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WHAT DOES IT MEAN TO STUDY MATHEMATICS?

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Abstract

In this article, four pervading universal themes of what we do when we study mathematics were identified. These four themes involve *ideas*, *representation*, *establishing relationships*, and *connecting knowns and unknowns*. Teachers need to focus explicitly on these themes while providing opportunities for students *to study* mathematics (instead of *to learn* mathematics), if the students have to go beyond *what* they learn to *how* and *why* they learn what they learn.

After several years experience of teaching mathematics and preparing student teachers of mathematics over 3 continents, let me share my views on *what it means* to study mathematics, NOT what it *should* mean to study mathematics. The purpose here is to get the mathematics education community, especially classroom teachers, to reflect on what we do in our classrooms. I have deliberately chosen very simple examples to illustrate the above claims. This is to avoid losing the essence of the article (which addresses some fundamental issues) in the complexity of the examples. In addition, notice that this is neither a theoretical nor research article per se, but an *experiential article*.

Let me first explain why I am using the word *study* instead of *learn*. Some people tend to use the words interchangeably, but most often, we hear of *to learn* mathematics, instead of *to study* mathematics. I believe to learn is an upshot, an end product that takes place after we study, which is a process. In other words, when we learn, we gain skill or knowledge *instantaneously* through study, which requires a continuous effort (and a disposition towards making such effort) to make sense of the skill or knowledge learned. For example, when we study the context and process of developing the Pythagorean theorem, we learn the rule $a^2 + b^2 = c^2$ and how to use it. Studying the theorem should sensitize us to explore "what if we do not have a right-angled triangle, that is if the angle is less or greater than 90°?" Some people might argue that it is just semantics. However, it can also be argued that an uncritical and excessive reliance on the word *learn* has led to a greater focus on demanding from our students *what* they learned, at the expense of *how* and *why* they learned what they learned.

Now, let me address the issue of *what it means* to study mathematics. Whatever different views people have regarding what it *should* mean to study mathematics, it can be argued that underlying all those views are *pervading universal themes* of what studying mathematics means. Whatever the views of what it *should* mean to study mathematics, they all amalgamate and crystallize into the following four themes. First, that mathematics is of ideas (or concepts). Second, mathematics involves representation of ideas. Third, mathematics involves establishing relationships between ideas, and fourth, problem solving which is the focus of mathematics involves expressing the unknown(s) in terms of what is (are) known. I invite views on any part of mathematics that cannot be reduced essentially to any one or all of the four universal themes.

Ideas

Mathematics involves ideas so whenever we study mathematics, we must focus on the ideas involved and what they mean (I acknowledge there could be multiple meanings). From number, the operations on number; point, line, plane, angle; variable, equation, functions; mean, median, mode, standard deviation; event, sample space, equally likely events; sine, tangent, cosine; and differentiation, integration, just to mention a few, are all ideas. In fact, there is nothing we do in mathematics that does not involve ideas. But how often do we emphasize mathematics as ideas rather than just a collection of symbols to be manipulated?

Representation

Whatever ideas we are dealing with, we try to represent them either in concrete, iconic, or symbolic forms, for purposes of communication. For meaningful communication, we attempt to be consistent (and introduce some form of order) in our representation and share the meaning of those representations. For example, we have 2, $+$, $\%$, Σ, x, ∞, as symbolic representations of mathematical ideas. Symbolic representation tends to dominate a larger part of mathematics, may be because of the economy of its use. What is important here is that as we study mathematics, we should not lose track of the close link between the ideas and their representation. For example, *twice a number* is linked with $2x$, *a number* *multiplied by itself* is linked with x^2, while *a number plus two* is linked with $x + 2$, where the number is x in each case. Notice that the representations of the ideas are different although they all have something to do with *two*.

Establishing relationships

Establishing (or discovering) relationships among mathematical ideas (and beyond) is at the heart of studying mathematics. It is here that we can portray the beauty and power of mathematics. It is here that we can demonstrate our mathematical power (NCTM, 1989; Zarinnia & Romberg, 1992). It is here that we can create *new* mathematics. A few questions are pertinent here. How are even and odd numbers related? How do points and lines relate? How do angles and the sides of triangles relate? What is the relationship between the radius of a circle and its area? Is there any relationship between differentiation and integration? What mathematical ideas are related (and how are they related) to baking, sports, music, space exploration, just to mention a few? More generally, what relationships are there between mathematical ideas and any other endeavors in life? To be successful at answering these questions, we need to study mathematics carefully and then use what we learned (from the study).

Connecting unknowns and knowns

The essence of studying mathematics is to solve *problems* (used here in a more generic sense), and the essence of solving problems is to connect the unknowns with (and express them in terms of) the knowns within the context of the problem. Sometimes, it may be necessary to go beyond the problem context to provide the connections for solving the problem. This is the case where relationships can be established either between the problem context and other topics in mathematics, or between the problem context and other subject areas, or between the problem context and real world experiences.

Connecting the unknowns to the knowns involves manipulations of the relationships so as to preserve the meaning of the original ideas involved in the relationships. For example, in the relationship $2x = 6$, we must manipulate properly to get $x = 3$, not $x = 4$ (where the original meaning of $2x$ is not preserved if it is taken to mean *2 plus x*). When we *mathematicize* problems (or issues), all we are trying to do in effect is to use mathematical ideas to represent the relationships involved and then to express the unknown in terms of the known.

Does it sound too simplistic? But is that not what problem solving (and for that matter mathematics) is all about? Notice however that we need to be mathematically literate (NCTM, 1989) to be able to mathematicize and communicate mathematically, all of which we can achieve if we study mathematics, not just learn mathematics.

Concluding remarks

Teachers must be predisposed towards focusing on the four universal themes in all their teaching of mathematics. Similarly, students must focus on these universal themes in their study of mathematics so that they learn a significant amount of mathematics and become mathematically powerful. Any reforms in mathematics education will have to consider seriously these four universal themes and integrate them explicitly as cores of the reform. Also, the preparation of textbooks and technology materials for the study of mathematics must address these four universal themes explicitly. We must remember, especially that technology is only a tool that we use to make the study of mathematics more meaningful and sometimes more convenient (and efficient). There are many good articles written for the journal *The Mathematics Educator*. However, I will like to see more written explicitly addressing the four themes so as to focus readers' attention on these universals for studying mathematics. This way, I hope we shall be making a concerted effort in ensuring quality mathematics education. We shall begin to see that generating the ideas, their representations, establishing relationships, and solving problems through the connections of the unknowns to the knowns is a creative human endeavor.

References

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