

Mathematical Problem Solving, Thinking and Creativity: Emerging Themes for Classroom Instruction

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Abstract

This article is based on an examination of classroom practice of a teacher who responded to the major change in the mathematics curriculum in Singapore. Classroom practice that aims to promote mathematical problem solving is scrutinized to unearth promising themes for mathematics teaching to encourage thinking and creativity. Features of classroom activities that seem promising for mathematics teaching in the next millenium are identified. Four promising themes emerged. Firstly, subtle changes in teaching style may lead to desired changes in learning outcomes. Secondly, problem posing can be a vehicle to promote thinking and creativity. Thirdly, thinking and creativity are not exclusive domains attainable only by academically abled students. Fourthly, opportunistic approach to teaching seems promising. An opportunistic approach involves the use of problems to create situations where there is a need for certain content knowledge and concepts. The teaching of content knowledge and concepts occurs when such a need arises. This is in contrast to a sequential approach which involves the teaching of content knowledge and concepts for the purpose of solving problems.

Introduction

Mathematics curriculum in Singapore experienced a major change early in this decade with the introduction of a curriculum that made problem solving its focus (Ministry of Education, 1990). With the decade drawing to a close, in accordance to the *Thinking Schools, Learning Nation* philosophy (Goh, 1997), the teaching of core thinking skills in school subjects, including mathematics, is encouraged (Ministry of Education, 1997). The teaching of core thinking skills in the dimensions of learning framework is based on the work of Marzano et. al. (1988). However, despite the aims of the curriculum to promote problem solving, thinking and creativity, mathematics lessons in Singapore classrooms are often expository with emphasis on procedures, answers and accuracy (Kaur & Yap, 1996).

This article is based on an examination of classroom practice of a teacher who responded to the major change in the Singapore school mathematics curriculum. Classroom practice that aims to promote mathematical problem solving is scrutinized to unearth promising themes for mathematics teaching to encourage thinking and creativity. Features of classroom activities that seem promising for mathematics teaching in the next millenium are identified. Four promising themes emerged. Firstly, subtle changes in teaching style may lead to desired learning outcomes. Secondly, problem posing can be a vehicle to promote thinking and creativity. Thirdly, thinking and creativity are not exclusive domains attainable only by academically abled students. Fourthly, an opportunistic approach (Yeap, 1997) to teaching seems promising.

Theme 1: Subtle changes in teaching style can lead to desired learning outcomes.

Teaching styles that have so far been successful in achieving the purpose of imparting basic mathematical knowledge are treasured but may be less successful in improving the ability of students to solve problems. However, subtle changes in current classroom practice may induce students to engage in processes that are required for problem solving and in creative thinking. Activity 1 and Activity 2 illustrate how lessons that encourage the development of thinking skills and creativity retain two features that are familiar to most teachers in Singapore schools – the use of drill-and-practice exercises and the use of routine textbook questions.

Activity 1 (*The Answer Is Three!*) (Yeap & Kaur, 1997) illustrates how a drill-and-practice session can simultaneously be a creative activity. The activity involves students writing possible questions to a given answer.

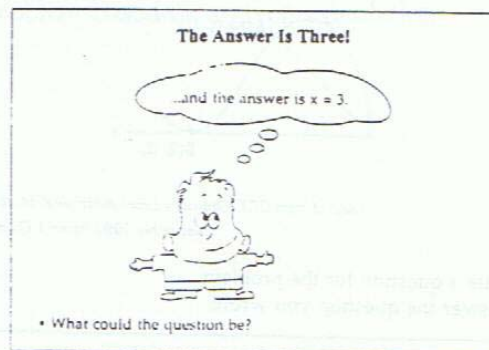


Figure 1 : Activity [The Answer Is Three!]

