement, Estimation and Number Sense; and (4) Boys tend to leave questions blank more often than girls.

Finally, the analysis of the data also shows that while students perform well in routine situations, they have difficulty using information in non-familiar contexts and making the transition from the enactive mode to the ikonic mode. This implies that there needs to be a focus on activities involving both familiar and non-familiar contexts as well as a specific emphasis on linking enactive and ikonic situations.

The first paper in the series (Kaur and Pereira-Mendoza, 1999) provided an overview of the Third International Mathematics and Science Study (TIMSS) and reviewed Singapore students performance in the areas on Whole Numbers and, Fractions and Proportionality. This paper will report on two other strands within the TIMSS study, namely (1) Geometry and (2) Measurement, Estimation and Number Sense. The following analysis is based *solely on the released items* (Website: http://wwwcsteep.bc.edu/timss; IEA: TIMSS, 1997).

The Geometry Data

There were a total of 10 released items on geometry (8 multiple choice and 2 short answer). The data shows similar trends to the Whole Number data.

(a) *Primary 3 versus Primary 4.* As would be expected, primary four students performed better than primary three on all the items (Table 1). On items J2 and T5 there is a substantial improvement in performance, i.e. 39% and 34% respectively. Both items involve symmetry, which is introduced to pupils in primary four for the first time (Curriculum Planning Division, 1995).

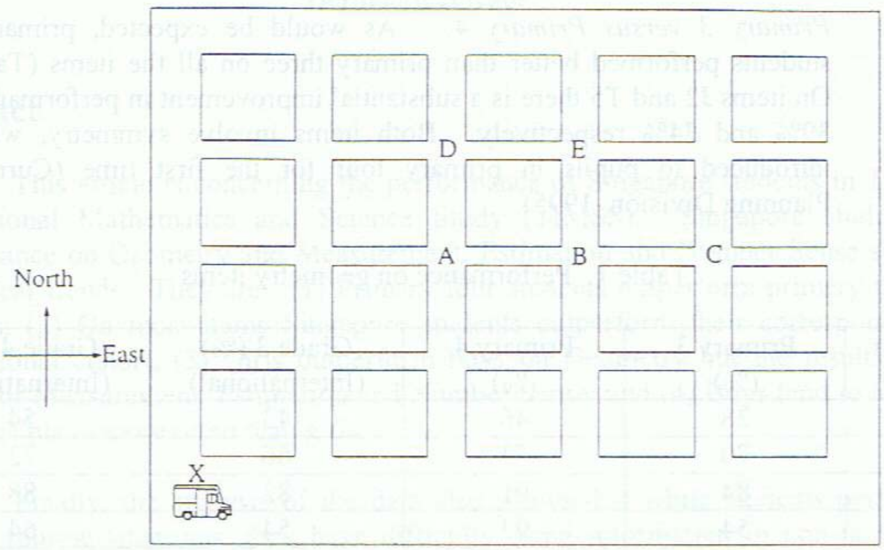
Table 1. Performance on geometry items *

Item	Primary 3 (%)	Primary 4 (%)	Grade 3 (%) (International)	Grade 4 (%) (International)
I1	28	46	43	54
I6	70	73	66	72
J1	84	91	82	88
J2	54	93	54	64
K1	54	63	55	65
K8	82	84	60	73
L3	83	91	80	88
L5	32	42	34	40
M4	36	55	30	42
T5	52	86	45	59

* All figures show the percentage of students responding correctly.

(b) *Singapore versus the International cohort.* On most of the items Singapore students outperformed the International cohort (Table 1). On items K1 – primary three and primary four and L5 – primary three where the International cohort outperformed the Singapore students the results were roughly equal. The only item, I1, where there was a relatively large difference in performance (15% at primary three and 8% at primary four) dealt with finding the location of a city block given specific directions. Students had to interpret the directional instructions and locate the city block on a grid. The context of the item appears to have caused difficulty and not the concept of co-ordinates, per se, since item L3 dealt with co-ordinates too, and the students did very well.

11. This map shows city blocks with a delivery truck at one corner.



The driver of the delivery truck starts at corner X. He goes 3 blocks east and 2 blocks north to get to the school. On what corner is the school located?

A. A

B. B

C. C

D. D

E. E

(c) *Performance of boys versus girls*. While no formal statistical analysis was done on the difference in performance between boys and girls in Singapore, the girls outperformed the boys on 7 of 10 items at primary three and 7 of 10 items at primary four (with one item equal). (Table 2). The differences are not large, but the trend implies that at this level girls do better than boys on geometry, which is a similar result to that for whole numbers (Kaur and Pereira-Mendoza, 1999).

