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Importance Of Algebraic Thinking For Preservice Primary Teachers

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Abstract

To develop and enhance the algebraic thinking of pupils in primary schools, the prospective primary teachers need themselves to exhibit high level of algebraic thinking skills and be able to articulate what it is that they are doing. The purpose of this study is to investigate the algebraic thinking skills of the prospective primary school teachers prior to any formal lecture in algebra.

Introduction

Algebraic thinking is a way of thinking, a method of seeing and expressing relationships. It is a way of generalizing the kinds of patterns that are part of everyday activities. It helps in describing and exploring the physical world. It embodies the construction and representation of patterns and regularities, deliberate generalization, and most important, active exploration and conjecture (Chambers 1994). Essentially, algebraic thinking lies at the heart of acquiring deep understanding in many areas of mathematics. In fact, it opens a door to organized abstract thinking and supplies a tool for logical reasoning.

Algebraic thinking has recently achieved considerable attention in the literature (Ruopp, Cuoco, Rasala, & Kelemanik, 1997; Usiskin, 1997, and others). Those who develop or acquire sound algebraic thinking and can apply it to a wide range of learning and problem solving situations are likely to be better learners, thinkers, reasoners, mathematicians, scientists, economists, businessmen, and in fact, better citizens. This also explains why algebraic thinking is becoming increasingly important in history, science, economics, grammar, military science, engineering, computer science, and business and in everyday life.

In primary schools, students develop algebraic thinking by "building meaning for the symbols and operations of algebra in terms of their knowledge of arithmetic" (Kieran & Chalouh 1993, p.179). Algebraic thinking begins to develop in children when they become aware of general relationships in arithmetic procedures, spatial patterns and number sequences. NCTM (1989) believes that the study of patterns is a productive way of developing algebraic reasoning in the elementary grades.

NCTM (1994) recommends that the notion of algebra be expanded to include a range of mathematical activity. It believes that all students can learn algebra and children can develop algebraic concepts at an early age. 'Algebra for everyone: start from Primary 1'. Those who support this statement don't visualize it as traditional high school algebra. They, in fact, believe that algebra can be informally introduced into the primary curriculum. One of the primary objectives of teaching informal algebra or algebraic thinking in Primary 1-6 is to provide opportunities for children to facilitate the development of their algebraic reasoning so as to be better prepared to study formal algebra in middle/high schools. When young children are presented with interesting problems in context, they observe patterns and relationships: they conjecture, test, discuss, verbalize, generalize, and represent those patterns and relationships. (Ferrini-Mundy, Lappan, & Phillips, 1997).

Knowledge of mathematics is obviously fundamental to being able to help someone else to learn it (Ball, 1988). Post, Harel, Behr, and Lesh (1991) claim that a firm grasp of the underlying concepts is an important and necessary framework for the elementary teachers to possess. A search of the literature reveals that content knowledge does influence teachers' decisions about classroom instruction, which in turn mediates student learning (Ball, 1991, Putt, 1995, Thipkong & Davis 1991).

To develop and enhance algebraic thinking of young children, primary school teachers need themselves to exhibit a high level of algebraic thinking skills, e.g. knowledge of structures, use of variables, understanding of functions, symbol facility/flexibility, looking for relationships and patterns, generalizing etc. They should be able to articulate what it is that they are doing.

There are evidences that many primary teachers have not been successful proponents of their own knowledge in algebra and skills. Many of them find it difficult to provide a mathematical environment in which pupils can develop algebraic thinking without reliance on rote learning or routine algorithms.

The main purpose of this article is to explore the preservice primary teachers' understanding of elementary algebra that might be important and necessary to develop and enhance their pupils' algebraic thinking. In particular, we examine whether the preservice primary teachers have acquired grounding in the fundamental algebraic concepts, such as symbol facility/flexibility, algebraic equivalence, use of variables and relationships, and knowledge of pre-algebra concepts.

Method

The subjects of this study were 162 preservice primary teachers enrolled in two different programs but four different classes: 68(42.0%) in Diploma in Education Year 1 (Dip 1), 45(27.8%) in Diploma in Education Year 2 (Dip 2), 24(14.8%) in BA/BSc with Diploma in Education Year 1 (BA/BSc 1), and 25(15.4%) in BA/BSc with Diploma in Education Year 2 (BA/BSc 2). The sample consisted of females 133(82.1%) and males 29(17.9%). We notice that the number of males is disproportionate to the number of females in the population because very few male students seek admission in these programs for preservice primary teachers.

