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Learning About Numbers In Kindergarten

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Abstract

The study was concerned with pre-school children whose school activities range from to 2 to 3 hours to full day, providing some play-based activities, but little informal activity and oral work.

Expectations of primary schools are high, and parents are anxious about preparation. Thus, many pre-schools emphasise accuracy and achievement, neglecting the individual needs of some children who later find the primary school curriculum daunting, especially mathematics.

Reported here are observations made during oral interviews with over 200 six year olds about strategies used to solve routine addition and subtraction tasks. The findings indicate that pre-school teachers may want to modify their mathematics teaching to take account of the heterogeneity in young children's numeracy development and the inconsistent strategies used to handle routine tasks. Also reported is an attempt to help the children form firmer mental images of the number system through oral encounters involving explanations and justifications of solutions, and, identification and analyses of misconceptions.

Introduction

In an attempt to alleviate some of the difficulties kindergarten children are found to encounter, as far as the teaching and learning of mathematics is concerned, this paper describes the structured observations of children discussing mathematics tasks. The children were interviewed just prior to starting primary school and then later on in primary school. The tasks involved counting, addition, and subtraction strategies typical of what would be expected of them in kindergarten and six months later in primary one.

Counting Practices And Addition And Subtraction

Some of the research conducted into children's use of counting reveals that in pre-school, whilst they are often quite competent at sharing, can count meaningfully, and know "more" and "less", some have difficulties applying these features in different situations. For example, Munn (1994) found that when children were asked for a set number of objects they grabbed a handful rather than using their knowledge of counting to count out the required number. Although most children are able to count and to know that the last count word represents the quantity of the set, Bryant (1995) shows that this may not help them to understand differences in quantity when two sets are compared. To understand the quantitative significance of number words, what is needed explains Bryant is an understanding of one to one correspondence between two sets. Definite strategies need to be learned. Munn (1994) found that when taught to represent a quantity e.g. four, by writing 1,2,3,4, children were able to link the number sequence to the operation of counting in one to one correspondence. They were able to relate the verbal to the written number system. They were thus able to understand that the order of words in the sequence represents their magnitude, the cardinal and ordinal properties of the number sequence.

Nunes and Bryant (1996) have suggested that there is an ordered sequence through which children progress in the development of early numeracy. Firstly, children learn to count with numbers in increasing order of magnitude, they are next able to count in one-to-one correspondence as a measure of set size, they then realise that numbers as measures of set size are related to addition and subtraction. This knowledge enables them to handle the complexities of problems represented in words with countables and /or pictures of things to count. Finally they are able to manipulate numbers related to set size, or transformations such as when handling money. Hence, simply being able to count doesn't necessarily mean that the purpose is understood or can be applied.

The usual sequence for teaching simple addition begins by counting all of the objects in the two sets to be added and then progresses to counting on from the set with a smaller number to the set with a larger number. With further practice there is an understanding of commutativity i.e. that two and four more is the same number as four and two more. This knowledge also enables similar strategies to be applied for subtraction where they need to either count up from the smallest number to the largest, or down from the largest number to the smallest. However, whilst children can use this knowledge of counting to solve simple addition and subtraction problems with countables, they have difficulties when problems are represented in words. They are uncertain of which strategy to use. In order to solve

such problems, children need to understand part – whole relationships between numbers and the inverse relationship of addition and subtraction and this is reflected in their ability to solve word problems presented in a variety of contexts (Bryant 1995).

Other studies have shown that when maths concepts arise from activities rather than being imposed on children, they are more successful in transferring skills and knowledge learned in one context to another (Brown et al. 1989, Lave 1988).

Furthermore, being able to match the teaching of appropriate strategies to children's attainment levels is clearly important for teachers as other studies have highlighted. Aubrey (1997) attributed a mismatch between pre-school children's responses and teachers' questions in structured mathematics teaching situations to teachers' lack of subject knowledge. Kleinberg and Menmuir (1995) noted teachers' inability to know when mathematics learning could arise incidentally during the preschool day. In this study, the naming of colours was noted as being mathematical, whilst the building of patterns was not. However, Peterson et al. (1989) found that the more teachers knew about their children's mathematical knowledge, the better the children were at solving word problems. They found that knowledgeable teachers questioned children about problem solving approaches and listened to responses whereas less knowledgeable teachers explained problem-solving processes or just observed their children's solutions. Hughes (1986) has also demonstrated the importance of teacher talk and observation to check pupil understanding.

