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Affecting K-9 Teachers' Beliefs To Improve Instruction In Mathematics

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Abstract

As part of her undergraduate honors project with me, Antonia, a K-9 mathematics specialist, assisted in analyzing data and writing about the beliefs of preservice teachers prior to and after taking a mathematics methods course taught by me. Our findings indicate that K-9 preservice teachers whose beliefs at the end of the course reflect a more cognitively based approach to teaching and learning than at the beginning of the course report they would make instructional decisions differently than when they initially held a more direct instruction approach to teaching mathematics. We discuss what beliefs changed, what we believe influenced those changes, and some things K-9 classroom teachers can do to affect changes in their own beliefs if they choose to teach using a cognitively guided approach.

We know what teachers believe about teaching and learning mathematics influences how they make instructional decisions, what they do in their classrooms, and what and how their students learn (Lubinski, 1994). We also know that even if teachers' beliefs about teaching mathematics change, they may or may not change their instructional practice (Franke, Fennema, & Carpenter, 1997). Furthermore, we know that most teachers believe mathematics is a set of rules to be learned and that the teachers' role is to explain those rules clearly (Ball & Mosenthal, 1990).

Currently there is much discussion in the mathematics education community about how mathematics should be taught (National Council of Teachers of Mathematics 1991; Mathematical Sciences Education Board, 1996). In reform-based classrooms, teachers are being encouraged to develop children's mathematical reasoning rather than telling children how to think. However, these same teachers have primarily learned mathematics by listening and mimicking procedures. Therefore, we ask how can they create reformed-based learning environments when they have never experienced one? Furthermore, how can we influence teachers' beliefs so that they can create reform-based learning environments?

The purpose of our work is to explore teachers' beliefs and discuss how changes in beliefs can be influenced. Our ultimate goal is to provide teachers with information on students' learning in order to positively effect the teaching and learning of mathematics within their own classrooms. In order to do this, we know it is necessary to challenge teachers' beliefs about the teaching and learning of mathematics.

Background

Our study focuses on K-9 preservice teachers (N=25 at the onset of the data collecting and N=20 at the end); however, we have similar findings from our work with experienced K-9 teachers and we will connect to those findings as appropriate. The preservice teachers in this study took a mathematics methods course at a mid-sized university in the midwest. One focus of the course was to provide information about how students learn mathematics in the content areas of whole number operations, geometry, and fractions. In addition, preservice teachers were required to read and discuss articles on students' learning, to develop assessment interviews to determine how individual students think about solving problems, to interview students, and to plan instruction based on the interview findings. Furthermore, preservice teachers were encouraged to be reflective about what they read and about what they observed in elementary classrooms. Journaling was an ongoing course requirement. Being able to write in-depth reflections about the teaching and learning mathematics process at the K-9 level was a course goal.

Two questions relate to our work: 1) What can a college mathematics methods instructor and a preservice undergraduate mathematics specialist learn about K-9 preservice teachers? and 2) How can what they learn help K-9 experienced teachers improve their mathematics instruction?

The Methods Course

The mathematics methods course provided information to the preservice teachers on children's thinking. Materials were used from the Cognitively Guided Instruction Project (Fennema & Carpenter, 1988), Nancy Mack's work on fractions, and various articles primarily from the National Council of Teachers of Mathematics journals. A partial list of articles is as follows:

1. Carey, Deborah A., "The Patchwork Quilt: A Context for Problem Solving"

2. Carey, Deborah A., "Students' Use of Symbols"
3. Fennema, E., & Carpenter, T. P. *Cognitively guided instruction: A program implementation guide.* Chapters 1 to 10.
4. Loef, Megan, et al., "Integrating Assessment and Instruction"
5. Lubinski, Cheryl A., "The Influence of Teachers' Beliefs and Knowledge on Learning Environments"
6. Mack, Nancy K., "Making Connections to Understand Fractions"
7. Nagasaki and Becker, "Classroom Assessment in Japanese Mathematics Education"
8. Skemp, Richard R., "Relational Understanding and Instrumental Understanding"
9. Thompson, Patrick W., "Concrete Materials and Teaching for Mathematical Understanding"
10. Vacc, Nancy Nesbitt, "Questioning in the Mathematics Classroom"
11. Vacc, Nancy Nesbitt, "Teaching and Learning Mathematics Through Classroom Discussion"

