The Mathematics Educator
1998, Vol. 3, No. 2, 38-53

Proportional Reasoning And Mathematical Beliefs Of Student Teachers From Singapore And Australia

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Abstract

Student teachers in Australia and Singapore were asked to solve a one-step ratio problem and to complete open sentences about the nature of mathematics and mathematics pedagogy. The purpose of this paper is to investigate the similarities and differences in proportional reasoning of the student teachers in Australia and Singapore through the approaches and strategies they used in solving the ratio problem. From their responses to the open sentences, this paper presents also the differences and similarities between the two country groups in their beliefs about the nature of mathematics and how mathematics is learned and taught. The relationships between beliefs about mathematics and variables in solving the ratio problem are discussed with particular reference to the curriculum and pedagogical contexts in Australia and Singapore.

Introduction

"The fact that many aspects of our world operate according to proportional rules makes proportional reasoning abilities extremely useful in the interpretation of real world phenomena" (Post, Behr, & Lesh 1988, p.79). Proportional reasoning involves more than setting up and solving a proportion. In the initial learning of the concept, students' involvement should include concrete experiences with proportional and nonproportional situations in which students collect data, build tables and determine the rule for relating the number pairs in the table. From there proportional situations are defined as those whose rule could be expressed in the form $y = mx$, where m is a constant factor relating the two quantities, x and y. However, most often textbooks emphasize the development of procedural skills rather than conceptual understandings. This tends to encourage rote learning and inhibits meaningful understanding of the multiplicative relationship between the quantities when expressed as an algebraic generalization. How students and adults use proportional reasoning and solve proportion problems has been the focus of a great deal of research (Fisher, 1988; Dube, 1990; Behr, Harel, Post & Lesh, 1992; Conroy & Sutriyono, 1993, Conroy & Perry, 1996).

Dube (1990) gave the following proportion task to 240 grade 12 students for them to write an equation to represent the statement:

> *"In a certain school there are 15 students to every teacher where S represents the number of students and T represents the number of teachers"*

This problem which we shall call the ***Students-and-Teachers*** problem, was replicated from other studies (Lochhead, 1980, Clement, 1982, Davis, 1984). Results from these studies indicated that most respondents, among them engineers, teachers and other professionals, as well as students of all levels made the reversal error of writing down *"15S = T"* as the answer. Data collected from these studies showed that errors in formulating the algebraic equation were not primarily due to syntactic translation and interference from natural language, but a lack of comprehension of relationships. In particular to this problem, conceptual understanding of ratio and proportional reasoning are prerequisites to successful solution. Lawton (1993) in a similar study on college students suggested that most of the students had relatively fragile understanding of proportion concepts and were easily influenced by structural variations in the problem. Aspects of natural language in which a mathematical relation is expressed may interfere with the process of translation into an algebraic representation. Kaput (1987) using a similar problem:" *There are six times as many students as professors"*, stated that the major cause of the reversal error, 6S = P, is the strong influence and automatic use of the natural-language rules of syntax where the tendency is to interpret "6S" as "six students".

Dube (1990) in her analysis of the student responses to the Students-and - Teachers problem, found that the solutions fell into two categories of approach, which she called holistic, and *analytic -synthetic.* In the holistic approach the students just wrote down the answer as the result of a global perception of the entire problem, whereas in the analytic-synthetic approach, the students showed explicit and careful defined steps. Further analysis of the analytic-synthetic approach showed that students applied cognitive strategies that could be categorised according to the ways the students organised their previously learned concepts and skills to get the required equation. There are three main strategies: 1. *Linguistic 2. Proportional* 3. *Functional.* The first strategy is based on translating meaning of the words in the problem, the second strategy is based on student's understanding of ratio and proportion and the last strategy on the use of function or other mathematical concepts.

Dube's classification of approaches and strategies will be used for the purpose of this present study to investigate similarities and differences in the ways that Australian and Singaporean student teachers approached the *Students-and - Teachers* problem.

Student Teacher Beliefs and Mathematics

A number of investigations (Mayers, 1994; Conroy and Sutriyono, 1993; Foong, 1993) have focused attention on teachers' beliefs about mathematics and the learning and teaching of it. Teachers' beliefs about mathematics have been shown to be particularly important in terms of the instructional practices they adopt. Studies have shown that teachers' instructional practices affect their pupils' perception of mathematics as a discipline (Schoenfeld, 1989). A traditional view of mathematics is known to predominate amongst teachers and pre-service student teachers (Thompson, 1992). They are known to regard mathematics either as a body of absolute truths, which exists independently of the learners or as a set of tools comprising facts, rules and skills.