Clearly, knowing how children use counting, how they handle addition and subtraction tasks, and how they apply these strategies to problems represented in words, and in real life situations, is crucial to the teacher in guiding children to select and apply appropriate strategies. Given these findings, what of the children observed in this study.

Initially, a small pilot study was conducted to identify some of the problems children were reported to have experienced with mathematics on entering primary school (Sharpe 1998). This study showed that the problems centred largely on the children's inability to select appropriate strategies for solving simple addition and subtraction activities. It was observed that given the limited time children spend in kindergarten, the teachers mostly relied on teaching to the workbook requirements in order to cover the syllabus which lasts for eight terms. They showed children how to complete the written activities rather than teaching sets of concepts and strategies in a developmentally appropriate way with things to

count and games to play. Rarely were children encouraged to explain solutions nor were tasks individualised. Consequently, some children had difficulties such as: seeing a relationship between one to one correspondence and counting, they were unable to adopt "count on" strategies for simple addition and they were unable to grasp the difference between the cardinal and ordinal values of numbers.

The purpose of the study reported here is to investigate more closely how children handle the counting, addition and subtraction tasks in kindergarten and whether the difficulties persist into primary school. For the purposes of this study, observations were made of children in kindergarten programmes immediately prior to entry to primary school, and, six months later in primary school.

Methodology

The Children in Kindergarten

The first set of interviews were conducted at 4 kindergartens with 158 children aged between 6 and 7 years during their last few weeks in kindergarten. All of the children had spent 2 years at the same 4 kindergartens and followed the same syllabus. The duration of the programme was 4 hours per day, for 5 days per week. All the children experienced 1 mathematics lesson per day.

A typical mathematics lesson would comprise an introduction to the workbook page. Here the teacher would describe what they should do on the page and would then go through examples of how to solve the tasks. The set text of the workbook is used by the majority of kindergartens in Singapore and follows a thematic approach, with the features of the particular theme dominating the way a concept would be taught.

For example, the unit on "Our Pets", is one of 3 units covered during the final term of their first year in kindergarten. The children would be typically five plus years old. It includes instructions and pictures of activities to be completed by the children to encourage "counting backwards", "taking away", "ordering sizes", "matching sizes", and "ordering sets". Mathematical language includes: small, smaller, smallest, big, bigger, biggest. The children are expected to complete activities involving these strategies by drawing lines, colouring, filling in the blanks or by using stickers. Similar strategies would be repeated with other units but not sequentially.

The pilot study however showed that there appeared to be little continuity and few opportunities for children to apply these strategies in realistic ways, such as in play, in real life situations or incidentally during the kindergarten day. As a result of these observations some of these tasks were selected for the study reported here. The purpose being to establish whether the children could in fact complete the tasks orally and whether they could select the appropriate strategy required for different activities. For the purposes of the interviews, the tasks were conducted individually and orally with the responses, both verbal and gestural, recorded verbatim.

The first sets of tasks were chosen to judge the extent to which the children could use counting successfully with objects to be counted. Conservation of number tasks were also included to assess whether the children had grasped the significance of comparing sets by one to one correspondence.

The first task involved counting an array of countables with no obvious starting point. The purpose of this task was to judge whether they could count and know that the last number name represents the cardinal value of the set. The next task involved producing a specified quantity of countables, and was to judge whether they could use their knowledge of counting. The next task required them to create an equivalent set when asked: "give me as many blue ones as red ones" This tested their ability to apply their knowledge of cardinal value. Finally, the number conservation task indicated whether or not the children had grasped the significance of counting which is knowing cardinal values and being able to mentally link one to one correspondence with cardinal numbers.

The next set of tasks was selected from the mathematics workbooks, which all of the children had worked through in sequence over the previous eight kindergarten terms. Rather than completing the tasks in the workbooks the children were asked to respond orally as the tasks were read out. The tasks comprised pictures for countables, and required the following skills:

1. count all (counting all the objects in the array),
2. count on (adding on to the first addend by counting on to the second addend),
3. counting up (counting up from the value of the numeral to be subtracted to the value of the numeral from which the subtraction is to be made),
4. counting down (beginning with the value of the numeral from which the subtraction is to be made, the child counts down or back to the value of the numeral which is to be subtracted),

5. counting on from a given set,
6. comparing the cardinal value of sets using "more than" and "less than",
7. shopping tasks involving subtraction sums with pictures of coin values, e.g." The bananas cost 17 cents. I give the fruit seller 20 cents. How much change will I get?"
8. word problems such as: "If you have 6 blocks and you hide 3 how many would be left?"