The entry requirement for 2-year Diploma and 4-year BA/BSc is Cambridge A-level or equivalent. However, the students' mathematical background ranged from 'Elementary Mathematics' at Cambridge O Level to 'Further Mathematics' at Cambridge A-level. Thus the entire population of 162 students were divided into 4 subclasses according to the highest level of school mathematics they studied prior to admission in their preservice programs: (i) 32(19.8%) with Elementary Mathematics at O-Level (EMaths), (ii) 72(44.4%) with Additional Mathematics at O-level/A-level (AMaths), (iii) 55(34.0%) with Higher Mathematics (called CMaths) at A-level, and (iv) 3(1.9%) with Further Mathematics (FMaths) at A-level. We notice that the size of the subclass (iv) is 1.9% because very few students with FMaths seek admission in preservice primary programs.

We further observe that 'FMaths' has more in-depth and more mathematical contents including algebra than 'CMaths' which in turn is more advanced than 'AMaths' and 'EMaths'. The prerequisite for 'FMaths' is 'CMaths' which in turn requires 'AMaths' at O-level or A- level.

The preservice primary teachers ranged from eighteen to thirty years in age. No algebra lesson was given to them prior to this study in their training programs.

The preservice primary teachers were given 30 minutes to complete an eight-item written test, *see* after next section. The items in the test were carefully selected in terms of difficulty and they covered four dimensions of algebraic thinking, viz. use of variables, understanding of algebraic equivalence, symbol facility/flexibility, and knowledge of some arithmetic concepts. The preservice primary teachers were directed to show all work and to write full explanations in response to the problems.

Each item was marked out of 10 points. A partial credit was given for a partially correct answer. The students were assigned levels based on the following guideline:

Level 1 : $0 \le x \le 40\%$
Level 2 : $40\% \le x < 60\%$
Level 3 : $60\% \le x \le 100\%$

Analysis and Discussion of Test Responses

In this section, the responses of eight test items will be both quantitatively and qualitatively analyzed.

Item 1: Solve for x: $0.5x + 0.01x = 51$

Correct answer: $x = 100$,
Count correct (%):
Male:22 out of 29(76%), Female: 104 out of 133(78%), All: 126 out of 162(78%).
Wrong answers: 36(22%).

Typical misconceptions in Pre-algebraic concepts: 27(16%).
Most of these prospective teachers correctly wrote $x = 51/0.51$. But, they did not know how to divide a whole number by a decimal number. Typical wrong answers were: x = 5100, 10, 510, 1000, 0.001, or 1. On the other hand, some students

thought that the given equation implies that $50x + x = 51$ and which gives $x = 1$. Other misconceptions: 9 (6%).

The written responses of Item 1 reveal that about one-sixth of all prospective teachers in the sample have had serious difficulties in their pre-algebraic concepts, in particular, decimals smaller than one. In a similar study on preservice primary teachers' misconceptions in interpreting decimal notation and operations with decimals, Thipkong and Davis (1991) found that 31% of their students had difficulties with decimals smaller than one.

Item 2: In a supermarket, apples cost 20 cents each and grapefruit cost 80 cents each. Annie buys some apples and some grapefruit and altogether it costs her $12. If A is the number of apples bought, and G is the number of grapefruits bought, write an equation using A and G.

Correct answer: A + 4G = 60, or any equivalent equation.
Count correct (%):
Male: 25 out of 29 (86%), Female: 93 out of 133 (70%), All:118 out of 162 (73%).
Wrong answers: 44(27%).

Typical wrong answers:

(i)	0.2A+0.8G=$12,	17(10%)
(ii)	20A+80G=$12,	6(4%)
(iii)	20A+80G=12,	5(3%)
(iv)	A(20¢)+G(80¢)=$12,	4(2%)
(v)	$(20A+80G)=$12,	3(2%),
(vi)	Others,	9(6%).

These errors indicate that about one-fourth of the students surveyed have a background of arithmetic which was built on a foundation in which the equal sign means "gives" or "makes", as in "5 plus 4 gives 9". It appears that these students did not have sound numerical thinking. Also, the use of letters ¢ and $ in an algebraic equation was highly abused.

***Item* 3:** *Solve* $\frac{2(x-1)}{4}+2=6$

Correct answer: x = 9
Count correct (%):

Male:17 out of 29 (59%), Female: 97 out of 133 (73%), All: 114 out of 162 (70%).
Wrong answers: 48(27%).

