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THE USE OF MAPLE IN FIRST YEAR UNDERGRADUATE MATHEMATICS

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Abstract

This paper describes our experience in incorporating Maple into a traditional First Year Undergraduate Mathematics course. We describe our approach to the overall organisation of the course, illustrating the use of Maple in specific topics. We also report on the results of a student survey on the use of Maple, noting a few areas for improvement. Overall, the non-intensive approach has had a positive impact on the course. Although this report is on a First Year Undergraduate course, the principles of the approach may be applied to the teaching of Mathematics at other levels as well.

Introduction

The purpose of this article is to describe the implementation of the Computer Algebra System (CAS), Maple V, in our First Year Undergraduate Mathematics programme at the National Institute of Education (NIE).

We believe that CASs are an important tool in the teaching and learning of mathematics at the tertiary level. There are several reasons for using CASs; firstly, it takes the drudgery out of burdensome calculations so that students can focus on mathematical ideas. Secondly, it encourages exploration and discovery when used along with properly designed worksheets. Furthermore, through the use of CASs, students may be led to see patterns and relationships, and form conjectures before they embark on formal proofs. Finally, interesting and more realistic problems may be considered without having to manipulate the data to make the algebra come out right.

As a first step, we introduced Maple V into the First Year Undergraduate Mathematics course comprising topics in Calculus and Linear Algebra. We hope that this will lead the way to further use of the CAS in upper level courses.

Background

This section describes the background and the logistic details of incorporating Maple into our Mathematics programme.

Logistics

At the NIE, computer laboratories are equipped with personal computers linked via local area networks. A typical laboratory has twenty PCs and an instructor PC. All PCs are IBM compatibles running Windows95 operating system with an Intel Pentium 200 MHz processor, 32 MB RAM and a relatively large hard disk space. However, students are reminded to copy their files from the hard disk to their own floppy disks after completing their work as these PCs are shared networked PCs.

The University has obtained a campus-wide site license from Waterloo Maple Software to install the complete professional version of Maple V (Release 4) on all machines in the campus. This gives our students ready access to the software. As our student enrolment is small, we required only one laboratory to conduct the laboratory sessions.

The Schedule

The First Year Mathematics course meets for two hours of lectures and one hour of tutorial a week for twelve weeks of the semester. Before the start of the academic year, we selected topics from Calculus and Linear Algebra which may be investigated readily using a CAS. The tutorials for these topics were assigned Maple laboratory sessions (see Table 1), with the remaining tutorials conducted in the regular way. As the computer laboratories tend to be heavily used during term time, we booked our sessions well before the start of the academic year.

Table 1: Schedule for the Maple Lab Sessions

Course	Week No.	Description of Maple Session
Calculus (Semester 1)	2	Introduction to Maple V
	5	Exploring Limits and Derivatives
	8	Exploring Integrals and Riemann Sums
	12	Solving Differential Equations
Algebra (Semester 2)	1	Systems of Linear Equations and Gaussian Elimination ▪ Polynomial Curve Fitting ▪ Heat Transfer Problem
	3	Matrix Algebra ▪ Defining limits in matrix algebra ▪ Verifying algebraic identities for matrices ▪ Modelling changes in cable television subscription
	5	Inverses of Matrices ▪ Conjectures concerning inverses of special matrices ▪ Cryptography
	7	Determinants ▪ Vandermonde and Hilbert Matrices ▪ Verifying properties of the determinant function ▪ Using determinants to define the path of an asteroid
	9	Vectors ▪ Balancing Chemical Reactions ▪ Hooke's Law and Lines of Best Fit ▪ Cross Products in $\Re^4$

The Introductory Maple Session

It was necessary to run an introductory session on Maple V as all the students had no knowledge of the software prior to the course. The introductory session was a 50-minute tutorial session during which students were taught the

basics of Maple V. This was done with the aid of a set of notes condensed from the Maple Learning Guide (Heal *et al*, 1996). Students were then given a worksheet to complete in their own time and submit it for marking.