Student teachers are on a course that takes them from the school situation where they have been pupils (for some, a long time ago) to a teacher education institution, and then back to the school situation, this time as teachers. Student teachers come to the training institutions directly from high school or from the university and they bring with them varying perceptions, attitudes towards and abilities in mathematics. Whatever beliefs they have about mathematics and mathematics pedagogy have been influenced not only by experiences and achievement in school mathematics but also by teachers, parents, employers and their peers.

One way of examining teachers' espoused beliefs about mathematics has been to categorise them into those related to the nature of mathematics, the learning of mathematics and the teaching of mathematics. In such investigations a belief can be defined as "*any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase: " I believe that...*" (Rokebach, 1968, p. 2). For the purpose of this study which is to investigate also the beliefs of student teachers in Australia and Singapore, the subjects were asked to complete open sentences about the nature of mathematics and mathematics pedagogy. Responses to these `beliefs' questions are examined for similarities and differences between the two country groups and identify possible links between these beliefs and the approaches the student teachers used in solving the ratio problem.

The Sample

The total sample consisted of 460 students who were in the first year of teacher education programs preparing them for careers in primary [elementary] schools.

Australia. This cohort comprised 178 student teachers from two universities [one Catholic and one secular] in Sydney, NSW. Both groups of students [46 and 132 respectively] were in the first semester of a six semester bachelor degree program, each with its own unique curriculum.

Singapore The cohort comprised 282 students from two distinct groups within a government university: 164 undertaking a two-year diploma program and 118 undertaking a one-year post-graduate diploma program. The students from Singapore have their education throughout using English Language as the medium of instruction and learning it as the first language, even though English is not their mother-tongue. Mathematics is learned and taught in English.

Table 1 shows the age composition of the cohorts in the two countries. The diploma in education (Dip-Ed) students in Singapore are compatible with the Australian cohort in age group, whereas the post-graduate diploma (PGDE) students are in the higher age group as they had already completed their university degrees.

Table 1. Age composition of cohorts (per cent of students in each cohort)

Age group (in years)	**Country** Australia	Singapore Dip-Ed	PGDE
Less than 18	15.2	nil	nil
18 or 19	67.4	58.5	nil
20 or 21	6.5	30.5	nil
22 or 23	6.5	4.9	65.3
24 and above	4.3	6.1	34.7

The Task

The students were presented with the *Students-and-Teachers* problem and were asked to complete it individually:

Please work the following problem as completely as possible:
'In a certain school there are 15 students for every teacher. If S is the number of students and T is the number of teachers, write down the equation, which represents the given situation.'

The problem was presented on a single sheet of paper and students were encouraged to write whatever explanation was necessary to support their answers. It is identical with the problem used in three previous studies (Dube, 1990 , Conroy & Perry, 1996 and Conroy & Sutriyono, 1993).

On a separate sheet, students were presented with three incomplete sentences about mathematics, which they were asked to complete in whatever way they felt appropriate. To encourage the maximum openness of response, no verb was included in the incomplete sentence, particularly not the verb `to be'. The incomplete sentences were as follows:

Please complete the sentences given:

Question 1.
In my opinion, mathematics..

Question 2.
In my opinion, mathematics in schools...

Question 3.
In my opinion, pupils involved in the process of obtaining mathematics knowledge...

Students were given sufficient space after each statement to write their ideas fully. They were not given a specific time limit for the tasks but, in general, took approximately half an hour to complete both.

Results

a) Proportional Reasoning Task

Table 2 shows the percentage of correct and incorrect responses given by the students. As well as obvious errors, an incorrect response covers any response that was not in the form of an equation (which the task required), any incomplete response or lack of it. Likewise, as well as the obvious correct equations, a correct response covers those cases which follow a correct line of reasoning, but may have a simple error near the end of this line.

Table 2. Percentage of students in each cohort responding correctly of incorrectly

Type of Response	Country		
	Australia	Singapore Dip-Ed	PGDE
Correct	**27.0**	**36.4**	**62.7**
Incorrect	73.0	63.6	37.3

The Dip-Ed cohort from Singapore (36.4% correct) performed somewhat better than the Australian group (27% correct). The Post-graduate diploma (PGDE) group outperformed the two groups with 62.7% giving correct equations.