Typical wrong steps in prospective teachers' responses:

(i) Careless errors: 13(8%)

Example: $x = \frac{18}{2} = 6$

(ii) $\frac{2(x-1)}{4} = 4 \Rightarrow 2(x-4) = 16$, 7(4%)

(iii) $\frac{2(x-2)}{4} + 2 = 6 \Rightarrow 2x - 2 = 4$, 4(2%)

(iv) $2x - 2 = 16 \Rightarrow 2x = 14$, 3(1%)

(v) $\frac{2(x-1)}{4} = 4 \Rightarrow 2(x-1) = 4 \div 4 = 1$, 3(1%)

(vi) Others: 18(11%).

The above-mentioned steps in the preservice teachers' responses reveal that some of them feel uneasiness when working with letters and symbols. They also have serious misconceptions in the interpretations and understandings of '=' symbol and of equation. It also appears that some of them are very weak in balancing techniques.

Item 4: If p is any number, write the number which is 25% bigger than p.

Correct answer: $\frac{5p}{4}$ or its equivalent forms.

Count correct (%):
Males: 23 out of 29 (79%), Females: 84 out of 133 (63%), All: 107 out of 162 (66%).
Wrong answers: 55(34%).

Typical wrong answers:

(i) p/4 or its equivalent forms, 17(11%)

(ii) p + ¼, 5(3%)

(iii) Letter evaluated: 10(6%)

e.g. answers such as 12.5, 25, 1.25, 125%, etc.

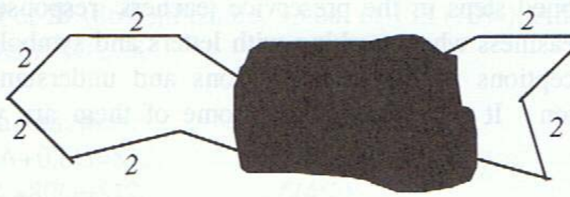
(iv) Other answers, for example: 18(11%)

$$125p, 1\frac{p}{4}, \frac{5p}{4} + p, 0.75p, 2.5p, 4p$$

(v) Not attempted: 5(3%).

These responses indicate that about one-third of the preservice primary teachers did not have a clear understanding of a variable. It appears from (iii) that they assumed an arbitrary numerical value for the letter p from the outset. This observation along with other wrong answers suggests that they still have numerical thinking rather than algebraic thinking. Also, many of them did not appear to have a good understanding of the concept of percent.

***Item 5**: What can you write for the perimeter of the following shape? Part of the shape is hidden, and there are n sides altogether, each of length 2 cm.*



Correct answer: 2n
Count correct (%):
Males: 17 out of 29 (59%), Females: 75 out of 133 (56%), All: 92 out of 162 (57%).
Wrong answers: 70(43%)

Typical wrong answers:

(i) Letter not used: 31(19%).
For example: 24 cm, 22 cm, 25 cm, 26 cm, and 46 cm, unknown.

(ii) Letter used as object or no meaning for it: 25(15%).
For example: 22+n, 22n+22, 22n-22, 22+2n, 10+2n, 2n cm^2, 2(22-n), 2n+x, etc.

(iii) Not attempted: 14(9%).

These wrong answers reveal that more than one-third of the preservice teachers either ignored the letter *n* or thought of it as a shorthand for an object or an object in its own right. They found it hard to see a relationship or pattern in the problem. Compare these findings with those of Kuchemann (1981) who, in fact, found that a majority of 13 to 15-year-olds were unable to cope with algebraic letters as unknowns or generalized numbers.

***Item 6:** Write an algebraic expression of the statement: "There are 8 times as many people in China as there are in England".*

Correct Answer: C=8E.
Count correct (%):
Male: 20 out of 29 (69%), Female: 65 out of 133 (49%), All: 85 out of 162 (52.5%).
Wrong answers: 77(47.5%)

Typical wrong answers:

(i)	Reversed equation: E=8C,	32(19.7%)
(ii)	An algebraic expression: 8E, 8C, 8C+E, E+8C or 8CE	32(19.7%)
(iii)	Just a ratio: 8E: E, 8C:C, 8C:E	4(2.5%)
(iv)	No meaning: $\frac{8}{E} = E$, $C = 8C$, $E = 8E$	3(1.9%)
(v)	Not attempted:	6(3.7%)

These findings indicate that about half of the preservice primary teachers had difficulties in translating the word problem into algebraic equation. They viewed the 'equals' as a kind of correspondence indicator and used the variables C and E as labels rather than thinking through the equality. The responses also reveal that about 20% of the errors were reversals: E = 8C (or an algebraically equivalent statement) instead of C = 8E where C = number of people in China and E = number of people in England. Surprisingly, many of those who got full or partial credit for this item wrote their answers as "1 China = 8 England" instead of 1C = 8E or C = 8E. It is important that preservice teachers should be more careful about translation, e.g. 8E means "8 times the number of people in England".