The Children In Primary One

Six months later, 126 of the original 158 children were interviewed again in their respective primary schools. All primary schools in Singapore follow the same mathematics syllabus, with the same topics and workbooks and lesson duration. The tasks selected were taken from an assessment paper all primary one children would have taken as part of their first end of term mathematics assessment.

The tasks selected for these interviews were without pictures as countables and were again conducted orally, and objects for counting were not provided. This was because the children would be expected to complete the tasks mentally by this stage. The tasks required the children to respond verbally demonstrating the following skills:

a) Knowledge of numbers in a series.
e.g.: **"Fill in the missing numbers: 10 __ __ 13 __ 15__ __ 18 19 __**

b) Comparison of sets using "less than" and "greater than" when comparing sets of dots.

c) Number decomposition.
e.g.: "Fill in the missing numbers: (9)← (15)→ ( )

d) Place values.
e.g.: "11 is 10 and ____

e) Creation of addition and subtraction sentences with numerals and symbols.
e.g. **Complete the number sentence about hearts**

(The child would be expected to complete the number sentence by filling in the blanks below:

4 + 5 = 9)

_____ + _____ = _____

f) An "equalise" word problem.
Iskander buys 6 fish. He puts 1 fish in each jar. He only has 4 jars.
How many more jars does he need ?
(The child would be expected to complete the number sentence by filling in the blanks below:

6 + 4 = 2)

_____ + _____ = _____

g) A "compare" word problem
How many coins must Siti give to Poh Cheng so that both of them have the same number of coins?
(0000000) Siti (000) Poh Cheng
Siti must give ____ coins to Poh Cheng

An Analysis Of Responses

A qualitative analysis of the children's performance on the two sets of tasks at kindergarten and primary school was conducted. It is not the intention at this stage in the study, to report any correlations between the performances, nor to suggest any causal relationships.

For the purpose of the comparisons the performances of the children on the kindergarten conservation tasks were analysed. This was because of the importance for children, of understanding number conservation for success in addition and subtraction. Three discernible groups were revealed. Of these, 40 children were non-conservers, 43 children were conservers who needed to count, and 43 children were able to conserve number mentally. All of the children were able to complete the initial counting tasks, and as expected, their grouped performances on all tasks revealed the superiority of the conserving children over

the non-conservers at kindergarten and in primary one, yet there were some interesting differences between groups on some of the tasks.

A glance at tables 1 and 2, which are represented graphically in figures 1 and 2, reveals that at kindergarten the conserving children relied less on the need to "count all", and "count on by set" when given pictures of sets to count. They also used fingers more than the non-conservers, who were not certain which kind of strategy to employ. The conservers demonstrated an understanding of "more than" and "less than". Very few of the children used "counting up" for simple subtraction tasks with pictures of countables, but more conservers were able to "count down". Just over half of the true conservers were able to complete the tasks involving coins and word problems.

Table 1. Observed responses to addition & subtraction tasks in Kindergarten, expressed in percentages (N = 126)

Kindergarten Tasks	F	CA	CS	CO	CU	CD	M/L	C	WP
Non – conservers N = 40	41	92	3	8	36	0	18	0	8
Counting conservers N = 43	70	72	28	56	40	23	51	33	42
True conservers N = 43	72	56	56	67	37	21	53	56	53

Key: F = used fingers
CA = counted all
CS = counted by set
CO = counted on
CU = counted up
CD = counted down
M/L= more/less
C = shopping tasks with pictures of coins
WP = word problems

Table 2. Observed responses to addition & subtraction tasks in Primary School, expressed in percentages (N = 126)

Primary One Tasks	O	M/L	EW	ND	PV	NS	CW
Non – conservers N = 40	90	69	21	46	23	8	0
Counting conservers N = 43	100	88	35	63	42	33	19
True conservers N = 43	100	86	42	81	53	40	47

Key: O = numbers in order
M/L= more/less
EW = word problem/equalise
ND = number decomposition
PV = place value
NS = number sentences
CW = word problem/compare