Table 2. Performance of Singapore Students (Geometry) – Boys versus Girls *

Item	Primary 3 **		Primary 4 **	
	Girls	Boys	Girls	Boys
I1	24	32	44	48
I6	75	65	75	71
J1	85	82	90	91
J2	52	55	94	92
K1	57	51	64	62
K8	85	79	85	83
L3	83	82	91	91
L5	33	30	43	42
M4	33	38	60	50
T5	53	51	88	84

* Number of boys and girls varies between items and levels. There were between 900-950 boys and 800 – 850 girls writing any individual item at a specific level.

** All figures show the percentage of students responding correctly.

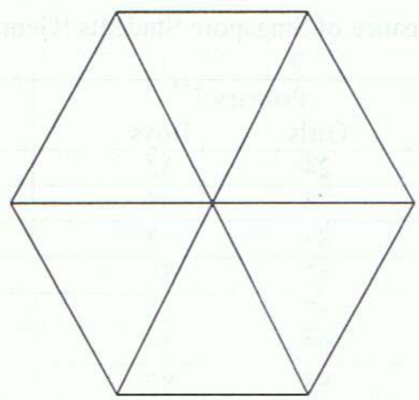
(d) *Leaving answers blank*. In most cases Singapore students answered the questions. There were relatively few blank answers, but again the data shows that girls are less likely to leave an answer blank than boys.

The authors will discuss some of the items, both where students did well and where the performance was relatively poor. The purpose of this is to see if there are any implications for the teaching of geometry.

Results On Some Specific Geometry Items

The following items, which were classified by TIMSS under the performance category: Knowing, have high levels of correct responses, over 80% for primary three and over 90% for primary four.

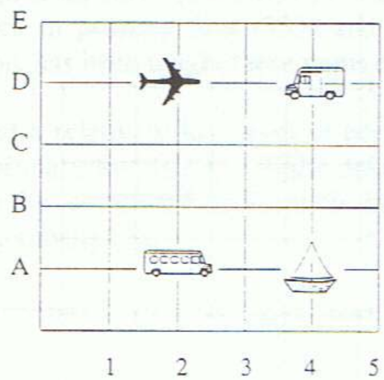
J1. Here is a hexagon



The hexagon is divided into six

A. triangles
B. squares
C. pentagons
D. rectangles

L3. This is a game board.



Which object is located at (2, D)?

A. The plane 

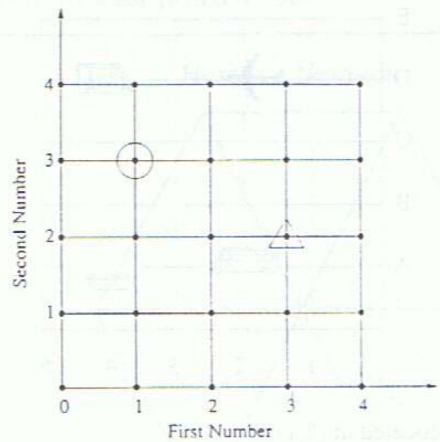
B. The truck 

C. The bus 

D. The boat 

The performance on the following item was considerably lower.

M4. On this grid, find the dot with the circle around it. We can describe where this dot is by saying it is at First Number 1, Second Number 3



Now find the dot with the triangle around it. Describe where the dot is on the grid in the same way. Fill in the numbers we would use:

First number ______________ Second Number ______________

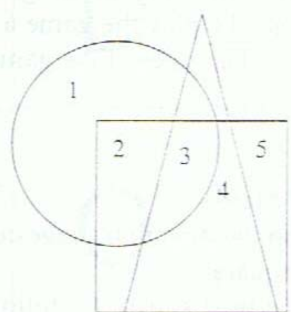
Items I1 and M4 have a low level of performance for geometry (Table 1). On the surface they seem similar to L3 where the performance was high and all three items have some relationship to the idea of co-ordinates. However, there are differences. In L3 even if one did not understand the role that order played in representing co-ordinates, there is only one object at the "intersection of 2 and D". This item was classified by TIMSS as knowing. As was discussed earlier ,I1 an item that was classified by TIMSS as Using Complex Procedures, involved the students having to interpret directional instructions to locate a block on a grid. Difficulty with any component associated with the directional instructions could lead to an incorrect response. On item M4, which was classified by TIMSS as Solving Problems, the students have to be able to derive the location of a point from the information provided. Even if one looks at students who label the point (2,3), which would imply that students have the ability to locate the point but do not comprehend the role of order, the "success" rate would only increase to 43% (primary three) and 64% (primary four).

Item L5 has a low level of performance. This item deals with the number of edges of a cube, a topic that is not covered in primary three or four.