Tasks for the course were carefully selected in order that preservice teachers reflect on the decisions they would make as teachers. They were encouraged to cite sources from their readings to support what they would do in their own classrooms. They were encouraged to reflect on the mathematics they were going to teach in order to develop their own in-depth understanding. For example, they were given the following assignment on division of fractions to be completed in three weeks:

> *Understanding division of fractions is difficult. Most people invert and multiply to obtain a correct answer. Discuss how you can develop your own understanding of division of fractions by providing a specific example that illustrates an in-depth understanding of division of fractions. Provide a variety of ways that you used to develop that understanding including pictures and algorithms that illustrate reasoning and depend on understanding. Make connections among your representations. If you are working in a group, you may not choose to use the same example to illustrate your point. That is if you use* $\frac{7}{8} \div \frac{3}{4}$ *(do not use these numbers, this is only an example) as your fraction numbers, others in your group must select different numbers.*

This task was completed by the preservice teachers in several different ways, reflecting various degrees of thought. Some did not go much beyond invert and multiply. Others drew pictures and explained how their pictures related to the answer (See Lubinski, Fox, and Thomason, 1998 for a more in-depth look into this task).

The course had an underlying objective that is reflected in Cooney's statement that describes the teacher's role as that of "helping students believe that mathematics truth is a function of reasoning rather than a declaration from authority" (Cooney, 1993, p. 43). Helping the preservice teachers to realize the meaning of this objective was part of every task assigned in the course. Typically mathematics is taught from an authoritarian perspective, focusing on right or wrong answers. Focusing on the reasoning process is not typical in the majority of mathematics classrooms. In this course preservice teachers were encouraged to elicit strategies from their students on how problems might be solved, not to provide strategies for their students to use to solve problems. This course focused on appropriate reasoning processes as well as correct answers.

Focusing on reasoning processes implies not only asking the elementary students to solve problems in a variety of ways, but also to represent their thinking symbolically and to make connections among the various representations from any given problem. For example, take the problem: If there are five shirts and each shirt has four buttons, how many buttons are there? Some students may count the buttons individually. Others may count by fours such as 4, 8, 12, 16, 20; 4 plus 4 is 8, 8 plus 8 is 16 and one more 4 is 20; or others might know five fours is 20. The representations for each of these processes differ and are respectively:

1) 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 +1 = 20;
2) 4 + 4 = 8, 8 + 4 = 12, 12 + 4 = 16, and 16 + 4 = 20 or 4 + 4 + 4 + 4 + 4 = 20;
3) 4 + 4 = 8, 8 + 8 = 16, and 16 + 4 = 20; and
4) 5 x 4 = 20.

It is important that students connect the 5 in the last equation to the number of shirts and the 4 to the number of buttons on each shirt. Equally important is for students to realize the connection of the five fours in the second example to the number of shirts and the 5 in the last equation. There are many symbolic connections that can and should be made with this one problem. For example, if the student thinks of just the first buttons on all shirts, then the second buttons on all shirts, then the third buttons on all shirts, and finally the fourth buttons on all shirts, the representation might be 5 + 5 + 5 + 5 or 4 x 5. Reflecting on this type of activity was important to developing the preservice teachers' content and pedagogical content knowledge (knowledge of how students learn and how to teach using students' thinking).

Collecting Information

In order to determine what impact the course had on the preservice teachers, we gathered information about their beliefs using several written surveys which will be described in detail. These surveys were administered at the onset of the course and again during the final week of the course.

The Mathematics Belief Survey (adapted from Fennema, Carpenter, & Peterson 1989)

The Mathematics Belief Survey consists of 48 statements related to the learning, teaching, and content of mathematics at the primary level. Responses were recorded on a five-point Likert scale that ranges from "strongly agree" to "strongly disagree." The survey can be divided into four constructs related to either teaching (Constructs II and IV), learning (Construct I), or the content of mathematics (Construct III). The Likert scale for each construct reflects a range in beliefs as follows:

> *Construct I goes from the belief that children receive knowledge from an authority figure (such as a teacher or textbook) to the belief that children construct their own knowledge of mathematics.*
>
> *Construct II goes from the belief that skills should be taught in isolation to the belief that skills should be taught in relationship to understanding and problem solving skills.*
>
> *Construct III goes from the belief that formal mathematics provides a basis for sequencing topics for instruction to the belief that children's natural development of mathematical ideas should provide the basis for sequencing topics for instruction.*
>
> *Construct IV goes from the belief that mathematics instruction should be organized to facilitate the teacher's clear presentation of knowledge to the belief that mathematics instruction should be organized to facilitate children's construction of knowledge (Fennema, Carpenter, & Peterson, 1989, p. 183).*

In summary, the constructs represent the more general beliefs that go from a less (reflected in a low score) to a more (reflected in a high score) cognitively based perspective. Overall scores range from 1 (lowest) to 5 (highest).