Responses were analysed according to the approach adopted by students, using the Dube (1990) mutually exclusive classifications of *holistic or analytic-synthetic* approach. A response was classified as *holistic* when the student just wrote down the equation correctly or incorrectly without any "working" as a result of a global perception of the entire problem. A response was classified as *analytic-synthetic* when there were carefully defined steps, evidence of analysis using semantic and mathematical reasoning, algebraic manipulations and arithmetical calculations. Tables 3 and 4 give details of the percentages of the students using either the holistic or analytic-synthetic approach or the percentages of success for each approach.

Table 3. Percentages of responses in each cohort using an holistic or analytic-synthetic approach

Approach Used	**Country**		
	Australia	Singapore	
		Dip Ed	PGDE
Holistic	**70.8**	**72.8**	49.2
Analytic-synthetic	23.6	27.2	**50.8**
Insufficient information	5.6	....	....

Table 3 shows that a large proportion of the Australian (70.8%) and the Singaporean Dip-Ed cohort (72.8%) used an holistic approach. However, the older PGDE group of Singapore was spread almost equally between the two approaches. A further breakdown of the data in Table 3 gives Table 4 which shows the percentages of correct and incorrect responses given for each of the two approaches.

Table 4. Percentages of correct **and incorrect** responses in each cohort using an holistic or analytic-synthetic approach

Approach Used	**Country**					
	Australia		Singapore			
			Dip-Ed		PGDE	
	correct [27%]	**incorrect [73%]**	**correct [36.4%]**	**incorrect [63.6%]**	**correct [62.7%]**	**incorrect [37.3%]**
Holistic	**68.8**	**71.5**	**59.3**	**80.6**	44.6	56.8
Analytic-synthetic	31.3	20.7	40.7	19.4	**55.4**	43.2
Insufficient info.	…	7.7	…	…	…	…

Table 4 shows that the Australian cohorts who gave either correct or incorrect responses more often than not used the holistic approach to solve the problem. The Singapore cohorts' preferred approach is different between the Dip-Ed group and the PGDE group. 59.3% of the Dip-Ed used the holistic approach for a correct response which is less than the Australian group, whereas more PGDE students favored an analytic-synthetic approach to obtain a correct answer.

Across all cohorts, larger proportions of student teachers using an holistic approach obtained an incorrect rather than a correct solution. The majority of the incorrect responses committed the reversal error of writing down *"15S=T"*. Other incorrect responses included such examples as *" y= 15S/T"; " S=T"*. For those correct responses using the holistic approach, the equations given were usually of the forms *"S/T=15"; "S=15T"; "S/15=T"*. Singaporean students who used the analytic-synthetic approach were more likely to produce a correct solution when they applied *the proportion strategy* than the Australian counterparts.

The types of cognitive strategies used by students in an analytic-synthetic approach were further analysed . Table 5 shows the percentages of correct and incorrect responses using the analytic-synthetic approach which had applied one or the other of Dube's (1990) three cognitive strategies: 1. *linguistic* 2. *proportional* 3. *functional.* The first strategy is based on translating the meaning of the words in the problem, the second strategy is based on the student's understanding of ratio and proportion and last strategy on the use of function of other mathematical concepts.

Table 5. Percentages of correct and incorrect analytic-synthetic responses showing cognitive strategies used

Cognitive Strategy Used	**Country** Australia		Singapore	
	correct	**incorrect**	**correct**	**incorrect**
Linguistic	**33.3**	7.4	16.2	16.1
Proportional	13.3	**59.3**	**71.6**	19.6
Functional	**53.3**	33.3	12.2	**64.3**

In Table 5 it is interesting to note that, overall, 71.6% of Singaporean students using the analytic approach and who produced a correct equation used the proportional strategy. The Australian students had difficulty using the proportion strategy, 59.3% of those who used it were unable to formulate a correct equation.

Among Australians using the analytic-synthetic approach, 33.3% of those obtaining a correct solution had used a linguistic strategy, 16.2 % of similar Singaporean students. 53.3% of the correct responses by the Australian students were obtained using the functional strategy. Although these Australian students did not use the ratio/proportion procedure they were able to use proportional reasoning to formulate the correct mathematical function for the equation.