The Maple Lab Sessions

The exercises chosen for the Maple laboratory sessions may be broadly categorised into the following types:

(i) Exercises to aid in conceptual understanding.
(ii) Exercises in forming conjectures.
(iii) Exercises in verifying theorems.
(iv) Computational exercises.
(v) Real-life problems.

Typically, each lab session would begin with 10 to 15 minutes of class discussion during which the instructor would select a problem for discussion. Students then spend 20 minutes working on the Maple activities. In the final 15 minutes, the class gathers for a final round of discussion during which the lesson was consolidated.

The Calculus Course

For First Year Mathematics, students are taught differential and integral calculus. The topics covered include functions and graphs, limits and derivatives in differential calculus and Riemann sums in integral calculus. These are topics which may be explored fairly extensively using a computer algebra system such as Maple. The textbook adopted is by Stewart (Stewart, 1995) but ideas for some of the Maple exercises were from various other sources (Anton, 1995; Harris & Lopez, 1995).

A sample worksheet on exploring the derivative of a function of the form a^x is used in the course. In this exercise, students use Maple V to find the derivative of say 2^x at a particular point using the definition for a derivative. They then check their answers using Maple's "limit" command. Next, they explore the derivative of a^x graphically for values of a between 2 and 3 using Maple's graphing capability. The idea is to get students to deduce from their graphs that

there is a number, say $a = e$, between 2 and 3 such that $\frac{d}{dx}(a^x) = a^x$. Since students are able to use Maple to plot the graphs of functions and their derivatives, they can concentrate on the concept of derivatives.

The Linear Algebra Course

We adopted the textbook by Anton and Rorres (Anton and Rorres, 1994) as there is comprehensive coverage of topics in Linear Algebra with a rich source of applications. However, we found the text short on exercises incorporating the use of technology. Before the course began, some effort was made looking for appropriate problems incorporating the use of technology. We drew our problems from a range of sources (see Bauldry *et al*, 1995; Deeba & Gunawardena, 1997; Grossman, 1994; Lay, 1994; Moore & Yaqub, 1998).

In addition to the scheduled Maple Lab sessions (see Table 1), students were given a Maple Lab assignment which included the following activities:

- *Matrices in Economics (Leontief Input-Output Model)*
- *Flight Connections Problem*
- *Genetically Transmitted Diseases*

As can be seen from the range of activities, most are of the applications type with a smattering of the verification and conjecturing type. This was to convey the immense usefulness of the subject, as well as the importance of technology in solving real-life problems. The focus in these problems was the formulation and the interpretation of the results generated by Maple. Questions in the worksheet often took the form of "Why would you expect this result?", or "What if a certain parameter is changed?", "Interpret the result", or "Comment on the significance of the result".

A sample worksheet on the Flight Connections Problem is given in Appendix C. In this exercise, students are required to formulate a mathematical model of a network of connecting flights between various cities. The objective is to analyze the different routes between every pair of cities using matrix algebra. Students use Maple to perform the matrix calculations quickly, allowing them to concentrate on the task of interpreting the operations of addition and multiplication of matrices in the given problem.

Additional Notes and Exercises

Besides preparing the worksheets, we also made some brief notes on the linear algebra package in Maple. Students were instructed to bring these notes with them for each lab session as a quick reference.

A typical tutorial worksheet contains two sections: pen and paper exercises taken from the textbook, and Maple exercises, indicated with **[M]** in the tutorial. As this is a first course in Calculus and Linear Algebra, we felt it was important for our students to acquire some degree of technical facility in solving problems. It was a requirement that the pen and paper exercises were attempted before the Maple exercises. Students were also encouraged to check their answers in this section with Maple.