Item 6 was first used by Clement (1982) who analyzed thought processes underlying a common misconception of 15 freshmen. Our findings are quite similar to Clement. In a similar study for an introductory college engineering class, 37% of the students could not write an equation expressing the idea that a college has six times as many students as professors (Clement, 1982; Clement, Lockhead, & Monk, 1981).

***Item 7**: In a school, there are two thirds as many girls as boys. Write an equation for the number of girls in terms of boys.*

Correct answer: 2B = 3G or any of its equivalent forms.
Count correct (%):
Male: 14 out of 29 (48%), Female: 46 out of 133 (35%), All: 60 out of 162 (37%).
Wrong answers: 102(63%).

Typical wrong answers:

(i) An algebraic expression: 34(21%)

$$\frac{2}{3}G,\ \ \frac{2}{3}B$$

(ii) Reversal: 2G=3B 23(14%)

(iii) Just a ratio/fraction: 5(3%)

$$\frac{2}{3}:1, 2:1, \frac{2}{3}, \frac{2}{3}B^2$$

(iv) No meaning: 30(19%)

$$2G + G = B, \frac{2}{3}B + B, B = B + \frac{2}{3}, \frac{2}{3}G = \frac{1}{3}B, G = \frac{5}{3}B, etc.$$

(v) Not attempted: 10 (6%).

A majority of these preservice teachers could not see that 3G stands for "3 times the number of girls". In fact, many of those who got full or partial credit for this item wrote their answers as: "2 Boys = 3 Girls" instead of "2B = 3G", where B = number of boys and G = number of girls. These findings also reveal that the prospective teachers had difficulties in proportional thinking. It follows from their typical responses in (iv) that there is a strong tendency for the prospective teachers to write a 'total' expression for 'there are … times as many …' statement. Moreover, a majority of the them had difficulties in dealing with the '='sign and equality. Kuchemann (1978), Kaur (1994), and Koay (1994) have also discussed a number of misconceptions concerning the meaning of algebraic equations.

Item 8: Write an equation using the variables V and N to represent the following statement: "At a Pizza shop, for every three people who ordered vegetarian pizzas, there were seven people who ordered non-vegetarian pizzas." Let V represent the number of vegetarian pizzas and N the number of non-vegetarian pizzas ordered.

Correct Answer: 3N=7V or any of its equivalent form.
Count Correct (%):
Male: 3 out of 29 (10%), Female: 16 out of 133 (12%), All: 19 out of 162 (18%).
Wrong answers: 143(88%).

Typical wrong answers:

(i)	Reversal errors: 3V=7N,	71(44%)
(ii)	Reversal and translation errors: 3V+7N=P, 3V=7P, 3N=7N, 3V=7V, P= # People.	28(17%)
(iii)	Just a ratio/fraction: 3V:7N,	20(12%)
(iv)	Others: N=3V, N-V=40, $3\frac{V}{10}+\frac{7}{10}N$, N+V=10.	12(7%)
(v)	Not attempted:	12(7%)

Out of the eight items in the test, this item appears to be most difficult for the students. We find that 44% of the students made errors of reversals: 3V = 7N (or an algebraically equivalent statement) instead of 3N = 7V. Interestingly, reversal errors in Item 6 and 7 were about 20% and 14%, respectively. A similar item was given by Clement (1982) to 150 freshman-engineering students and he found that reversals were as high as 73%. In fact, he also found that these reversal errors are not a careless mistake but a strong misconception that had developed in them over a period of time.

After having analyzed the patterns of errors in Item 8 made by the students, we agree with Clement (1982) that most of their errors were due to a difficulty in translating words to algebraic equations rather than a difficulty with simple algebraic manipulation skills or with simple ratio reasoning. However, some of the students may also have difficulty in ratio and proportionate thinking.

Results

Based on the level descriptions defined earlier in the section on Method, 162 test papers were classified into three groups. These groups were modified until they were judged to characterize the range of typical responses for each level. This helped us to characterize their algebraic thinking according to the level as follows:

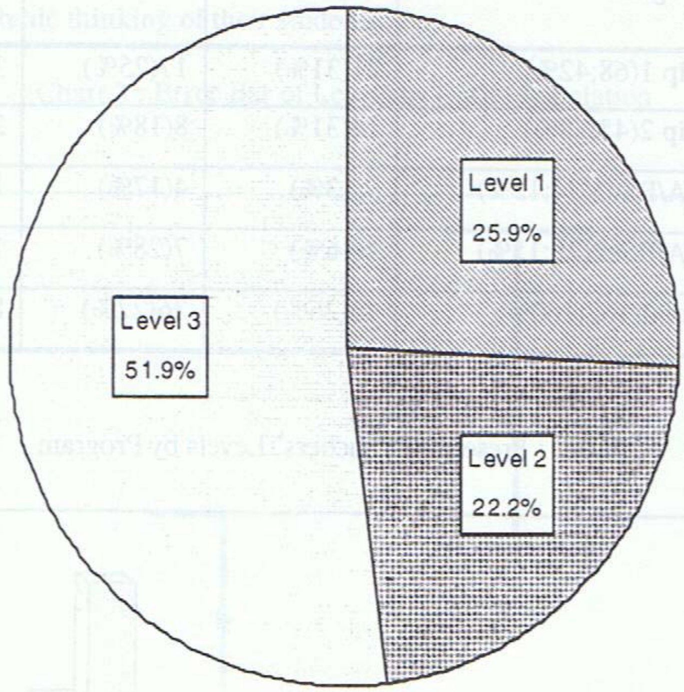
Level 1: Pre-algebraic thinking

The preservice teachers (25.9%) in this group had serious difficulties in grasping the notion of letter as a generalized number. They ignored the letter, or at best, acknowledged its existence but without giving it a meaning. Some of them gave an arbitrary value to the letter or used it as a shorthand for the name of an object. They found it hard to see relationships or patterns even in simple problems. Such preservice teachers faced serious difficulties in translating a word problem into an equation. They lacked confidence in basic arithmetic skills in topics such as fractions, decimals, ratio and proportion. However, some of them were able to use letters as unknowns in, say, equation solving and other routine problems, e.g. *Item 1 or Item 3.*

Level 2: Concrete semantic algebraic thinking

The preservice teachers who were classified at this level (22.2%) could use letters as specific unknowns only when the item-structure was simple. They could observe a pattern or get a relationship only in simple problems. They might have developed sound numerical thinking because they could work out numerical examples and conclude from those examples. For example, most of them could solve Items 1-3 without any error. Some of them could also work out Items 4 and 5. But, they could not consistently cope with specific unknowns, generalized numbers or variables. They were not able to cope with a problem, which had a complex structure (e.g. *Items* 6-8). It seemed that these students were still at the level of early formal algebraic thinking.

Chart 1: Preservice Teachers' Levels of Algebraic Thinking



Level 3: Abstract semantic algebraic thinking

The preservice teachers who were grouped at this level (51.9%) have had developed formal algebraic thinking. They could see a letter as a variable. They could visualize a letter representing a range of unspecified values. They seemed to believe that arithmetic could be generalized. They have had experience in using algebraic symbolism as a tool with which to think about and to express general relations. For instance, they could cope with problems such as *Items* 7 and 8. They could see relationships or patterns in problems of the type *Item 5*. In fact, most of them could solve at least seven items in the test, except for a few minor errors.

Table 1: Preservice Teachers' Levels by Program

Program	Level 1	Level 2	Level 3
Dip 1(68;42%)	21(31%)	17(25%)	30(44%)
Dip 2(45;28%)	14(31%)	8(18%)	23(51%)
BA/BSc1(24;15%)	3(13%)	4(17%)	17(71%)
BA/BSc2(25;15%)	4(16%)	7(28%)	14(56%)
Total(162;100%)	42(26%)	36(22%)	84(52%)