Of the Singaporean students, 64.3% of those who used a *functional* strategy failed to produce a correct equation. The majority applied inappropriate mathematical functions like addition and other algebraic skills, which indicated that they did understand not the ratio and proportional relationship in the problem.

b) Student -Teachers' Beliefs

Question 1: ***In my opinion, mathematics..***

Responses were grouped into five main categories; namely, mathematics viewed as:

a. an affect (enjoyable, interesting, confusing, difficult etc);
b. being useful (important, necessary, beneficial in daily life etc.);
c. a body of knowledge (related to other sciences, possessing broad content, explaining things in general etc.);
d. an exact science (concerned with true results, calculations, formulas, technical terms etc.);
e. a way of thinking (needing rational thought, gaining confirmation through proof, concerned with how to know and define etc.);

Responses sometimes combined two or more of these ideas or gave ideas that fell outside the categories.

Table 6. Students' responses to the open sentence: "In my opinion, mathematics...."

Response	Country	
	Australia	Singapore
An affect	31.5	**45.7**
Being Useful	**35.4**	12.1
A body of knowledge	6.2	15.4
An exact science	0.6	3.6
A way of thinking	10.7	**20.0**
Two or more of above	10.7	……
Other	5.1	3.2

Table 6 indicates that Singaporean student teachers (45.7%) refer more frequently to the relation between mathematics and attitudes than the Australian counterparts (31.5%). Examples of their expressions include: "*mathematics can be interesting and challenging*"; "*is one of the hardest subject*"; "*gets more and more and more difficult at higher levels*" etc.

However Australian students (35.4%) give more consideration to the usefulness of mathematics than the Singapore students who might have taken its usefulness for granted. When these responses are analysed in terms of the approach taken by students to solve the ratio problem, there is negligible difference between students taking an holistic approach and those taking an analytic-synthetic approach.

More of the Singapore students (20%) see mathematics as *a way of thinking*. They responded with statements such as: " *is not just getting the answer but it must be done with understanding*"; " *it tests the flexibility and speed of thinking*"; "*developing the mind to analyse and see connection....*" etc. A larger percentage of the Singaporean students who used the analytic-synthetic approach in solving the ratio problem viewed mathematics as a way of thinking than did those students who used an holistic approach.

Question 2: ***In my opinion, mathematics in schools ....***

Responses were grouped into six main categories; namely, school mathematics is viewed as:

a. having utilitarian value;
b. affecting attitudes;
c. having broad cognitive implications (e.g. it develops thinking);
d. depending on teaching for its quality;
e. needing to match the interest, abilities and understanding of students;
f. depending on the quality of the curriculum.

Table 7. Students' responses to the open sentence: "In my opinion, mathematics in school..."

Response	Country	
	Australia	Singapore
Have utilitarian value	**27.0**	15.4
Affect Attitudes	16.9	**25.4**
Have broad cognitive Implications	2.2	6.4
Depend on teaching	**19.1**	**40.0**
Need to match interest	4.5	4.8
Depend on curriculum	11.8	7.2
Two or more of above	14.6	.....
Other	4.0	2.4

Similarly as in the previous responses of their beliefs about the nature of mathematics (Table 6), Australian students consider `utility' as of more relative importance in school mathematics than other considerations. Again for the Singpapore students `usefulness' or `utility' is not a major consideration as compared to `affect' when they thought about school mathematics.

Singapore students (40%) strongly believed that school mathematics is very much dependent on teaching. Some were of the following opinions: *"mathematics in school can be interesting if the teacher is able to explain the concept clearly and be creative....."; " school mathematics is not taught in the complete way. Sometimes the teacher just show you how to do the sum but does not explain in details"; "...teachers nowadays are using more varied methods to bring forward a mathematics concept"* etc. Also Australian students (19.1%) ranked this factor second to `utility'. Australian students who believe mathematics depends on the teaching, all used the holistic approach to solve the ratio problem while those Australian students who used the analytic-synthetic approach were more likely to believe mathematics in school is useful.

There is little difference between the groups of Singaporean students taking an holistic approach and those taking an analytic-synthetic approach in their various opinions about mathematics in school. The 40% who strongly believed that school mathematics is linked with teaching were about equally divided in their approaches i.e. 40.4 % of those in the holistic group and 39.5% of the analytic-synthetic group shared the same view.