In Activity 1, the responses generated could have ranged from a simple $x - 3 = 0$ to a more difficult $x^2 - x = 6$ to $x^3 - x^2 - 9x + 9 = 0$ to $\sqrt{x+1} = 2$ to a fairly complex $\lg(x+7) = 1$. Students not only get the opportunity to practise solving equations as they generate and check questions to see if their responses fit the given answer, but they also get the opportunity to take stock of and use their existing knowledge space. Integrating knowledge is one core thinking skill. Ability to generate many, as well as many types of, responses is an aspect of creative behaviour.

Such activities are simply created by selecting an answer such as *the area is 154 cm^2* or *the derivative is $2x^2 - 3$* , which could be answers to a host of questions, and requiring students to generate as many, and as many varieties of, possible questions to the answer.

Activity 2 (*The Missing Question*) (Yeap & Kaur, 1997) illustrates how a routine textbook question can be used to develop more than just instrumental understanding (Skemp, 1976). The activity involves students writing a, hopefully, elegant question to a given problem situation and, subsequently, solving the problem to answer the question posed.

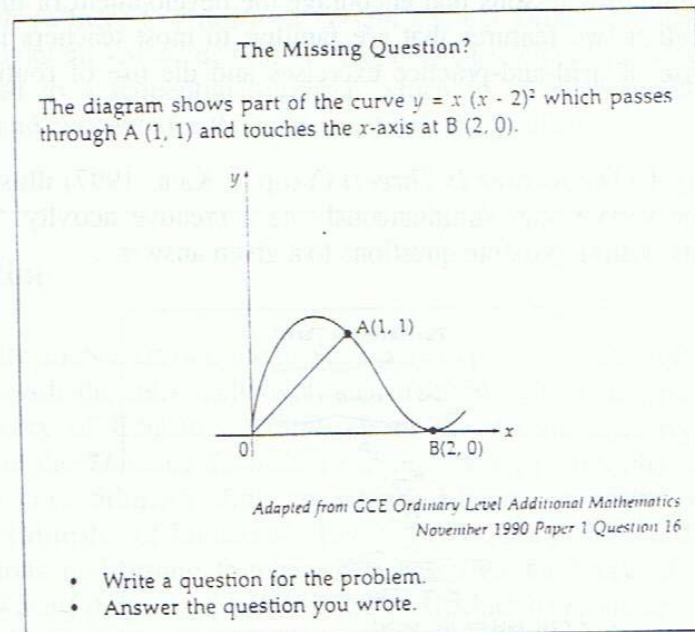


Figure 2 : Activity 2 [The Missing Question]

Such activities are created by deleting questions from typical mathematics problems. The problem situation should ideally be rich in information to allow the generation of responses that range from simple ones to complex types.

In Activity 2, the questions generated could vary from simple ones such as *find the gradient of the curve at A* to more challenging ones such as *find the intersection point between the tangent at A and normal at B* to complex ones such as *find the ratio of area P to area Q*. This activity allows students to demonstrate what they know. The act of posing a question that they can solve evokes a whole range of processes that we hope to develop amongst students to promote critical and creative thinking. Students are required to critically examine all the information in a given problem situation to pose a question in a creative way that would acknowledge the richness of information in the problem situation while maintaining its elegance.

Theme 2: Problem posing can be a vehicle to promote thinking and creativity.

Problem posing is the generation of new problems and the reformulation of given ones (Silver, 1994). Some examples of problem posing activities include the generation of a problem (Ellerton, 1986), the generation of a problem based on a given context (Silver and Cai, 1993), the generation of a problem based on a given calculation (Green and McCaan, 1991), the generation of a problem given the answer, the generation of sub-problems in the solution of a larger problem, the generation of questions to problems with unstated questions (Kruestskii, 1976), and the generation of what-if questions (Brown & Walter, 1985). Activity 1 and Activity 2 are examples of problem posing activities.

Activity 3 (*My Dog Ate My Homework*) illustrates another problem posing activity that seems to promote thinking and creativity. This activity requires students to write a problem based on a given calculation.