Chart 2 : Preservice Teachers' Levels by Program

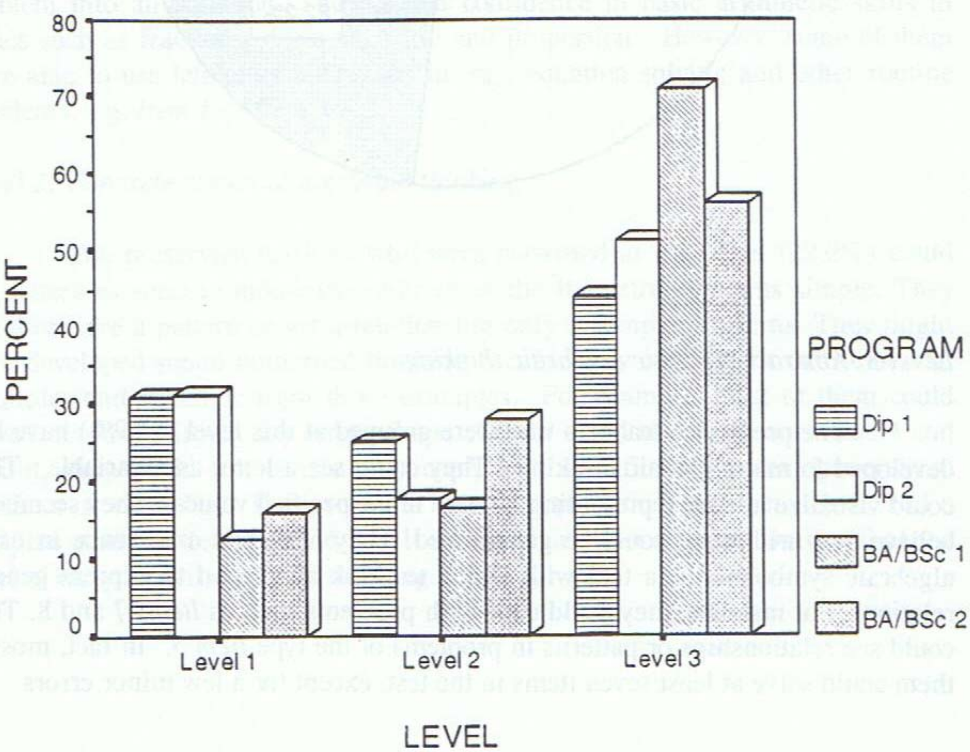
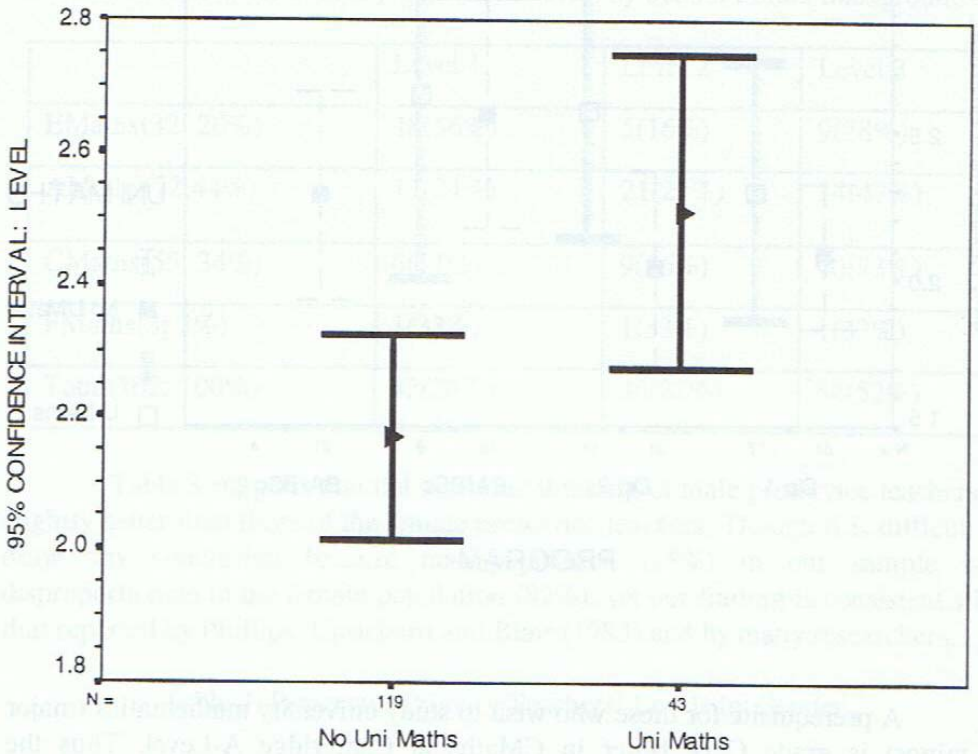