Question 3: ***In my opinion, pupils involved in the process of obtaining mathematics knowledge..................................***

Responses were grouped into four main categories; namely, how children learn mathematics is influenced by:

a. affective factors (children's interest, motivation, enjoyment etc.);
b. its activeness and relatedness to daily life;
c. various cognitive and developmental factors (levels of ability, thinking skills etc.);
d. its reliance on memorisation and practice.

Table 8. Students' responses to the open sentence: "In my opinion, pupils obtaining mathematics knowledge..."

Response	Country	
	Australia	Singapore
Affective factors	**28.7**	**29.6**
Activeness & relatedness	**21.9**	15.0
Cognitive factors	**17.4**	**23.2**
Reliance on memory & practice	10.7	**24.6**
Two or more of above	6.2	....
Other	7.3	7.6

Table 8 suggests that student teachers in the two countries share the belief in roughly equal proportions that children learning mathematics are influenced by affective factors. This view is held regardless of the approach to the ratio problem in Singapore. For Australian students, those using the holistic approach (34%) are more likely to hold this view than those using the analytic-synthetic approach (16%).

The belief that children's mathematics learning needs to be active and related to daily life is more likely to be found among Australian students. In the two country groups, there is negligible difference between students using either approach to the ratio problem.

The belief that mathematics needs to be related to children's cognitive levels is expressed more frequently by the Singaporean group (23.2%) than the Australians (17.4%); and more often by those using the analytic-synthetic approach (27.2%) than those using the holistic approach (20.5%). Also 24.6% of the Singaporean students are of the opinion that largely memorisation and practices influence children learning. Two opinions offered are".... *may not understand the concepts but perhaps attained it through memorisation"* and *"..often need a lot of practices on different questions (though it's of the same concept) before they actually obtain it"*. Singaporean students who used the holistic approach are more likely to express this belief than those using the analytic-synthetic approach. Only 10.7% of the Australian students hold this view.

Conclusion

Proportional reasoning involves an understanding of the mathematical relationships embedded in proportional situations such as in the *Students-and-Teacher* problem used in this study. Context or numerical complexity should not influence a person who reasons proportionally. The results of this study revealed that 73% of the Australian undergraduates, 63.6% of the Singapore's Diploma-in Education students and 37.3% of the Postgraduate students were unable to solve the single-step ratio problem. This suggests that proportional reasoning, an abstract thinking skill, is not well developed in learners even though they had gone through at least junior high or `O' level mathematics. The majority of these unsuccessful problem-solvers used the holistic approach to produce an incorrect equation which showed that they had been influenced by the natural-language rules of syntax where they interpreted "15S" as "fifteen students" and "T" to represent "teacher" instead of "the number of teachers". Hence many produced the reversal error of "15S=T" as the answer. The data also suggest that educational background could be a factor affecting success. The Singapore PGDE cohort who were university graduates with at least senior high or `A' level mathematics have a higher success rate than the other cohorts. On the whole, more Australian than Singapore students used the holistic approach. More Singapore students used the proportional strategy routinely in an analytic-synthetic approach to obtain a correct response than the Australian students who used more linguistic and functional strategies to produce a correct response. At this juncture, one may ask why the differences exist between the two country groups in their approach and strategies? It would be interesting to further study and compare the curricula, the textbooks and the pedagogy used in Australia and Singapore.

Data from the beliefs statements of the Australian and Singapore student-teachers could give some indications as to how the students' perceptions of mathematics and the pedagogy were influenced by the mathematics curriculum of their respective country. The results of this study also reveal that most Australian and Singapore students relate mathematics to the affective domain of learning, more so for the Singaporean. Australian students placed more emphasis on the utilitarian value of mathematics and its learning. Could it be that they were more exposed to examples of the real-life usage of mathematics? The higher proportion of Australian students than Singapore students believing that children learning mathematics need to be active and relate to daily life could confirm this. On the other hand Singaporean students emphasised more than Australian students the need for children's learning to be related to their cognitive levels and more believed that children's mathematics learning is largely influenced by memorisation and practice. More Singaporean students than Australian students viewed mathematics as a way of thinking. Could these beliefs explain why Singaporean students are more inclined to approach the ratio problem in an analytic-synthetic method and use the routine ratio and proportion strategy to effect a correct response whereas the Australian students are more inclined to a variety of strategies although, not as efficiently?

In conclusion this comparative study between Australia and Singapore student-teachers raises more questions that need to be answered by further in-depth research to test the many conjectures that arose from interpretations of the data.

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