My Dog Ate My Homework

This is all I can show my mathematics teacher on Monday morning because my dog ate my homework.

$$\begin{aligned}
 &1.75 + \frac{1}{5} \times 0.25 \\
 = &1.75 + \frac{0.25}{5} \\
 = &1.75 + \frac{1}{10} \\
 = &1.75 + 0.3 \\
 = &2.05
 \end{aligned}$$

Now the teacher wants me to write a problem that can be answered by the remnants of my mathematics homework.

You are to help me write the problem. More credit is given to more difficult problems.

Figure 3 : Activity 3 [My Dog Ate My Homework]

The activity effectively requires students to translate a piece of information expressed in mathematical language into verbal language. Relational understanding (Skemp, 1976) of the mathematics in the given calculation, an essential component of mathematical thinking, is drawn upon. Students need to understand the meaning of the mathematics involved in the given calculation to pose a problem that fits the calculation. Moreover, the open nature of the activity induces students to engage in self questioning and reflection. The open nature of the activity also allows students to be creative in constructing a response.

Theme 3: Thinking and creativity are not exclusive domains.

Certain classroom tasks seem to be able to motivate weak mathematics students, not only to learn mathematics, but also to engage in mathematical thinking normally associated with good mathematics students.

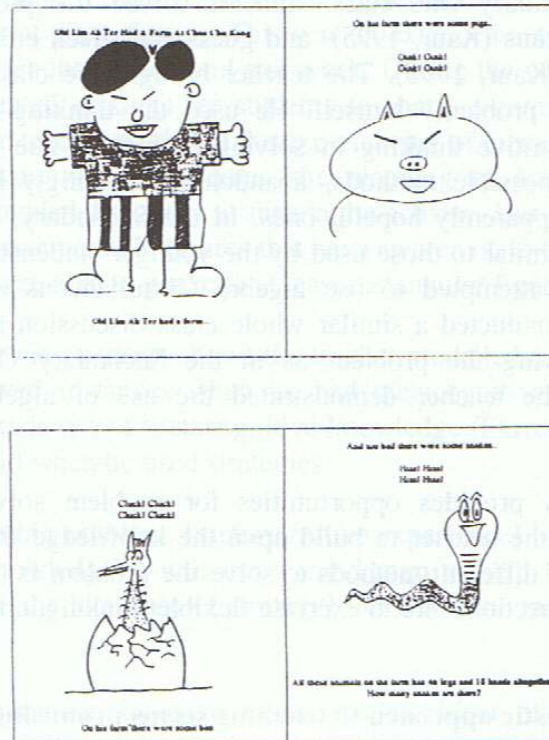


Figure 4 : Activity 4 [Old McDonald Had A Farm]

Activity 4 (*Old Lim Ah Ter Had A Farm in Choa Chu Kang*) (Yeap & Kaur, 1996) illustrates a classroom activity that involves significant aspects of thinking and seems to be motivating even to mathematically weak students. The activity has a familiar and humorous context which puts many students at ease. The activity is also in a pictorial form with few words. These characteristics make the activity less threatening and allow the weaker students, in particular, to be less anxious and to focus on the task with a clearer mind.

This activity was given to two groups of academically weak Secondary One (Normal Technical) and Secondary Three (Normal Academic) students. The activity was used at the start of the academic year in the Secondary One class while in the Secondary Three class, students attempted this activity after learning how to solve linear simultaneous equations. In both instances the students were shown colourful illustrations using an overhead projector. They were then instructed to solve the problem in pairs in 20 minutes. The teacher then conducted a whole class discussion to share students' solutions. Both the classes had about 40 students each.

In the Secondary One class, students solved the problem mainly by modelling with diagrams (Kaur, 1995) and guess-and-check either systematically or unsystematically (Kaur, 1995). The teacher brought the class discussion to a close by solving the problem himself. He used the thinking aloud method to demonstrate metacognitive thinking in solving problems. The students saw the teacher thinking of possible methods, abandoning seemingly hopeless solution paths and pursuing apparently hopeful ones. In the Secondary Three class, most pairs used methods similar to those used by the younger students. However, a few pairs unsuccessfully attempted to use algebra (simultaneous equations) (Kaur, 1995). The teacher conducted a similar whole class discussion and demonstrated his approach to solving the problem as in the Secondary One class. In the subsequent lesson, the teacher demonstrated the use of algebra to solve the problem.