<u>Learning Context Questionnaire</u> (Chapman & Griffith, 1982)

The Learning Context Questionnaire (LCQ) is based on the work of William G. Perry (1970). The questionnaire has 50 items and teachers choose responses from a six-point Likert scale that ranges from "strongly agree" to "strongly disagree". Sample statements are "I like classes in which students have to do much of the research to answer the questions that come up in discussions" and "I am often not sure I have answered a question correctly until the teacher assures me it is correct". Scores reflect a perception of learning that is termed Dualistic, Multiplistic, Relativistic, or Dialectic. A low score reflects immature cognitive development and describes preservice teachers who look to authority figures for correct answers for making decisions (such as a textbook). The Multiplistic preservice teachers realize there may be different perspectives to learning and teaching; however they do not differentiate or evaluate among perspectives. Anything goes! They still tend to look to authority figures for help in finding answers and they have little confidence in their own ability to find solutions. At the upper end of the scale preservice teachers take a more critical look at situations and rely on their own ability to find solutions to problems. They have developed their own view by which they evaluate information, they recognize diversity of opinions but come to their own views through assessment of those opinions.

<u>What Is Mathematics Survey</u> (Lubinski & Otto, in press)

The Survey on Mathematics was developed at Illinois State University. The survey is essay in format and poses the following open-ended questions:

- *1.* *How would you respond to someone in this class who asks, "What is mathematics?"*
- *2.* *What does it mean to understand mathematics?*
- *3a.* *How do you best learn mathematics?*
- *3b.* *What is your approach for beginning a word problem that uses mathematics?*
- *4.* *Explain the role of reasoning when doing a problem that uses mathematics.*
- *5a.* *Describe the role of a mathematics teacher.*
- *5b.* *Describe the role of a mathematics student.*
- *6.* *Describe a memorable mathematical experience you have had.*
- *7.* *What is your mathematics background? List/describe the mathematics courses that you have taken (7-12 and college).*

This survey provided us with information on how the course affected the way the preservice teachers' beliefs changed on many issues, such as the role of a mathematics teacher and the mathematics students.

Difficulty & Ranking Task (adapted from Fennema & Carpenter, 1988)

The "Difficulty and Ranking Task" is another instrument used in which a series of mathematics problems appropriate for the elementary classroom are presented. Teachers are to rank order the problems according to their perceived level of difficulty and then assign grade levels at which each problem would be appropriate to use. Two example of problems on this survey are: "Kaitlin has seven dollars. How many more dollars does she have to earn so that she will have eleven dollars to buy a puppy?" and "There are 19 children going to a circus. If 5 children can ride in each car, how many cars will be need to get all 19 children to a circus?" This instrument provides information about what teachers believe children are capable of doing or thinking about at various grade levels.

Findings

The course had a significant impact on the way the preservice teachers answered the questions.

The Mathematics Belief Survey

Since the course focused on children's reasoning abilities and how to teach using those abilities, it is not surprising that all preservice teachers scores increased on this instrument, reflecting a more cognitively based approach to teaching and learning. The range of scores initially was from 2.73 to 4.51 with 3.44 being the median score. At the end of the course scores ranged from 3.86 to 4.88 with 4.405 being the median. This change is significant. This finding reflects a similar pattern found among inservice teachers who have had access to information about children's learning. However, we find preservice teachers scores typically reflect more extreme changes quantitatively than inservice teachers.