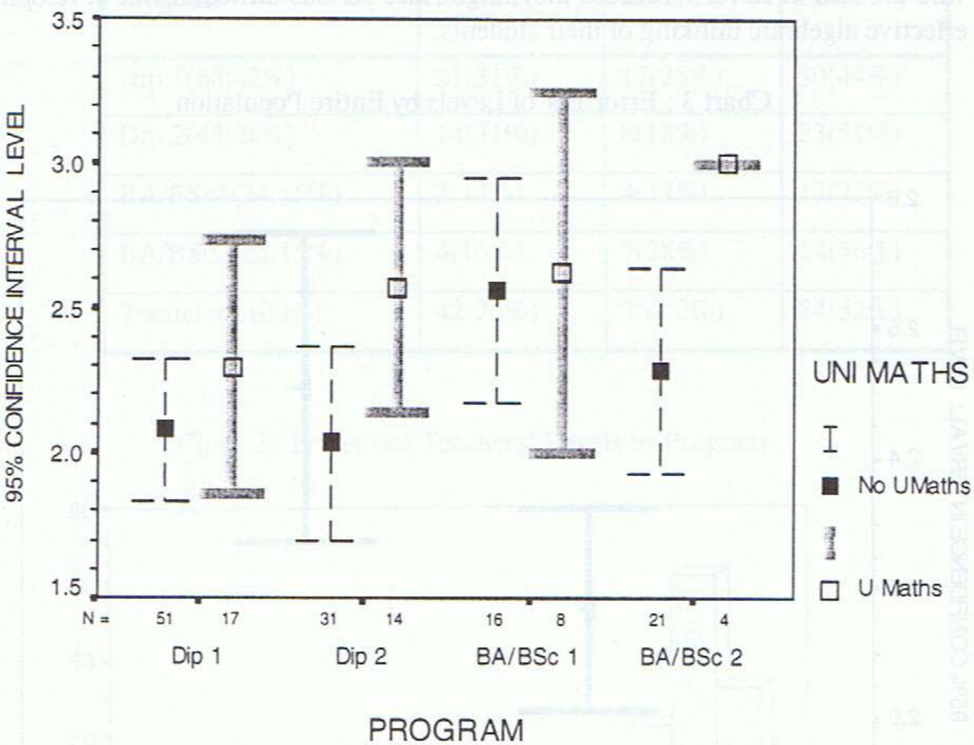
Chart 1 shows that only half of the preservice teachers have had Level 3 algebraic thinking. A cause of concern is one-fourth of the prospective teachers who are still at Level 1 because they might face serious difficulties in developing effective algebraic thinking of their students.

Chart 3 : Error Bar of Levels by Entire Population



Both Table 1 and Chart 2 indicate that the performance of BA/BSc subgroup was far better than those of Diploma subgroup. This was expected because of the higher entry requirements for preservice primary teachers in the BA/BSc program than those in the Diploma program. However, it was surprising to notice that the algebraic thinking of BA/BSc 1 subgroup (with 71% at Level 3) was far better than those of BA/BSc 2 subgroup (with 56% at Level 3).

Chart 4 : Levels by Programs and University Maths



A prerequisite for those who wish to study university mathematics (major or minor) is grade C or better in CMaths at Cambridge A-Level. Thus the preservice primary teachers with university mathematics are expected to be better in algebraic thinking than those who do not or cannot take university mathematics. The error bars in Chart 3 confirm this fact.

In particular, Chart 3 reveals that 95% CI level and 5% trim for preservice primary teachers with university mathematics in all programs are (2.28, 2.75) and 2.57, respectively, against those of (2.01, 2.32) and 2.19 for those without university mathematics. This shows that those who study university mathematics develop a higher level of algebraic thinking than those who study mathematics only up to their Cambridge O or A-level.

Since the size of the subgroup of preservice primary teachers with FMaths is just 2% of the population, this subgroup will be ignored for further consideration. Thus, except for FMaths, Table 2 and Chart 5 confirm that the algebraic thinking of a preservice teacher depends on his/her background in school mathematics. For example, 73% of those who did CMaths were classified at Level 3 as compared to 47% with AMaths and 28% with EMaths at Level 3.

Table 2: Preservice Primary Teachers' Levels by School Maths Background

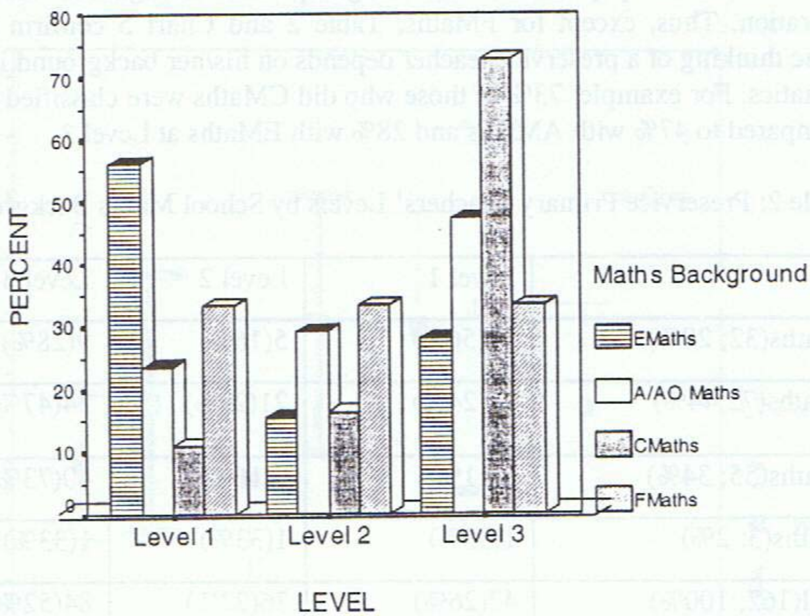
	Level 1	Level 2	Level 3
EMaths(32; 20%)	18(56%)	5(16%)	9(28%)
AMaths(72;44%)	17(24%)	21(29%)	34(47%)
CMaths(55; 34%)	6(11%)	9(16%)	40(73%)
FMaths(3; 2%)	1(33%)	1(33%)	1(33%)
Total(162; 100%)	42(26%)	36(22%)	84(52%)