This activity provides opportunities for problem solving, for making connections, and for the teacher to build upon the knowledge that students bring into class. The use of different methods to solve the problem is a suitable way for students to make connections and to exercise flexible thinking, a feature of creative behaviour.

Theme 4: Opportunistic approach to teaching seems promising.

In an opportunistic approach (Yeap, 1997) to teaching, problems are used to create situations when some mathematical knowledge or concepts are needed. In a sequential approach (Yeap, 1997), which is the traditional approach to mathematics teaching, mathematical knowledge and concepts are delivered topically for the purpose of solving problems. The opportunistic approach involves teaching mathematics through problem solving while the sequential approach involves teaching mathematics for problem solving.

Activity 5 (*One Five Four*) (Yeap, 1997) illustrates an activity that was used to teach the topic area of plane figures in an opportunistic manner.

One Five Four

A figure has an area of 154 cm^2 .

Draw possible shapes that this figure can be.

Figure 5 : Activity 5 [One Five Four]

In a Secondary One (Express) class, this activity was used to teach the entire unit of area of plane figures. This was an extended activity which alternated between whole class instruction and pair work. During the whole class instruction, the teacher facilitated sessions for students to share, summarize and consolidate their findings. The teacher modelled the experiencing of metacognitive experiences, the use of metacognitive knowledge, the planning of actions and the use of strategies, and provided directions to sustain the activity. During pair work students attempted to generate possible figures that have an area of 154 cm^2 . It was observed that all students were able to generate at least rectangular figures.

The following excerpt shows the teacher modelled a good problem solver. The teacher showed instances when he had metacognitive experiences (Flavel, 1981), when he made use of metacognitive knowledge (Flavel, 1981), when he did some planning, and when he used strategies.

Teacher: This is a big problem. [metacognitive experience] I better solve bit by bit [metacognitive knowledge] I will use a mind map to help keep track of the big problem as I solve the bits. [use of strategy] Put down the main problem first. (See Figure 6a.)

Teacher: The figure can have so many shapes. Most of you have generated rectangular figures after ten minutes of discussion. Do rectangles first. (Drew a rectangle on the board.) [planning] Hmmm. What can the sides be? (Called on student who have worked systematically.)

Student 1: Could be 1 by 154, 2 by 77, 7 by 22 and 11 by 14.

Teacher: Better be systematic otherwise we may miss out possible answers [metacognitive knowledge], so start with the smallest to the largest. [use of strategy] But should I skipped 3, 4, 5, 6 and so on? [metacognitive experience] (Called on another student who has worked systematically.) What did you all did?

Student 2: 1 times 154, 2 times 77, 3 times $51 \frac{1}{3}$, 4 times $38 \frac{4}{5}$... (Continued reading from his list.)

Teacher: In fact couldn't it be 1.1 times something? 1.11 times something? [metacognitive experience] So what can we say? (Students' responses were basically that there were infinite number of possibilities.) When the problem is too big, we should set criteria (wrote the word criteria on the board) so that we can handle the problem. [metacognitive knowledge] Let us assume that the sides are

whole numbers; that is our criteria. [use of strategy] But they can be decimals as well, one of them a decimal or both of them (wrote on the board). (See Figure 6b.)

Later students suggested other possible shapes. At times some students taught the others how to find the areas of such shapes, while at other times the teacher showed the students how to find the area of shapes which they were unable to. The method to find the area of a particular shape was shown when it was asked for by some students. This excerpt illustrates an opportunistic approach that involves teaching a piece of knowledge when the need arises.