Items J2 and T5 both of which deal with symmetry have a relatively low level of performance in primary three (54% and 52% correct, respectively) and a high level of performance in primary four (93% and 86%, respectively). This suggests that once the topic has been taught these items cause little difficulty.

Item K1 also had a relatively low level of performance for both primary three (54% correct) and primary four (63%). While the item is labelled knowing in geometry, it appears to be associated more with logical thinking than mere recognition of shapes in geometry.

K1. Here is a figure.



Which number is in the square and the circle but is NOT in the triangle?

A. 2

B. 3

C. 4

D. 5

Implications of the findings for teaching geometry

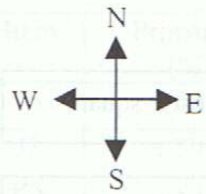
Singapore students did relatively well in geometry and better than their International counterparts. They do well on routine geometric items. However, the performance on an item such as M4 may be indicative of an underlying difficulty. The item looks relatively easy. All the information is given in the problem. Even if a student had no experience with co-ordinates it is possible to solve the problem based solely on the information provided.
The performance on this item, together with the performance of the secondary students on similar items (Kaur and Yap, 1998) seems to suggest difficulty in using information provided in non-familiar contexts.

The geometry data does not indicate specific difficulties with geometric concepts, per se. The following activity focuses on developing the skill of using information.

This activity in the form of a board game involves moving on a grid following a set of instructions. To play the game a student needs a board and a set of cards as shown in Figure 1. The rules of the game are:

Shuffle the cards.
Select 5 cards.
Begin at the square marked "Start".
Use the first card and move in the direction suggested.
Write down the letter in the square.
Move from that square to the next square by following the directions on the next card.
Write down the letter in the square.
Finish the five moves and try to make a word using the letters. You get one point for each letter used.
Shuffle the deck and play again.
See how many games it takes to get 20 points.
If following the instructions on a card would take you off the grid stay where you are and write down the letter of that square.

Figure 1



A	H	B	C	A
S	E	D	I	E
N	Start	O	E	A
A	S	E	N	T
M	A	T	M	K

Figure 1 (…cont'd)

Sample cards:

East 2 squares	North 1 square	West 3 squares
South 3 squares	South 2 squares	West 2 squares
East 1 squares	East 3 squares	North 2 squares

The game can be adapted to new situations by moving the Start Square to different locations. Also, the NS-WE axes can be rotated through 90°, 180° and 270°, making the students interpret the movements in a new situation. This game is an adaptation of "How did the thief escape (Wagstaff, 1990).

The Measurement, Estimation And Number Sense Data

There were a total of 11 released items on Measurement, Estimation and Number Sense (7 multiple choice, 3 short answer and 1 extended response). As with the geometry data there are some clear trends.

(a) *Primary 3 versus Primary 4.* As with geometry, primary four students performed better than primary three on all the items (Table 3). On item K7 the performance of primary three was 28% below that for primary four. On J8 the performance differential was 41%. An analysis of item K7 shows that it involves the concept of the perimeter of a rectangle. Although the concept of perimeter is taught in primary three, pupils are only expected to find perimeters of regular figures whose dimensions are given. In primary four, when the topic of perimeter is revisited pupils are often given questions like item K7 to do. Item J8 involves finding the best estimate and it may have been a combination of the concept of estimation and the language in the problem that caused difficulty.

Table 3. Performance on Measurement, Estimation and Number Sense*

Item	Primary 3 (%)	Primary 4 (%)	Grade 3 (%) (International)	Grade 4 (%) (International)
J6	63	75	61	72
J8	39	80	33	52
K5	87	90	69	77
K7	19	47	21	23
L6	54	59	41	55
L8	18	24	21	32
M7	38	44	30	38
S5	32	46	34	48
T3	40	50	32	47
V5	36	42	31	49
U1	47	65	36	50

* All figures show the percentage of students responding correctly.

K7. A thin wire 20 centimeters long is formed into a rectangle. If the width of this rectangle is four centimeter, what is the length?

A. 5 centimeters
B. 6 centimeters
C. 12 centimeters
D. 16 centimeters

(b) *Singapore versus the International cohort.* The primary four cohort outperformed the International cohort on all items and the primary three cohort outperformed the International cohort on 8 of 11 items (Table 3). On the three items where the performance of Singapore students was lower than the International cohort the difference was only marginal - two percent.

(c) *Performance of boys versus girls*. This comparison produced different results from that for geometry. Girls outperformed boys and vice-versa on approximately the same number of items (Table 4). As was the case with geometry the differences are not large, but the trend is different.