Table 3 suggests that the algebraic thinking of male preservice teachers is slightly better than those of the female preservice teachers. Though it is difficult to draw any conclusion because male-population (18%) in our sample was disproportionate to the female-population (82%), yet our finding is consistent with that reported by Phillips, Uprichard and Blair (1983) and by many researchers.

Table 3: Preservice Primary Teachers' Levels by Gender

Sex	Level 1	Level 2	Level 3
Males(29;18%)	6(21%)	6(21%)	17(59%)
Females(133;82%)	36(27%)	30(23%)	67(50%)
Total(162;100%)	42(26%)	36(22%)	84(52%)

Chart 5: Preservie Teachers' Levels by Maths Background



Conclusions and Implications

In order to develop and stimulate algebraic thinking of Primary1-6 children, it is essential that all preservice primary teachers should have a minimum of Level 3 algebraic thinking. On the other hand, this study shows that about half of the prospective teachers surveyed are at Level 1 or Level 2. The findings reveal that a majority of these future teachers cannot see algebra as a generalized arithmetic. In fact, lack of understanding of variable is the most important factor for these prospective teachers. They interpret algebraic expressions incorrectly. They believe that letters in algebraic expressions stand for names of things (rather than for numbers). For example, a majority of Level 1/Level 2 preservice teachers visualized that the letter 'C' (in *item* 6) stands for "China" rather than say that 'C' stands for "number of people in China". They face serious difficulties in translating a word problem into an equation. They find it hard to see patterns or relationship in a problem. It appears that as secondary school students, these preservice teachers could not develop their own algebraic thinking. We, therefore, agree with Kuchemann (1981) who found that many secondary school students learn manipulation rules without reference to the meanings of the expressions being manipulated.

It is a matter of concern that when those who are not operating at Level 3 become teachers, they may face several barriers in developing and stimulating algebraic thinking of their pupils. The teacher educators should help their preservice teachers to develop their knowledge base related to algebra. They should be engaged in exploring traditional algebraic concepts and procedures from new perspectives. In order to help those who are at Level 1 or Level 2, there should be an activity-based course in algebraic thinking. They should be given enough experience at using algebraic notations. Information technology should also be used to provide them a deeper understanding of algebraic concepts and develop algebraic reasoning. There should be more emphasis on the importance of problem solving and its relationship of the teaching of higher order thinking skills.

The findings further reveal that many preservice teachers have difficulties in pre-algebraic concepts, such as fractions, decimals, percentage, ratio and proportionate thinking. They should, therefore, be provided opportunity to develop sound numerical thinking and conceptual understanding of pre-algebraic concepts by using concrete materials, pictorial representations, and a calculator.

Notable in the study is the trend for BA/BSc 1 subgroup to outscore Diploma or even BA/BSc 2 subgroup. It should be interesting to find out why the performance of BA/BSc 2 subgroup was less than those of BA/BSc 1. However, the algebraic thinking of preservice teachers in Diploma is a cause of concern because a majority of them do not operate at Level 3.

This study proves that the algebraic thinking depends on the level of mathematics studied in the school prior to joining a preservice program. About seventy-five percent of those preservice teachers who studied only EMaths at their Cambridge O-Level were operating at Level 1 or Level 2. On the other hand, about seventy-five percent of those who studied CMaths at their Cambridge A-Level have had Level 3 algebraic thinking. These findings, therefore, reveal that those who did not (or were not allowed to) take CMaths in junior colleges have weak conceptual backgrounds in algebra. This observation implies that CMaths (with grade C or better) should be one of the admission requirements for all preservice primary teachers. Alternatively, prospective teachers with EMaths or AMaths as background should be provided an opportunity to upgrade their knowledge base in mathematics.

Our study also suggests that the preservice primary teachers should be encouraged to take university mathematics. All those who have weak backgrounds in mathematics should be motivated to take remedial mathematics followed by some modules on number theory, algebra, geometry, calculus, and statistics. In particular, they should be helped to raise their level of confidence as algebraic thinkers, so that they, in turn, could develop and enhance algebraic thinking of their pupils.

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