Figure 1: Observed Responses to Addition and Subtraction Tasks - Kindergarten

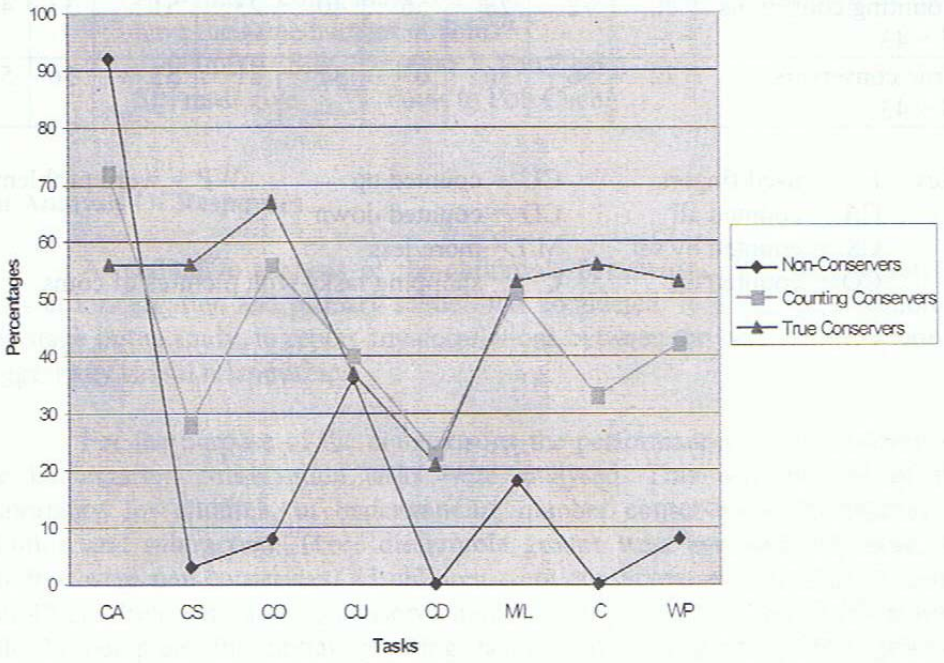
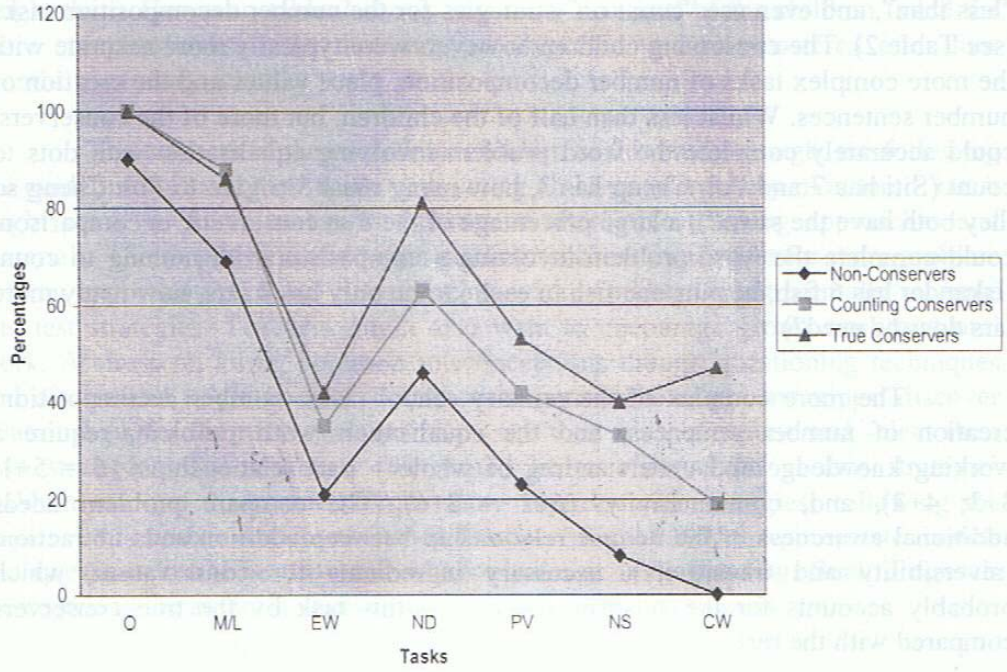


Figure 2: Observed Responses to Addition and Subtraction Tasks - Primary 1



The tasks relating to real life situations, the shopping tasks with pictures of coins required the children to demonstrate knowledge of part/whole relationships. These tasks were beyond the majority of children, many of whom could count on from a set for addition tasks, who were unable to see the inverse relationship between addition and subtraction in these tasks. The children were observed to look at the pictures of the coins and count each as one cent instead of recognising the differences in the values between one cent, five cents, and ten-cent coins. Furthermore, the word problems required the children to compute addition or subtraction sums with very small numbers and they were encouraged to use countables. Even so, less than half of conserving children were successful.