Table 4. Performance of Singapore Students - Measurement, Estimation and Number Sense – Boys versus Girls *

Item	Primary 3 **		Primary 4 **	
	Girls	Boys	Girls	Boys
J6	63	63	74	75
J8	39	38	81	78
K5	90	85	92	89
K7	19	18	43	50
L6	53	54	59	59
L8	16	19	22	26
M7	36	40	45	43
S5	29	34	46	46
T3	39	42	45	55
V5	36	36	43	41
U1	48	45	66	63

* Number of boys and girls varies between items and levels. There were between 900-950 boys and 800 – 850 girls writing any individual item at a specific level.

** All figures show the percentage of students responding correctly.

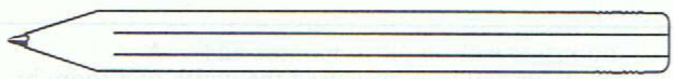
(d) Leaving answers blank. As with geometry there were relatively few blank answers, but again the data shows that girls are less likely to leave an answer blank than boys.

Again the overall data shows that Singapore students did well relative to the International cohort, but the general level of performance was lower than that for geometry (Tables 1 and 3). The authors will now focus on some specific items.

Results on some specific Measurement, Estimation and Number Sense items.

The only item with a very high success rate at both levels was K5. This is a routine situation for Singapore students.

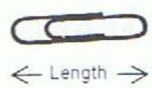
K5. About how long is this picture of a pencil?



A. 5 cm
B. 10 cm
C. 20 cm
D. 30 cm

There were many items where the performance was not particularly good. In the following discussion the authors would like to emphasise that since the students were not interviewed the reasons are only speculative. '

S5. Here is a paper clip.



About how many lengths of the paper clip is the same as the length of this line?

Answer ________________

To solve item S5 correctly the students have to visualise the paper clip being placed against the line. This is a more difficult task than if the paper clip was actually given to the students, and they may have had difficulty imagining placing the clips end-to-end.

L8. Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

Who has the longest pace?

A. Stephen
B. Erlane
C. Ana
D. Carlos

In item L8 the word pace might not have been familiar to many students and have caused difficulty. Furthermore, this item deals with an inverse relationship (i.e. the larger the pace the less steps required) which could also have been the cause of difficulty. This conclusion seems reasonable in light of the fact that virtually all students who got an incorrect answer chose A as their response, implying that the larger the pace the more steps required.

V5. How many millimeters in a meter?

Answer ________________

> M7. Which of these would most likely be measured milliliters?
>
> A. The amount of liquid in a teaspoon
> B. The weight (mass) of a pin
> C. The amount of gasoline in a tank
> D. The thickness of 10 sheets of paper

Item V5 involves the relationship between millimeters and meters. While the conversion of liters to milliliters is specifically taught in the primary three programme, the corresponding relationship between meters and millimeters is not. If students understand the role of the prefix milli they should be able to transfer their knowledge, but without this understanding of the language this conversion is impossible.

For item M7 the most common incorrect response was C – the tank of gasoline, although substantial numbers of students chose the other distracters. The word milliliters may have focused the students on the answer being either A or C, but they would be unfamiliar with the term gasoline, so would not know if a tank of gasoline was more appropriate than a teaspoon of liquid. In addition, car ownership is not as common in Singapore as in the West, so many students are less likely to have experience of purchasing a tank of gasoline (petrol).

Finally, item T3 involves a word problem on time, and the difficulty could be generically associated with word problems, a difficulty mentioned in the first part of this series (Kaur and Pereira-Mendoza, 1999).

Implications Of The Findings For Teaching Fractions And Proportionality

There appears to be a variety of difficulties with the Measurement, Estimation and Number Sense items. As with the geometry items, these difficulties may have less to do with the topic, per se, than be indicative of students underlying understanding of mathematics. First, the use of unfamiliar words causes problems. As teachers we need to be cognisant of the fact that words that are well defined for us, may be unfamiliar or ill-defined for students. Second, asking students to go from an enactive mode, such as actually measuring a line using paper clips, to doing the same activity in the ikonic mode (Bruner, 1966) can be a difficult task. The transition from the enactive mode to the ikonic mode, as suggested at the beginning of many teachers' guides (Curriculum Development Institute of Singapore, 1993a, 1993b), is a difficult one.

As with geometry, the following activity deals with the topic in a generic sense, rather than developing specific concepts.

Have students measure lengths and widths of objects using some selected non-standard measuring "units" (could be a paper clip, pencil…). Once they are familiar with the procedure, select some other objects to measure using one of the "units". Ask them to look at each object closely and predict how many "units" represent its length, width or depth.

Finally, when teaching measurement stress the meaning of centi, milli, etc. and have students apply these prefixes to other measurements and generate conversions. This type of activity focuses on relational understanding rather than instrumental understanding (Skemp, 1976).

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