Learning Context Questionnaire

From the LCQ, we found that in the beginning of the semester the preservice teachers' scores ranged from Dualistic to Relativistic. Most were Multiplistic, which is what we have also found among the experienced teachers with whom we have worked. By the end of the semester, two moved to the most mature level of cognitive development which is Dialectic. The number of Relativists moved up by two. There were still a majority at the Multiplistic level. The number of Dualistic changed from seven to two. By the end of the semester many of the preservice teachers' scores reflected a higher level of cognitive development than at the beginning of the semester.

What Is Mathematics Survey

Initially, most preservice teachers (N=12) wrote that mathematics is using numbers, equations, and shapes to solve problems. Understanding "why" was mentioned only once. When asked what it means to understand most preservice teachers (N=11) wrote that following directions to find the most efficient way to solve problems showed understanding. Most (N=15) believed that following directions, seeing examples and repetition was the best way they learned mathematics. After the course, most preservice teachers (N=13) wrote that mathematics is a system of numbers or problems or formulas and how they relate. Understanding "why" was mentioned by four. When asked what it means to understand mathematics, most (N=15) wrote about understanding concepts, knowing how and why one is going about solving a problem, struggle, application, and personal understanding. Only one believed that following directions, seeing examples and repetition was the best way for her to learn mathematics. Most preservice teachers (N=11) mentioned tactile experiences, doing mathematics first hand, and personal understanding as the way they best learn mathematics.

The role of reasoning changed after the class and this change can be best exemplified by comparing one preservice teacher's response at the onset of the course when she wrote:

> *Reasoning is to figure out a problem in a logical way, to figure out how the numbers work together in problems, to figure where numbers go and when, what is right and what is wrong, how to get a correct answer.*

to her response after the course when she wrote, "Reasoning helps to give explanation and support for thinking."

Initially, 11 preservice teachers stated that the role of the mathematics teacher is to help students solve problems on their own, discover things themselves, and steer students to the correct path. Most (N=15) believed a mathematics student is to follow directions or listen then apply or practice and participate. After taking the course, 14 preservice teachers wrote that the role of a mathematics teachers is to help students solve problems on their own or discover things themselves or steer students to the correct path however, in regard to students, most preservice teachers (N=14) wrote abut having them struggle through problems, reach their own understandings, or continuously ask the teacher "why?" Prior to the course several wrote about teachers making math fun. This was not mentioned after the course. (Aside: We had discussed in class about fun being an outcome, not a goal. That is, mathematics can be fun for the student if the student is being challenged and is achieving. A teacher does not have to plan fun activities.)

Questions 5a, 5b, and 6 were analyzed for preservice teachers perceived locus of authority. That is, we wanted to determine who they saw as the authority, was it internal (themselves) or external (e.g. the textbook or their teachers). Doing this would provide additional data that could be compared to their beliefs about teaching primary-age students (Belief Survey) and to their beliefs about their own learning (LCQ).

Preservice teachers' responses were categorized by two university instructors and two undergraduate mathematics specialists as either Internal, External, or Mixed (or unclear) in relation to their locus of authority. In the initial survey preservice teachers were evenly distributed among the three categories in regard to the role of the mathematics teacher. In the final survey, all but one response was categorized as Internal. An example of a typical response at the end of the course was,

> *A mathematics teacher's role is that of a facilitator. They are to present students with opportunities that will help them gain an in-depth understanding of mathematics. They are to foster students ideas. They are to present real-world contexts that are relevant to students.*

For the question about the role of the mathematics student, again the preservice teachers' responses initially were distributed evenly among the categories. At the end of the semester, all but two responses were categorized as Internal. A typical end-of-course response was, "To develop understanding and to reason and think through. To share their ideas and strategies. To listen to others' strategies."

Initially for the question about "your most memorable mathematics experience" most responses were categorized as External. For example one preservice teacher wrote,

> *I am very good in math and in 5th and 6th grade that is when I really started believing it. Our class always played "Around the World." each student would compete against another and whoever*

> *won moved on to the next student. I was always proud of myself for always knowing my math facts.*

The same preservice teacher referring to the division of fraction assignment wrote at the end of the course,

> *Understanding the reasoning behind the division of fractions. I actually understand how it works instead of just doing the computational. I can think about it and explain why.*

At the end of the semester all responses except one were Internal.

We concluded that preservice teachers' beliefs about teaching and learning mathematics changed as did their views about their own learning. End-of-course responses reflected taking more responsibility for their own learning than beginning-of-course responses. Furthermore, they placed more importance on understanding mathematics in-depth after completing the course. By the end of the course, ten of the preservice teachers listed the fraction task as being their most memorable mathematics experience.