Clearly, at the end of kindergarten, less than half of the total group (only 56% of conservers solved coin problems, and 53% word problems) were able to complete the tasks expected of them by the end of kindergarten. What then are the implications of these observations for the children's performances on primary school tasks?

When interviewed later on at primary school, most of the children could arrange numbers in a series and demonstrate an understanding of "more than" and "less than", and even use "count on" strategies for the number decomposition tasks (see Table 2). The conserving children however, were typically more accurate with the more complex tasks of number decomposition, place values and the creation of number sentences. Whilst less than half of the children, but more of the conservers, could accurately complete the word problem involving equalisation with dots to count (Siti has 7 and Poh Cheng has 3, how many must Siti give to Poh Cheng so they both have the same?), a large percentage of the true conservers, in comparison, could complete the word problem involving a comparison with nothing to count (Iskander has 6 fish, he puts one fish in each jar, he only has 4 jars, how many more jars does he need?)

The more complex of the primary school tasks: number decomposition, creation of number sentences, and the equalisation word problem, require a working knowledge and understanding of whole – part relationships: (6 = 5+1; 3+3; 4+2), and, commutativity: (6+2 = 2+6). The compare problem needs additional awareness of the inverse relationship between addition and subtraction, (reversibility and transitivity), necessary ingredients for conservation, which probably accounts for the superior success on this task by the true conservers compared with the rest.

The success on these more complex tasks by the majority of the conserving children compared with the non-conservers can perhaps be explained by the increasing sophistication of the strategies they use to find solutions. Although discouraged by their teachers for doing so, they are more likely to use their previously reliable tools, their fingers, and count by sets when "counting on", "up" and "down" in kindergarten. The unsuccessful children were by and large confused or uncertain as to which strategy to employ. For the compare word problem, many children were unable to recognise the need to subtract, or arrive at the correct solution verbally. Some children found the correct solution but provided an inaccurate number sentence. As noted earlier (Bryant 1995), tasks such as word problems tells us much about how children represent maths tasks to themselves mentally and a glance at Figures 1 and 2 shows that the gap between the three groups' performances on the word problems at primary school and kindergarten remains the same. It may well be that their earlier exposure to inappropriate mathematical experiences may have contributed to the their lack of knowledge and their inability to mentally apply what they know about addition and subtraction.

In sum, a comparison of performances of the same children at kindergarten and primary one, shows that all of the children have made some

progress, especially with tasks involving "more/than" and "less/than", and an understanding of number decomposition which their primary school teachers have taught them. However, table 2 shows that more than half of the children failed the more complex tasks, tasks that they are expected to have mastered after four months in primary school.

The teachers of the pre-school children in this study though, are following the guidelines set and their work with young children is regularly monitored, but it might prove useful to assess the impact of alternative teaching approaches and materials. The emphasis could be on guiding teachers to provide for more problem solving opportunities in real life situations where children might mentally rehearse and test strategies. Teachers might also want to encourage group work and oral work. A stress on airing common misconceptions though questioning techniques and discussions might enable children to negotiate their own meanings, discover relationships for themselves, learn rules, ask their own questions and select their own strategies. The teacher's role would be to observe the various strategies children use and direct attention to more sophisticated strategies, following the sequence of number development noted earlier (Bryant 1995). Appropriate models and questions could be provided which challenge understanding. Such proposals are central to the author's intervention study currently in progress.

Conclusion

In order that pre-school teachers might better prepare children for the more complex thinking skills required of them in primary school, they might want to consider alternative teaching approaches and materials at kindergarten. They might also want to pay more attention to the individual differences in how children think about mathematical problems and the strategies they employ. This will require from teachers, both a revision of expectations of children at the different stages in their mathematical understanding, and a thorough grasp of mathematical knowledge in order to present concepts meaningfully to children in relation to the real world.

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