Student Feedback

At the end of the programme, a questionnaire adapted from Fulton (Fulton, 1994) was administered to obtain student feedback on the use of Maple. 35 out of 37 students responded to the survey. Generally the feedback was favourable, with the majority of students recognising the usefulness of Maple as a tool for solving problems with burdensome calculations. It is surprising to note that while 60% of the students agreed/strongly agreed that Maple had helped them see new approaches to problem solving, only 23% agreed/strongly agreed that their problem solving skills had improved with its use. This could be because while students are able to *see* new ways of solving problems, they may not have *acquired* the necessary skills to solve those problems.

It is also worth noting that only 34% felt confident enough to use Maple independently. In order to master a CAS like Maple, one needs to spend a fair amount of time on it. It is therefore unlikely that students will feel confident enough to use Maple on their own if they have used it only during the scheduled lab sessions.

Despite some negative feedback, on the whole, students appeared to have enjoyed the course, and would prefer to continue the use of Maple in future programmes.

Future Considerations

In general, our experience with using Maple in the First Year Mathematics programme has been positive and consistent with what we had expected. We have noted some areas for improvement and have listed the following points for future consideration:

- *Care must be taken when choosing and designing activities involving the use of Maple. This is particularly important on the choice of exercises taken from textbooks. Simply lifting exercises from texts may not yield the best results. Often, one has to modify the exercises to suit one's needs. We also feel that the choice of textbook should complement the use of technology.*

- *Many students pointed out that they would like to work on the Maple assignments at home. Although the full Maple version is too costly for students to purchase, the "Student Edition" is affordable. We have observed greater confidence in the use of Maple amongst those students who purchased the Student Edition of the software. We hope that more students may be persuaded to purchase the software.*

- *While our students have made a good start at using Maple in their first year, we hope that they will continue using the CAS in subsequent courses. This may be done by getting some agreement among staff members to use Maple in their courses too.*

We are pleased with the results of our first attempt at introducing Maple into our Mathematics programme. The overall feedback from students has been encouraging and we will continue to use the CAS to improve the quality of mathematics teaching and learning at the institute.

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Appendix A

Feedback on the use of Maple V in First Year Mathematics

No.	Item	Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
1	The introductory Maple session in the computer lab taught me enough about Maple to be able to use it effectively in the course.	2	18	11	3	1
2	The use of Maple in lectures helped me to better understand the concepts of Calculus.	3	12	15	5	0
3	The use of Maple in lectures helped me to better understand the concepts of Algebra.	3	21	8	3	0
4	The Maple exercises were reasonable in length and difficulty.	2	22	10	1	0
5	Working on the Maple exercises on my own was helpful and worthwhile.	6	12	9	8	0
6	Working the Maple projects helped me to better understand the concepts in Calculus.	1	13	17	4	0
7	Working the Maple projects helped me to better understand the concepts in Algebra.	3	17	12	3	0
8	I used Maple often to check my homework or to help understand material from class.	3	14	8	8	2
9	I prefer to use Maple on my PC rather than on one of the networked computers in the computer lab.	6	15	11	2	1
10	Maple enables me to solve some problems that would be almost impossible by hand calculations.	22	11	2	0	0

No.	Item	Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
11	By using Maple, I am able to see new approaches to solving some problems.	2	19	12	2	0
12	Maple was easy to apply to problems of various kinds.	0	10	21	4	0
13	The use of Maple in this course has improved my problem-solving skills.	2	6	24	3	0
14	The use of Maple revealed aspects of Calculus that I hadn't thought about before.	1	8	25	1	0
15	The use of Maple revealed aspects of Algebra that I hadn't thought about before.	4	15	16	0	0
16	The use of Maple was satisfactorily interwoven with the rest of the course.	3	17	15	0	0
17	I have learnt Maple well enough to feel confident using it on my own.	2	10	18	5	0
18	My knowledge of Maple will probably help me in other science courses as well as in my profession later.	2	14	15	4	0
19	I enjoyed the course.	4	17	14	0	0
20	I would rather have had a Mathematics course that did not use Maple.	0	3	12	11	9