<u>Difficulty & Ranking Task</u>

The preservice teachers' decisions changed in regard to the difficulty of problems from the beginning of the semester to the end of the semester. The results of the Difficulty and Ranking Task showed that in the beginning of the semester most preservice teachers were recording that the problems were appropriate for children in grades K-8. Many recorded that simple addition word problems could not be handled by young students. Others felt that multiplication concepts could not be understood until later in the elementary curriculum.

Upon completion of the course, there was a major shift in the grade-level range of the problems that were ranked. Most preservice teachers wrote that the problems could be completed at approximately the K-2 level. At the end of the course more preservice teachers than at the beginning of the course stated that even multiplication word problems could be solved by kindergartners.

These findings indicate that the preservice teachers changed their initial assessment to reflect the belief that students at much younger ages could complete the same problems that were in the Difficulty and Ranking Task at the beginning of the semester. This finding is similar for experienced teachers.

Discussion

At this time, we return to the two questions posed at the beginning of the article, 1) What did we learn about preservice teachers? and 2) How can what we learned assist experienced teachers with their teaching? We will discuss each question in relation to teachers' beliefs about students' learning as reflected in responses on the Belief Survey and the Difficulty and Ranking Task and in relation to teachers' beliefs about their own learning as reflected in responses on the LCQ and What is Mathematics Survey.

We learned that preservice teachers' beliefs can be influenced by reading and discussing articles on how children learn. The changes in their beliefs reflected a cognitively guided perspective of teaching and learning that influenced the decisions about instruction they report they would make. Furthermore, preservice teachers can learn to believe that young students can solve a variety of problems at earlier ages and that young students can achieve much more mathematically than once thought possible. During the course, manipulatives were used for representing thought processes. Preservice teachers observed that when young students used manipulatives in a way that had meaning for them they could often solve problems that might not be introduced until much later in their curriculum.

These findings have implications for instruction for experienced teachers. Teachers need to have their students solve a variety of word problems. Students will learn their basic facts as they do so. Teachers need to listen to their students' solution strategies and assist their students in making connections among strategies. For example, one of our first grade teachers with whom we work has her students solve problems involving addition, subtraction, multiplication, and division. Her students also solve problems involving fractions and geometry. A typical problem for her class in early November was, If there are three pumpkin pies and 12 people, how much pie does each person get if each person gets the same amount? Students drew pictures of either dividing each pie by fourths or twelfths. The students discussed the relationship of one-fourth to three-twelfths. We have noted that the more this teacher learns about children's thinking the more she modifies her decisions about curriculum. Furthermore, her own knowledge of mathematics has improved. For example, after reading the book, *Sea Squares* (Hulme, 1991), she had her students represent square numbers with blocks (e.g. 4x4, 6x6, etc.) so that they could see the geometry of square numbers. This instructional decision reflected a knowledge of mathematics not observed in our initial work with this teacher.

We also learned that reflection is very important to the changing beliefs process. We found that the preservice teachers who reflected in depth about the mathematics they learn, as for example on the division of fraction task, came to believe that a high level of understanding is important. The fraction task was the single-most mentioned task that influenced preservice teachers' ideas about what it means to learn and completing the task broadened their view about what it means to understand.

The implications of this finding to experienced teachers relates to an awareness of one's own level of thinking in order to move towards a more reflective level. Reflecting on how one makes decisions about teaching for understanding is important and requires one to be aware of one's own thinking. Teachers need to experience talking about their own thinking, listening to the reasoning of others, making connections among a variety of solution strategies in order to improve their own understanding of mathematics. Furthermore, in order to teach for in-depth understandings of concepts and procedures, it is necessary for teachers to reflect on their role in the classroom. Many times, facilitating learning is more beneficial to students than providing information. Finally, we feel that reflecting on the students' and teacher's roles in a mathematics classroom is important to attaining greater learner outcomes.

Reading, reflecting on what is read, and experiencing similar results as our teachers (both preservice and experienced) will also allow teachers to change their beliefs, modify their instructional decisions, and ultimately challenge their students mathematically. It is important to note that all of this takes time, but the results for both teachers and their students are worth it. We believe this because this is what we learned from our work with teachers.

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