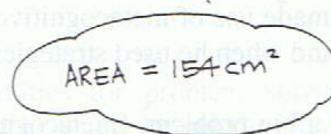


Figure 6a

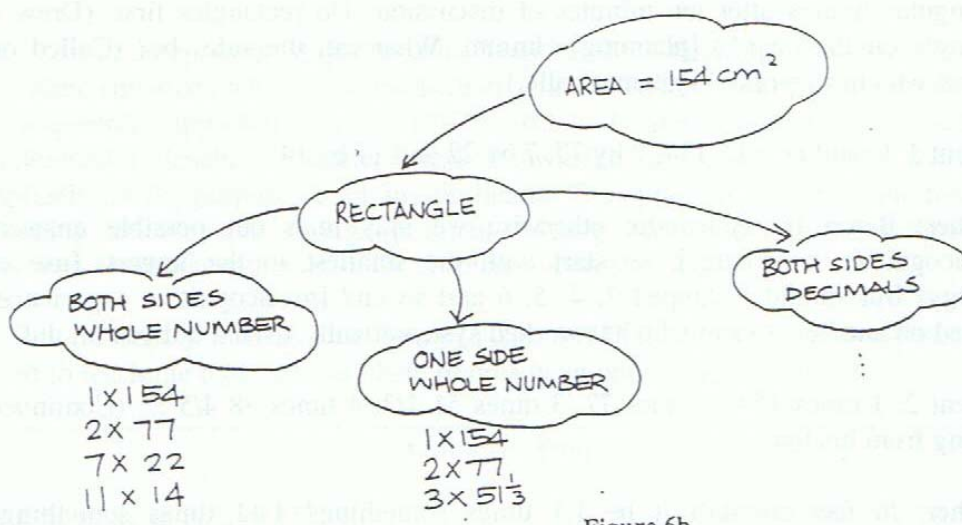


Figure 6b

Figure 6 : The teacher solving the problem One Five Four

In another classroom, a teacher initiated a series of lessons on differentiation by showing the differentiation of two simple functions, $f(x) = 4x^2 + 3$ and $g(x) = 5x - 2$. By encouraging the students to ask what-if questions (Brown & Walter, 1985) and solving them, students generated other functions, some of which required the use of rules that were yet to be taught. Whenever students discovered that the existing rule cannot be used to obtain the derivative of the new function posed by asking what-if questions, the teacher showed the use of a suitable rule for that function. As a result of posing what-if questions, students generated functions such as $(4x^2 + 3)(5x - 2)$, $(4x^2 + 3)^{5x-2}$, $4x^{5x-2} + 3$, $(4x^2 + 3) + (5x - 2)$, and a host of other functions that require the use of the chain rule, the product rule and the quotient rule. The teacher captured the opportunity to show the use of a rule when the rule is required to find the derivative of a function posed by the students. While the students get the opportunity to engage in the creative activity of posing new functions, the teacher get the opportunity to teach content knowledge when it is needed to solve the questions posed. The process of creating the functions themselves seems to help students relate a particular differentiation rule to the structure of the functions.

Conclusion

After a decade of implementing a problem solving curriculum in the classroom, it is timely for us to reflect on features of instructional practice that are consistent with curriculum changes and that would sufficiently prepare students for the new millenium where thinking and creativity are assets.

Mathematics classrooms in Singapore typically have about forty students each. The activities described in this article were used in such large classes and were found to engage students in problem solving, thinking and creative behaviour even in such classrooms.

One important feature of the activities decribed in this article is the provision for students to choose and make decisions. This feature seems to intrinsically motivate students, especially mathematically weak students, to engage in significant aspects of mathematical thinking. This feature allows most students to be successful at their ability level. Weaker students were able to be successful while good students were not left unchallenged.

In summary, this article suggests that thinking and creativity must be and can be developed amongst all students. This can be achieved without radical

changes to present, time-tested teaching methodology. Problem posing activities and an opportunistic approach to teaching, where content knowledge and concepts are taught through the use of problems, hold significant potential in mathematics classrooms of the new millenium.

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